Phase shifting interferometry*

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Abstract. Interferometric methods permit us to measure the shape of an optical surface with high accuracy, using the wavelength as a unit of length. On the other hand, spherical surfaces are the most common, but aspherical surfaces *i.e.*, not spherical, are more and more popular due to their great design advantages. Aspherical surfaces generally have the shape of a conic of revolution. The procedure to determine the quality of an aspherical surface is not an easy one. However, these methods are absolutely necessary in order to manufacture the very precise modern optical surfaces.

Phase shifting interferometry started about ten years ago. It has many great advantages with respect to the traditional methods, mainly when it's applied to the testing of optical surfaces. Here, we present a review of these advances.

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1. Introduction

Optical surfaces are generally spherical, but aspherical surfaces, *i.e.* not spherical, are becoming more and more frequently used, because of their great design advantages. The reason is that one aspherical surface can eliminate or reduce aberrations as well as three or four spherical surfaces do. In most cases an aspherical surface is a conic of revolution, like a paraboloid, hyperboloid or ellipsoid.

On the other hand, as it may be easily understood, an aspherical surface is more difficult to manufacture. In this process, the most difficult step is its testing. It has even been said that an asphere can be made as good as it can be tested.

As it is to be expected, the traditional testing methods [1] are nearly always interferometric, due to their intrinsic accuracy. If the surface is spherical, a typical two wave interferometer is like that shown in Fig. 1, produced by the interference between two wavefronts. The first one, ideally with a flat shape, comes from the surface or lens under test, but very likely it has some deformations due to the imperfections in this surface. The second wavefront does not pass through the lens under test and is flat, to serve as a reference in the test. If the fringes are straight, parallel and equidistant from each other, the wavefront under test is flat as the
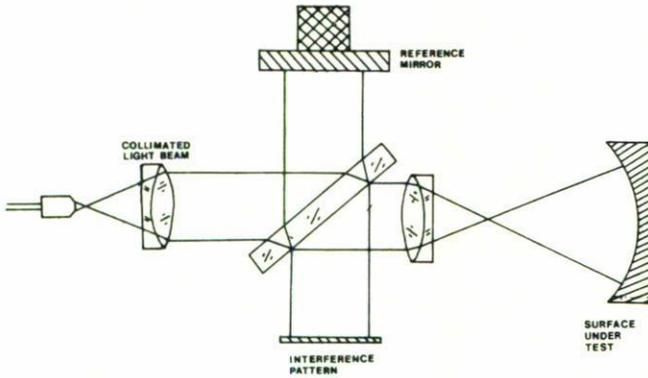

FIGURE 1. Twyman-Green interferometer.

reference wavefront. Hence the optical element under test is perfect. In the case of a perfect optical surface, with no errors and in perfect focus, the fringes are straight, equidistant and parallel.

If the surface under test does not have a perfect shape, the fringes will not be straight and their separations will be variable. The deformations of the wavefront may be determined by a mathematical examination of the shape of the fringes. By introducing a small spherical curvature on the reference wavefront or by changing its angle with respect to the wavefront under test, the number of fringes in the interferogram may by changed, in order to reduce its number as much as possible. The greater the number of fringes, the smaller the sensitivity of the test.

If the surface under test is not spherical, and its spherical aberration is not compensated (the usual case), the reflected wavefront will not be spherical (a flat is a particular case of sphere with an infinite radius of curvature). In this case, as we have explained before, the fringes in the interferogram will not be straight, neither will have constant separations between them. In an interferogram of an aspherical surface, the number of fringes may be adjusted, by changing the angle between the reference wavefront and the wavefront under test (tilt), and the curvature of the spherical reference wavefront (focusing), but this number of fringes can not be smaller than a certain minimum, that in general is a very large number. Since the fringe separation is not constant, in some places the fringes will be quite spaced, but in some others the fringes will be too close together.

