

A method for high precision normalization of cross sections

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(Recibido el 29 de noviembre de 1988; aceptado el 3 de abril de 1989)

Abstract. A method is devised in which a minimum of three monitors must be used to eliminate the dependence on equipment-alignment and beam focusing conditions in normalizing cross sections. A 4-monitor system is designed which in addition allows us to account for the possible error sources. As an example of the technique, an analysis of a series of experimental data is made. A typical precision of 1% in the corresponding normalization factors was obtained. The superiority over methods using 1 or 2 monitors is demonstrated theoretically and illustrated experimentally.

PACS: 29.90.+r; 25.70.-z

1. Introduction

The measurement of nuclear reaction cross sections is probably the main way to obtain experimental information about nuclei. The more precise a measurement is, the higher will be its probability of making a relevant contribution to the understanding of the studied phenomenon. It is thus important to have a good means of minimizing errors in the measurement of quantities such as integrated beam intensity, Q , and target thickness, t , which affect the absolute normalization of the cross section.

For a wide range of systems and energies, reaction cross sections can be normalized by measuring, simultaneously with the reaction products of interest, the elastic yield at some forward angle where Coulomb scattering is certain to dominate. Comparison with Rutherford formula gives then the desired normalization. The usefulness of this method, in which the product Qt is directly determined, has been recognized for a long time [1,2] and is particularly suitable for measuring heavy-ion induced reactions, for which the technique of charge collection for measuring Q is usually unreliable. In case that the energies of interest are above the Coulomb barrier, the method can still be used to get a relative normalization if a high- Z

*Partially supported by CONACYT under contract 140105 G102-139.

**Partially supported by NSF under contract INT86-02209.

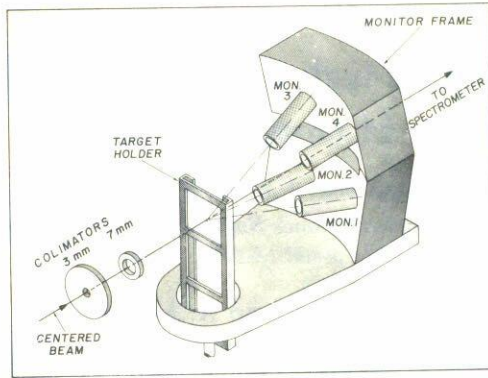


FIGURE 1. Experimental arrangement.

element is added to the target [3]. In a related application, angular distributions of elastic scattering in heavy targets have been used to check the alignment of the experimental arrangement [4].

One or, at most, two fixed-angle detectors are normally used to monitor the beam for normalization purposes. As we show here, the fast variation of the Rutherford cross section at small angles makes the results obtained with such configurations strongly dependent on equipment-alignment and beam-focusing conditions, especially for the most widely used case of one single detector. We report in this work on a simple system of four fixed-angle monitors which eliminates this strong dependence and, with the help of a computer code, permits substantial improvement in the determination of the normalizing factor Qt . Experimental results obtained for a variety of combinations projectile-target at different energies are presented.

2. Experimental method

The experimental device consists of four Silicon surface-barrier (SSB) detectors placed symmetrically at $\theta_m = 15^\circ$ with respect to the nominal beam direction (Fig. 1). The four detector holder is a rigid frame designed in such a way that even an error of 0.5° in the building tools would produce an error of less than 0.1° in θ_m . For each detector, a 0.71 mm collimator placed at 51 mm from the target center defines the corresponding solid angle. The whole frame can be horizontally rotated around the target and must be optically aligned with the beam-line axis. Two collimators in front of the target are used to define the beam, which is focused by minimizing the current in the first collimator and simultaneously maximizing it at a beam collector placed behind the target. By following this procedure several times while observing a quartz crystal placed in the beam path, we were able to estimate the uncertainty in the beam location on the target.

This device has been used in a recoil velocity spectrometer to normalize data for

projectile	target	target thickness ($\mu\text{g}/\text{cm}^2$)	process measured	laboratory energies (MeV)
^{27}Al	^{70}Ge	587	FUSION	72,73,...,85
^{27}Al	^{72}Ge	565	FUSION	68,69,...,85
^{27}Al	^{74}Ge	226	FUSION	68,69,...,85
^{27}Al	^{76}Ge	386	FUSION	67,68,...,85
^{35}Cl	^{58}Ni	187	FUSION	100.33
^{37}Cl	^{58}Ni	187	FUSION	94,95,...,108
^{37}Cl	^{60}Ni	220	FUSION	93,94,...,108
^{37}Cl	^{62}Ni	63	FUSION	91,92,...,108
^{37}Cl	^{64}Ni	160	FUSION	89,90,...,108
^{81}Br	^{60}Ni	220	ELASTIC	42,45
^{103}Rh	^{60}Ni	220	ELASTIC	36,39,42