The traditional interferogram analysis method requires the measurement of the position of several data points located on top of the fringes. These measurements are made in many ways, for example, by means of a measuring microscope, by means of a digitalizing tablet or video camera connected to a computer, or many others. The sensitivity of the test depends on the separation between the fringes, because an error of one wavelength in the wavefront distorts the fringe shape by an amount

equal to the separation between the fringes. The fringe deformations can in general be measured with an accuracy of one tenth of their separation, assuming that this separation is large enough to be measured with acceptable accuracy. The sensitivity to wavefront deformations is then limited to about one tenth of a wavelength, if there is a large fringe separation. If the fringes are very close to each other, the sensitivity is directly proportional to their separation. Where the fringes are wide separated, the sensitivity of the test is about one tenth of a wavelength, but the sampled points will be quite separated from each other, leaving many zones without any information. On the other hand, where the fringes are very close to each other, there is a high density of sampled data points, but the precision of the measurements are much lower than one tenth of a wavelength.

All these problems of the traditional interferometric methods have been overcome by the phase shifting interferometric techniques, where the density of sampled data points as well as the sensitivity and accuracy of the test is constant over the wave front.

Besides, phase shifting interferometry is simple and fast, thanks to the modern tools like the array detectors (CCD) and the microprocessors. Most of the conventional interferometers, like the Fizeau, Twyman-Green, etc., have been adapted to perform the phase shifting techniques to be described here.

2. Phase shifting interferometry

The phase shifting interferometric techniques have their first indirect antecedent in the works by Carre [23], but they really started less than twenty years ago, with Crane [3], Moore [4], Bruning *et al.* [5] and many others. The popularity of these techniques and its impact has been so great that about one hundred papers have been published on this subject, in the last decade, including some review papers [6]. In the phase shifting interferometers the reference wavefront is moved along the direction of propagation, with respect to the wavefront under test, in this manner changing their phase difference. The interference fringes then change their position, hence the initial name of these techniques, namely "fringe scanning".

By measuring the irradiance changes for different phase shifts, it is possible to determine the phase for the wavefront under test, relative to the reference wavefront, for that measured point over the wavefront. By obtaining this phase for many points over the wavefront, the complete wavefront shape is thus determined.

If we consider any fixed point in the interferogram, the phase difference between the two wavefronts has to be changed. We might wonder how this is possible, because relativity does not permit any of the two wavefronts to move faster than the other, because the phase velocity is c for both waves. However, it has been shown [7], that what really takes place is the Doppler effect, with a shift both in the frequency and in the wavelength. Then, both beams with different wavelengths interfere, producing beats. These beats can also be interpreted as the changes in the irradiance due to the continuously changing phase difference. In other words, these two conceptually different models are physically equivalent.
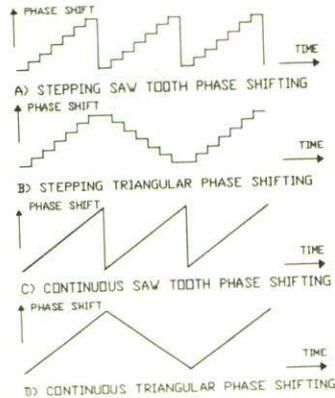
FIGURE 2. Four different manners to do the phase shifting, when the frequency of the reference beam is not permanently modified.

Then, the change in the phase may be accomplished only if the frequency of one of the beams is modified in order to form beats. Of course, as we will see below, this is possible in a continuous fashion using certain devices, but only for a relatively short period of time with some other devices. This has lead to a semantic problem: if the frequency can be modified in a permanent way, some people say that it is an "AC interferometer", an "heterodyne interferometer" or a "frequency shift interferometer". Otherwise, they say, it is a "phase shifting interferometer". In the author's point of view this distinction made by a few authors may confuse more than clarify concepts. Here, we will refer to all of these instruments as "phase shifting interferometers".

3. Techniques to shift the phase

The procedure just described can be implemented in almost any kind of two beam interferometer as for example, in the Twyman-Green shown in Fig. 1. The phase may be shifted, or equivalently, the frequency of one of the beams may be changed in many ways, as reviewed in a paper by Wyant and Shagam [8] and by Creath [6]. As pointed out before, the phase may be shifted in: a continuous fashion, by introducing a permanent frequency shift in the reference beam; or in a discontinuous manner, by periodically increasing and decreasing the optical path difference, with an oscillation of the phase in any of two ways: *a)* In a saw tooth manner as in Figs. 2.a and 2.c, and *b)* in a triangular manner as shown in Figs. 2.b and 2.d.

The first method that can be used to shift this phase is by moving the mirror for the reference beam along the light trajectory, as shown in Fig. 3.a. This can be done in many ways, for example, with a piezoelectric crystal or with a coil in a
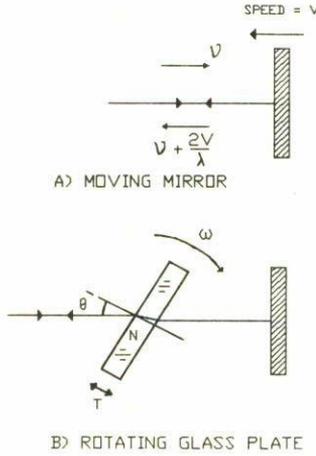
FIGURE 3. Obtaining the phase shift by means of a moving mirror or a rotating glass plate.

magnetic field. If the mirror moves with a speed V, the frequency of the reflected light is shifted by an amount equal to $\Delta\nu = 2V/\lambda$.

Another method to shift the phase is by inserting a plane parallel glass plate in the light beam, as shown in Fig. 3.b. The phase is shifted when the optical path changes. The optical path difference OPD introduced by this glass plate, when tilted an angle θ with respect to the optical axis is given by

$$\text{OPD} = T(N\cos\theta' - \cos\theta), \tag{1}$$

where T is the plate thickness and N is its refractive index. The angles θ and θ' are the angles between the normal to the glass plate and the light rays outside and inside the plate, respectively. A rotation of the plate increasing the angle θ, increases the optical path difference. Thus if the plate is rotated with an angular frequency ω, the frequency ν of the light passing through it is shifted by an amount $\Delta\nu$ given by

$$\Delta\nu = \frac{T\nu\omega}{c}\left[1 - \frac{\cos\theta}{N\cos\theta'}\right]\sin\theta, \tag{2}$$

where c is the speed of the light. The only requirement in this method is that the plate has to be inserted in a collimated light beam, to avoid introducing aberrations.

The phase may also be shifted by means of the device shown in Fig. 4. If a beam of circularly polarized light goes through a half wave retarding phase plate, the handness of the circular polarization is reversed. If the half wave phase plate rotates, the frequency of the light changes. This frequency change $\Delta\nu$ is equal to twice the frequency of rotation of the phase plate ν_p, that is: $\Delta\nu = 2\nu_p$. This arrangement works if the light goes through the phase plate only once. However, in
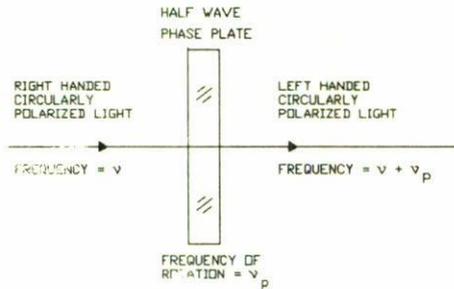
FIGURE 4. Obtaining the phase shift by means of phase plates and polarized light, with a single pass of the light beam.
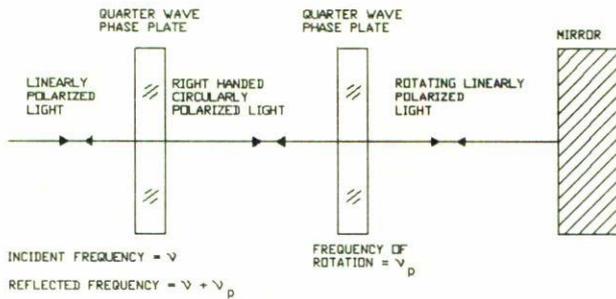

FIGURE 5. Obtaining the phase shift by means of phase plates and polarized light, with a double pass of the light beam.

a Twyman-Green interferometer the light passes twice through the system. Thus, it is necessary to use the configuration in Fig. 5. the first quarter wave retarding plate is stationary, with its slow axis at 45 degrees with respect to the plane of polarization of the incident linearly polarized light. This plate also transforms the returning circularly polarized light back to linearly polarized. The second phase retarder is also a quarter wave plate, it is rotating, and the light goes through it twice. Hence it really acts as a half wave plate. The shift in frequency is limited by the maximum mechanical speed of rotation of the phase plate, which is only one or two kHz.