TABLE I. Experiments performed using the normalization method described in this work.

elastic scattering and sub-barrier fusion measurements for several systems [5,6]. The results obtained for the normalization of these data will be analyzed in Section 4. The experiments, described in Table I, were performed with beams from the 3-stage accelerator at the University of Notre Dame. The target thicknesses reported in Table I were obtained in a separate experiment by measuring sub-Coulomb scattering of ^{16}O beams on each target and were used only for energy loss calculations.

The elastic scattering of Br and Rh was measured at the spectrometer angle $\theta = 10^\circ$. For all the experiments, several fixed-energy measurements were made for some of the energies listed in Table I, either because an angular distribution was being measured or because the transmission of the spectrometer was being studied. All these data points will be included in the analysis of Section 4.

3. Theoretical analysis

For scattering of a projectile of energy E at an angle θ in the laboratory reference system, the Rutherford cross section can be written as

$$\sigma_R(\theta) = (1 + \gamma)^2 \left[\frac{Z_p Z_t e^2}{4E} \right]^2 \frac{(1 + \gamma^2 \cos 2\theta)(1 - \gamma^2 \sin^2 \theta)^{-1/2} + 2\gamma \cos \theta}{\sin^4 \left(\frac{\theta + \sin^{-1}(\gamma \sin \theta)}{2} \right)}, \quad (1)$$

where $\gamma = (A_p/A_t)$ and $Z_{p,t}$, $A_{p,t}$ are the atomic and mass numbers of projectile and target, respectively.

We choose a coordinate system in which the target center is at the origin; the z -axis points in the nominal beam direction (the geometric axis of the beam-line) and the x - y plane coincides with the plane of the target, the y -axis pointing downwards (Fig. 2). The scattering angle, θ_i , into the i^{th} monitor, located at point (x_i, y_i, z_m) , depends on the coordinates (x_0, y_0) of the beam spot on the target, and on the

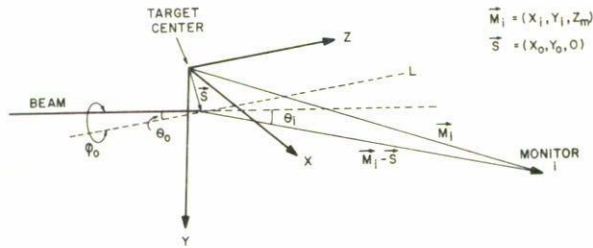


FIGURE 2. Coordinate system used. M_i defines the position of the i^{th} monitor and S determines the position of the beam spot on the target. The auxiliary line L , parallel to the z -axis, is drawn to define the beam direction, (θ_0, φ_0) .

direction (θ_0, φ_0) of the beam, through the expression

$$\theta_i(x_0, y_0, \theta_0, \varphi_0) = \cos^{-1} \left[\frac{(x_i - x_0) \sin \theta_0 \cos \varphi_0 + (y_i - y_0) \sin \theta_0 \sin \varphi_0 + z_m \cos \theta_0}{[(x_i - x_0)^2 + (y_i - y_0)^2 + z_m^2]^{1/2}} \right]. \quad (2)$$

On the other hand, if Y_i is the yield of particles elastically scattered into monitor i , the experimental cross section is given by

$$\sigma_{\text{exp}}(Qt, \Omega_i) = \frac{Y_i}{Qt\Omega_i}, \quad (3)$$

where Ω_i is the solid angle seen by the monitor at the beam spot position. Calling A_i the area of the corresponding collimator, Ω_i can be expressed as

$$\Omega_i(x_0, y_0) = A_i \left[\frac{x_i(x_i - x_0) + y_i(y_i - y_0) + z_m^2}{(x_i^2 + y_i^2 + z_m^2)^{1/2} [(x_i - x_0)^2 + (y_i - y_0)^2 + z_m^2]^{3/2}} \right]. \quad (4)$$

For the case of dominant Coulomb scattering, we thus have

$$\sigma_{\text{exp}}(Qt, \Omega_i(x_0, y_0)) = \sigma_R(\theta_i(x_0, y_0, \theta_0, \varphi_0)). \quad (5)$$

Clearly, at least five detectors are necessary in order to solve for all the unknowns. Although this can be done [7], we will show here that for the range of θ_0 values of interest, it is sufficient to assume $\theta_0 = 0^\circ$ since any departure from this value can be compensated for by a shift in (x_0, y_0) , the Qt value remaining essentially unchanged. This would tell us that, in principle, three detectors should be enough to determine Qt . However, as we will show later, it is convenient to use a fourth detector in order to have a reliable determination of the associated uncertainty. With this in mind, we will analyze from the beginning the case of four detectors, which includes also the 3-detector case.