Another manner to obtain the shift of the phase is by means of a diffraction grating moving perpendicularly to the light beam, as shown in Fig. 6.a. It is easy to notice that the phase of the diffracted light beam is shifted m times the number of
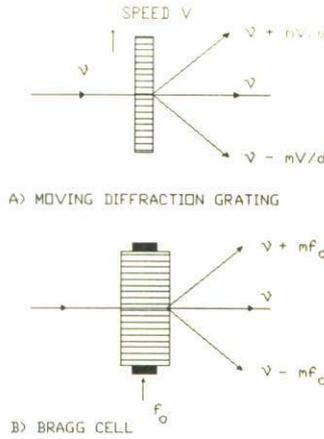
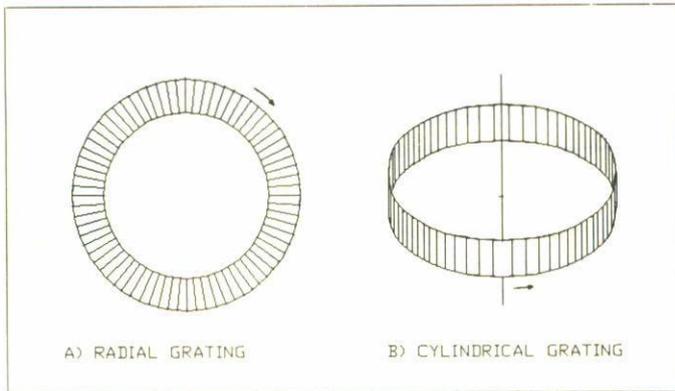FIGURE 6. Obtaining the phase shift by means of diffraction.

FIGURE 7. Two continuously moving diffraction gratings.

slits that pass through a fixed point. The letter m represents the order of diffraction. Thus, the shift in the frequency is equal to m times the number of slits in the grating, that pass through a fixed point in the unit of time. To say it in a different manner, the shift in he frequency is equal to the speed of the grating, divided by period d of the grating. Thus, we may finally write: $\Delta\nu = mV/d$. It is interesting to notice that the frequency is increased for the light beams diffracted in the same direction as the movement of the grating. The light beams diffracted in the opposite direction to the movement of the grating decrease their frequency. This method to shift the phase may be implemented by means of oscillating gratings, by means of a rotating grating, with their slits in the radial direction, or by means of a cylindrical grating, as illustrated in Fig. 7. As it is to be expected, the direction of the beam is changed

because the first order beam has to be used, and the zero order beam must be blocked by means of a diaphragm.

The diffraction of light may also be used to shift the frequency of the light, by means of an acoustic optic Bragg cell, as shown in Fig. 6.b. In this cell an acoustic transducer produces ultrasonic vibrations in the liquid of the cell. These vibrations produce periodical changes in the refractive index, moving in the liquid. These periodical changes in the refractive index act as a diffraction grating, diffracting the light. The change in the frequency is equal to the frequency f of the ultrasonic ave, times the order of diffraction m. Thus, we may write: $\Delta\nu = mf$.

The last method to be mentioned here for shifting the frequency of the light beam is a laser emitting two frequencies, ν and $\nu + \Delta\nu$. This is a Zeeman split line, in which the frequency separation $\Delta\nu$ may be controlled with a magnetic field. the two frequencies have orthogonal polarization, so, in order to be able to interfere, their plane of polarization is made to coincide. The non shifted frequency is used for the testing wavefront and the shifted frequency is used for the reference wavefront.

4. Measurement of the phase

Once the phase shift method is implemented, it is necessary to determine the procedure by which the non-shifted relative phase of the two interfering wavefronts is going to be measured. This is done by measuring the irradiance with several pre-defined and known phase shifts. Let us assume that the irradiance of each of the two interfering light beams at the point x, y in the interference pattern are $I_1(x, y)$ and $I_2(x, y)$ and that their phase difference is $\phi(x, y)$, it is then easy to show that

$$I(x, y) = I_1(x, y) + I_2(x, y) + 2\sqrt{I_1(x, y)I_2(x, y)} \cos \phi(x, y). \qquad (3)$$

This is a sinusoidal function describing the phase difference between the two waves, as shown in Fig. 8. It has maximum and minimum values given by

$$I_{\max}(x, y) = I_1(x, y) + I_2(x, y) + 2\sqrt{I_1(x, y)I_2(x, y)} \qquad (4)$$

and

$$I_{\min}(x, y) = I_1(x, y) + I_2(x, y) - 2\sqrt{I_1(x, y)I_2(x, y)} \qquad (5)$$

respectively, and an average value given by $I_0 = I_1 + I_2$.

As explained before, the basic problem is that of determining the non-shifted phase difference between the two waves, with the highest possible precision. This may be done by any of several different procedures to be described next. The best method to determine the phase, depends on how the phase or frequency shift has been made. As pointed out by Moore [4], basically, there are three different possibilities: a) the frequency is permanently shifted and hence the output is a continuous
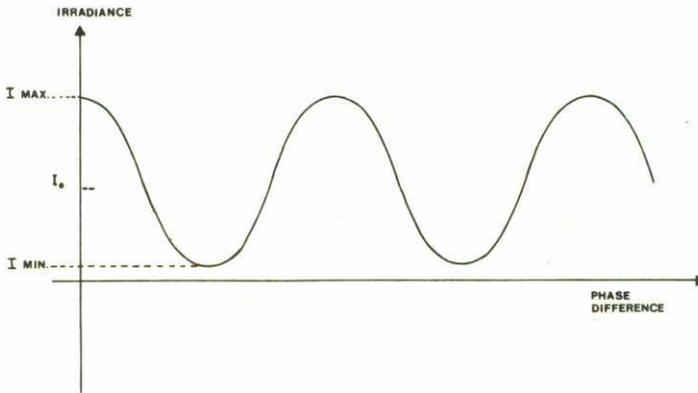
FIGURE 8. Irradiance signals in a given point in the interference pattern as a function of the phase difference between the two interfering waves.
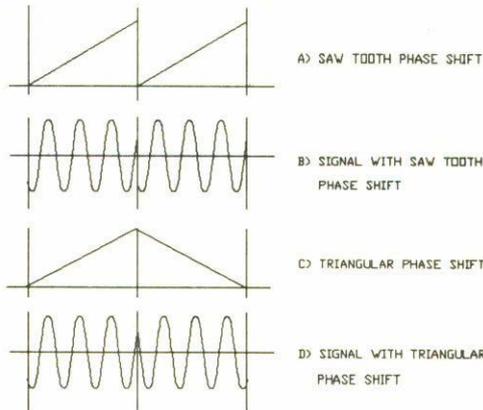

FIGURE 9. Signals obtained for saw tooth and triangular modulations of the ' iase.

sinusoidal signal, b) the phase is changed in a saw tooth manner, as shown in Figs. 9.a, obtaining the signal in Fig. 9.b, and c, the phase is changed in a triangular manner, as Figs. 9.c, obtaining the symmetrical signal in Fig. 9.d. All three methods have been used.

4.1. Zero crossing

This method detected when the irradiance plotted in Fig. 8 passes through zero when changing the phase difference between the two interfering waves. This zero does not really mean zero irradiance, but the axis of symmetry of the function, which

has a value equal to I_0, or almost any other intermediate level between the maximum and the minimum irradiance. The points crossing the axis of symmetry can very easily be found by amplification of the irradiance function to saturation levels. In this manner, the sinusoidal shape of the function becomes a square function. the zero crossing technique has ben used by Crane [3] and by Moore [4].

The wavefront shape is obtained by measuring the phase Φ at some reference point on the wavefront (reference signal) and then at several other points on the wavefront (test signal). A practical implementation of this method uses a clock that starts when the reference signal passes through zero and stops when the test signal passes through zero. The ratio of the time the clock runs to the known period of the irradiance signal gives the wavefront deviation respect to the reference point.