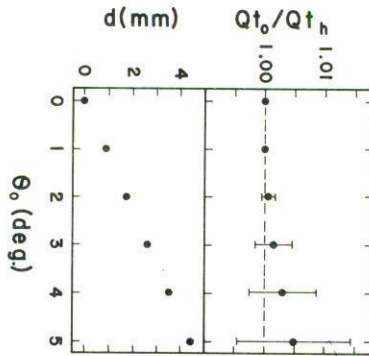


FIGURE 3. (Up) Ratio of calculated to hypothetical Qt as a function of the beam inclination. Yields at the monitors were numerically generated for $^{103}\text{Rh} + ^{60}\text{Ni}$ at 39 MeV with x_0, y_0, φ_0 and Qt equal to 0, 0, 30° and 100 mb^{-1} , respectively. (Down) Beam spot shifting, d , that our method uses to compensate for the beam inclination θ_0

In order to show that the assumption $\theta_0 = 0$ gives a good approximation, we have made a numerical simulation of different conditions of the incident beam. Yields are generated for each of the 4 monitors described in Section 2 by assuming hypothetical values for Qt , x_0 , y_0 , θ_0 , φ_0 , and using formulas (1) to (5). These yields are then taken as “experimental” values and used to establish equation (5) with $\theta_0 = 0^\circ$ (which also causes φ_0 to disappear) for each monitor. These equations are simultaneously solved for x_0 , y_0 , Qt and this last quantity is compared to the hypothetical value of Qt .

We actually solved, for each hypothetical beam, four 3-equation systems, each one corresponding to a different triplet of successive monitors (Fig. 1) and took the mean, Qt_0 , of the four Qt values so obtained as the final result; x_0 and y_0 were also determined in this way. The corresponding sample standard deviation, s , is taken as an estimation for the error at this stage. Note that this way of estimating the error makes sense only for four (or more) detectors and one can expect that s will be a measure of the goodness of our approximation. In addition, for actual experimental data, s will also include the effects of possible unknown error sources. In fact, this is the main reason for our using 4 monitors instead of 3. The treatment of errors for experimental data will be discussed in detail later. The procedure described is carried out by means of the code DETECT, which uses the method of Ref. [8] for solving systems of non-linear equations.

The upper part of Fig. 3 shows, for a typical case, the ratios of calculated to hypothetical Qt as a function of θ_0 . We see that even a beam inclination as large as 5° is compensated for with good precision in this method, the deviation in Qt being of only 0.5% for this case. Furthermore, the actual values are always well within the error bars so that our method safely takes into account possible deviations due to beam inclination effects. The lower part of Fig. 3 shows the shift, d , from the original to the calculated beam spot position, which the method has to assume in order to compensate for the given beam inclination. It is interesting to note that,

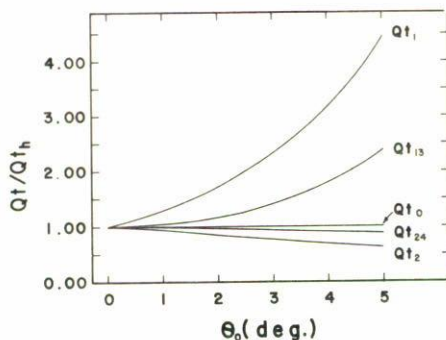


FIGURE 4. Comparison of Qt ratios (calculated to hypothetical) generated from 1, 2 or 4 detectors, as a function of the beam inclination, for the same case of Fig. 3. The subindex i , ij or 0 in Qt indicates that the Qt values were determined using monitor i , monitors i and j or the four monitors, respectively.