4.2. Phase lock

The phase lock method can be explained with the help of Fig. 10. Let us assume that an additional phase difference is added to the initial phase $\Phi(x, y)$. The additional phase being added, has two components, one of them with a fixed value and the other with a sinusoidal time shape. both components can have any predetermined desired value. Thus

$$\phi = \Phi(x, y) + \delta(x, y) + a \sin \omega t \qquad (6)$$

then, the irradiance $I(x, y)$ would be given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos[\Phi + \delta + A \sin \omega t] \qquad (7)$$

and this function can be expanded in series as follows:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \Big\{ \cos(\Phi + \delta)[J_0(A) + 2J_2(A) \cos 2\omega t + \ldots]$$

$$- \sin(\Phi + \delta)[2J_1(A) \sin \omega t + 2J_3(A) \sin 3\omega t + \ldots] \Big\}. \qquad (8)$$

Here, J_n is the Bessel function of order n. the first part of this expression represents harmonic components of even order, while the second part represents harmonic components of odd order.

Let us now assume that the amplitude of the phase oscillations $A \sin \omega t$ are much smaller than π. If now we adjust the fixed phase δ to a value such that $\Phi + \delta = n\pi$, then $\sin(\Phi + \delta)$ is zero. Hence, only even harmonics remain. This effect is shown in Fig. 10, near one of the minima of this function. This is done in practice by slowly changing the value of the phase δ, while maintaining the oscillation $A \sin \omega t$, until the minimum amplitude of the first harmonic, or fundamental frequency, is obtained. then, we have $\delta + \Phi = n\pi$. Since the value of δ is known, the value of Φ has been determined.
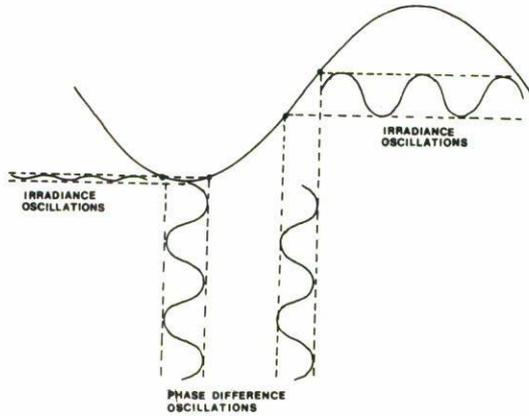
FIGURE 10. Methods to find the phase with a small sinusoidal modulation of the phase.
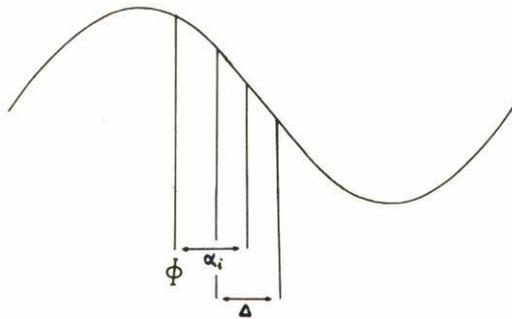

FIGURE 11. Averaged signal measurements with the integrating phase shifting method.

Another equivalent manner, would be to find the inflection point for the sinusoidal function, as shown in Fig. 11, by changing the fixed component of the phase, until the first harmonic have their maximum amplitude. Then, $\sin \delta = 1$.

4.3. Phase stepping

This method consists in measuring the irradiance values for several known increments of the phase. there are several versions of this method that will be described later. The measurement of the irradiance for any given phase takes some time, since there is a time response for the detector. Hence, the phase has to be stationary during a short time in order to take the measurement. Between two consecutive

measurements, the phase may change as fast as desired in order to get to the next phase with the smallest delay. Let us assume that an irradiance I_i is measured when the phase has been incremented from its initial value Φ by an amount α_i, as shown in Fig. 11. Thus

$$I_i = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\Phi + \alpha_i). \tag{9}$$

The mathematical treatment will be completed in the next section, when describing the integrating phase shifting method, because the phase stepping method may be considered as a particular case of the first.