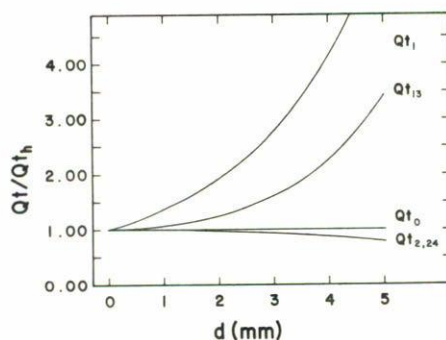


FIGURE 5. Comparison of Qt ratios (calculated to hypothetical), obtained from 1, 2 or 4 detectors, as a function of the distance beam spot-origin. The simulation corresponds to the same system of Fig. 3, but here $\theta_0 = 0$ and the beam spot was moved along the $x_0 = y_0$ line toward monitor 1. The labeling of curves is the same as in Fig. 4.

even though the hypothetical values x_0 , y_0 , φ_0 were all varied, this plot remained unchanged for all cases studied. This seems to indicate that for a given θ_0 the compensating shift is always the same, no matter what the beam spot position or the azimuthal direction of the beam. The orientation of the shift, of course, changes with these parameters.

To discuss the case of three detectors, we first note that, as mentioned before, there is no way of assigning error bars to the corresponding points in Fig. 3 (not shown). However, the calculated results are still very close to the hypothetical Qt (the maximum deviation is only 1.7% for $\theta_0 = 5^\circ$).

The power of the 4-monitor method is further illustrated in Figs. 4 and 5, where a comparison is made with the results when using only one or two monitors. In Fig. 4 the beam inclination is varied, while all other parameters are kept fixed. The superiority of the 4-monitor method is quite obvious in this figure, specially for

large θ_0 . For a reasonably well aligned system, however, we wouldn't expect beam inclinations larger than about 0.2° or 0.3° . If we assume that the combined effect of the chamber alignment and detector alignment could give a maximum angular error of 0.5° , this would produce an uncertainty of about 14% in the normalization if only one monitor was used. By doing the alignment very carefully, though, it is feasible to reduce appreciably this uncertainty.

More uncontrollable is the effect of a shift in the beam spot position with respect to the center of the target, which is illustrated in Fig. 5. By following the procedure described in Section 2, we estimated that our focusing method (which we consider a good method), can locate the beam at the target with an uncertainty of about ± 1 mm. According to Fig.5, this would produce a maximum error of about 36% or 6% in the normalization if only one or two detectors were used, respectively. For higher shifts, these errors grow up very steeply while the 4-monitor method gives always exact results.

As for the 3-detector case, the corresponding curves in Figs. 4 and 5 are essentially the same as the ones corresponding to Qt_0 , the 4-detector calculation. So, the conclusions drawn in the two previous paragraphs for the 4-detector method are also valid for three detectors.

Summarizing, the generalized use of one or two monitor systems can not account for the effects of beam inclination or beam spot shifting. This shortcoming is overcome by using three detectors. By adding a fourth detector, we can estimate the absolute uncertainty in Qt , including the effects of unknown error sources. Accordingly, in the analysis of experimental data we will restrict ourselves to the 4-monitor method and will compare only with the 1 and 2 detector results.

4. Analysis of experimental results

Before presenting the experimental results, a few words must be said about the corresponding analysis. Energy loss in the target was taken into account by using in (1) the average energy weighted by the Rutherford cross section. Explicitly,

$$E = E_i E_f \frac{\ln(E_i/E_f)}{(E_i - E_f)}, \tag{6}$$

where E_i is the lab bombarding energy and E_f is the beam energy after traversing the target, calculated according to Ref. 9.

As for the error analysis, we already showed that the standard deviation, s , corresponding to four determinations, properly accounts for errors coming from a possible beam inclination. The errors coming from counting statistics, however, are not well treated in s since the method always looks for the pair (x_0, y_0) that better solves equations (5) (with $\theta_0 = 0$) and this could in some cases compensate for statistical deviations in the yields, so underestimating the corresponding error.

In order to further investigate this, we studied the statistical behavior of the method by generating 10000 quartets of random numbers distributed according to

$\overline{Qt} \pm \text{s.d.}$	100.00 ± 0.18
$\overline{s} \pm \text{s.d.}$	0.16 ± 0.12
$\overline{e} \pm \text{s.d.}$	0.18 ± 0.00

TABLE II. Statistics of results from DETECT for a simulation of 10000 cases with statistical yields generated out from the hypothetical beam parameters $x_0 = y_0 = \theta_0 = 0$ and $Qt = 100 \text{ mb}^{-1}$. All quantities (defined in the text) are given in mb^{-1} .

a Gaussian around a mean yield of 78620, with a width equal to the square root of this number. This yield was calculated, in the way previously described, for $^{103}\text{Rh} + ^{60}\text{Ni}$ at 39 MeV by assuming $x_0 = y_0 = \theta_0 = 0$ and $Qt = 100 \text{ mb}^{-1}$. Each quartet was then used as an input for DETECT and the respective values of Qt and s were calculated.

The corresponding averages and standard deviations are presented in the first and second lines of Table II. Since the only source of error in these synthetic data comes from counting statistics, the large spread in s confirms our assertion that the statistical error is not well treated in s . This error may be estimated from the standard deviation of the mean yield, \overline{Y} , by the expression

$$e = \frac{Qt}{(N\overline{Y})^{1/2}} = \frac{Qt}{\left(\sum_{i=1}^N Y_i\right)^{1/2}}, \quad (7)$$

where N is the number of detectors, 4 in this case. This quantity was also calculated for each case and the corresponding average and standard deviation is given in the last line of Table II. Comparison with the first line shows that this is indeed a good estimation.

By assuming a maximum beam inclination of 0.3° , which is actually an overestimation for our experimental set-up, we were able to estimate an upper bound for the error due to this effect. To this end, we simulated beams at 0.3° for some typical systems from Table I, taking the minimum and maximum energies indicated for each case. The beam spot position was taken either at the origin or at $d = 2 \text{ mm}$ from it and φ_0 was varied from 0° to 45° for each case, using always $Qt = 10 \text{ mb}^{-1}$. The calculated error, s , proved to be essentially independent of the particular system and energy, with a weak dependence on φ_0 and a stronger dependence on d . The maximum error, corresponding to $x_0 = y_0 = 1.414 \text{ mm}$ and $\varphi_0 = 45^\circ$, was of only 0.05%.

By quadratically adding this to e from (7), we obtain an estimation for the error in Qt that includes effects from both, counting statistics and beam inclination. The maximum between this and the s calculated from the experimental data, which might include effects from other error sources not treated explicitly, was finally taken as the experimental error, δQt .

In applying our normalization method to the experiments of Table I, we must be cautious since some of the energies there indicated are slightly above the Coulomb

system	No. runs	\bar{x}_0 (mm)	\bar{y}_0 (mm)	$\overline{\delta Qt}$ 4 det. (%)	$\overline{\delta Qt}$ 2 det. (%)	$\overline{\delta Qt}$ 1 det. (%)
$^{27}\text{Al} + ^{70}\text{Ge}^*$	26	-2.67 ± 0.32	-0.60 ± 0.15	0.95	30.51	47.42
$^{27}\text{Al} + ^{72}\text{Ge}^*$	34	-2.66 ± 0.15	$-.042 \pm 0.31$	0.54	25.32	54.43
$^{27}\text{Al} + ^{74}\text{Ge}^*$	31	-3.05 ± 0.33	-0.67 ± 0.24	1.21	40.38	53.47
$^{27}\text{Al} + ^{76}\text{Ge}^*$	28	-2.76 ± 0.19	-0.57 ± 0.26	0.73	30.88	50.49
$^{35}\text{Cl} + ^{58}\text{Ni}$	32	-0.34 ± 0.21	-0.82 ± 0.21	0.99	4.94	20.51
$^{37}\text{Cl} + ^{58}\text{Ni}$	83	-0.81 ± 0.70	-0.18 ± 0.40	0.74	3.38	19.44
$^{37}\text{Cl} + ^{60}\text{Ni}$	28	-0.12 ± 0.62	-0.25 ± 0.54	0.81	2.85	16.75
$^{37}\text{Cl} + ^{62}\text{Ni}$	33	-0.77 ± 0.22	-0.43 ± 0.26	1.81	3.79	21.91
$^{37}\text{Cl} + ^{64}\text{Ni}$	52	-0.93 ± 0.92	-0.16 ± 0.41	1.38	5.22	24.65
$^{81}\text{Br} + ^{60}\text{Ni}$	30	-0.63 ± 0.05	-0.54 ± 0.30	0.79	3.37	28.41
$^{103}\text{Rh} + ^{60}\text{Ni}$	52	-0.96 ± 0.16	-0.20 ± 0.28	0.49	3.46	28.18

* The monitor frame was slightly rotated with respect to the beam.

TABLE III. Statistics of experimental results for the systems of Table I. \bar{x}_0, \bar{y}_0 are the average coordinates of the beam spot. the last 3 columns give the average error in Qt as obtained from 4, 2 or 1 detector, respectively.

Barrier. However, optical model calculations indicate that the corresponding deviations from Rutherford scattering are negligible and therefore we will apply our method to all cases without any further consideration.

The main features of the normalization procedure for the experiments of Table I are summarized in Table III. All quantities here refer to an average over the total number of runs measured for each experiment. With the exception of the $^{37}\text{Cl} + ^{58,64}\text{Ni}$ systems, whose respective measurements were made in separate occasions, all the runs for each system were measured during the same experimental session, with the same alignment and similar focusing conditions. The rather small spread obtained for x_0 and y_0 indicates that with our focusing method we can actually locate the beam on target with more precision (≈ 0.4 mm) than the 1 mm estimated with the quartz viewer. The sample standard deviation for the four determinations of x_0 and y_0 , calculated for each experimental run, was typically of the order of 10^{-2} mm, which shows the consistency of the method. The rather large negative values of x_0 obtained for the Al + Ge systems are mostly due to a slight misalignment of the monitors which we actually noticed on opening up the chamber after completing the experiments.

The fifth column of Table III gives the typical error in Qt obtained for each system. The low values obtained, of the order of 1%, illustrate the high precision of the method. Notice that the mentioned misalignment for the Al + Ge experiments had no effect here, the corresponding errors being comparable to the rest. It is worth mentioning that, for the vast majority of experimental runs, the dominant error was the standard deviation associated to the four determinations (one for each triplet of monitors), which we have called s , rather than the error calculated from the combined effects of counting statistics plus beam inclination (see paragraph below (7)). Thus, our usage of a fourth detector turns out to be essential for the uncertainty determination.

For the purpose of comparison, columns 6 and 7 give the typical maximum errors for the case of using 2 or 1 detector, respectively. These errors were estimated from the corresponding deviations with respect to Qt_0 , the 4-detector result, and represent the maximum of the averages obtained for monitors (1,3) and (2,4), or 1 and 2, respectively. As expected, for the Al + Ge experiments both methods give large errors, which in addition differ from each other, within the same column, by up to 15% or 5% for the 1-detector or the 2-detector method, respectively. This provides experimental evidence that these methods, in contrast to ours, are highly sensitive to alignment and/or focusing conditions, as predicted in Section 3. For the remaining systems, the two-detector method gives pretty good results, with a maximum error of about 5% while the one-detector calculation still gives too large errors. The superiority of the four-detector method is evident for all cases.

Summarizing, the analysis of a large sample of experimental data allows us to conclude that, using our particular design of a 4-monitor system, the method typically gives 1% precision in the normalization factor, independently of the particular conditions of equipment alignment and beam focusing.

5. Conclusions

A 4-monitor system and an associate computing code have been designed to determine the normalizing factor, Qt , in the measurement of nuclear cross sections. A detailed numerical simulation was made which shows that, in principle, three detectors suffice to deduce Qt with good precision, even though five parameters need to be determined in the most general situation. In practice, a 3-monitor system can be used and it will give much better results than using only one or two monitors, but an independent way of estimating the uncertainty must then be devised. We used a fourth detector, which allowed us to obtain a reliable estimation for this uncertainty. It was theoretically proved that, in contrast to the usual method where only one monitor (or the less usual one with two monitors) is used, our method gives results which are stable to variations in the equipment-alignment or beam-focusing conditions.

The gained stability is clearly displayed by the statistics of experimental data presented in Table III, where the errors corresponding to the 1 and 2 monitor methods are excessively large for the first 4 systems, while those corresponding to four monitors remain always around 1%. As the table shows, the typical precision of our system is about 23 (4) times better than that of the 1 (2)-monitor method under reasonably good alignment conditions.

The method developed here is not limited to the field of nuclear physics but can also be applied, for instance, to the measurement of atomic cross sections. All that is needed is for the bombarding energy not to exceed too much the corresponding nuclear Coulomb barrier.

Acknowledgments

We wish to thank G. Murillo, R. Tigh, R. Krieger and S. Graaf for their help in data taking.

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Resumen. Se ideó un método en el cual al menos tres monitores deben ser usados para eliminar la dependencia de las condiciones de alineación del equipo y de enfoque del haz en la normalización de secciones eficaces. Se diseñó un sistema de 4 monitores el cual además permite tomar en cuenta las posibles fuentes de error. Como un ejemplo de la técnica, se hizo un análisis de una serie de datos experimentales. Se obtuvo una precisión típica de 1% en los correspondientes factores de normalización. La superioridad sobre métodos que usan 1 o 2 monitores se demuestra teóricamente y se ilustra experimentalmente.