4.4. Integrating phase shifting

This method, also called integrating bucket, is very similar to the phase stepping method, with the only difference that the phase changes continuously and not by discrete steps. The problem with the phase stepping method is that the sudden changes in the mirror position may introduce some vibrations to the system.

In the integrating phase shifting method the detector continuously measures the irradiance during a fixed time interval, without stopping the phase, but since the phase changes continuously, the average value during the measuring time interval is measured. Thus, the integrating phase stepping method may be mathematically considered a particular case of the phase stepping method, if the detector has an infinitely short time response, such that the measurement time interval is reduced to zero. Like in the phase stepping method, the phase may be shifted using a saw tooth profile as in Fig. 2.c, or a triangular profile, as in Fig. 2.d, to avoid sudden changes.

Let us assume, as in Fig. 11 that the average measurement is taken from $\alpha = \alpha_i - \Delta/2$ to $\alpha = \alpha_i + \Delta/2$, with center value α_i. Then, the average value of the irradiance would be given by

$$I_i = \frac{1}{\Delta} \int_{\alpha - \Delta/2}^{\alpha + \Delta/2} \left[I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\Phi + \alpha) \right] d\alpha$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2} \,\mathrm{sinc}(\Delta/2) \cos(\Phi + \alpha_i) \tag{10}$$

As it is expected, this expression is the same as Eq. (9) for the measurement with phase steps, when $\Delta = 0$, since the sinc function has a unit value. If the integration interval Δ is different from zero, the only difference is that the apparent contrast of the fringes is reduced. Then, we will consider this expression to be the most general.

a) Four Step Method. The values of the irradiance are measured using four different values of the phase, α_i, equal to: 0, $\pi/2$, π, $3\pi/2$, as shown in Fig. 12.a. In this manner, the following four values of the irradiance are obtained: I_A, I_B, I_C,
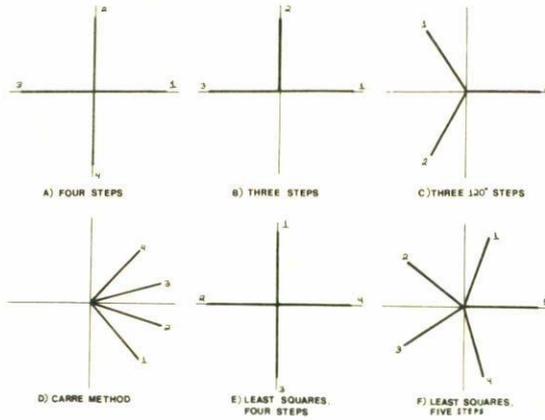
FIGURE 12. Six different manners to shift the phase using phase steps.

and I_D

$$I_A = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{sinc}(\Delta/2)\cos\Phi,$$
$$I_B = I_1 + I_2 - 2\sqrt{I_1 I_2}\,\mathrm{sinc}(\Delta/2)\sin\Phi,$$
$$I_C = I_1 + I_2 - 2\sqrt{I_1 I_2}\,\mathrm{sinc}(\Delta/2)\cos\Phi,$$
$$I_D = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{sinc}(\Delta/2)\sin\Phi,$$

(10)

From these equations we may obtain

$$\Phi(x,y) = \tan^{-1}\left\{\frac{I_D(x,y) - I_B(x,y)}{I_A(x,y) - I_C(x,y)}\right\},$$

(11)

It is interesting to notice that this result is independent of the integration interval Δ.

b) Three Step Method. Since we have only three unknowns I_1, I_2 and Φ in Eqs. (9), three measurements would be enough to determine the phase Φ. (Fig. 12.b). Hence, we may write

$$I_A = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{sinc}(\Delta/2)\cos(\Phi + \pi/4),$$
$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{sinc}(\Delta/2)[\cos\Phi - \sin\Phi]/\sqrt{2},$$
$$I_B = I_1 + I_2 - 2\sqrt{I_1 I_2}\,\mathrm{sinc}(\Delta/2)\sin(\Phi + 3\pi/4)$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\,\text{sinc}(\Delta/2)[-\cos\Phi - \sin\Phi]/\sqrt{2},$$

$$I_C = I_1 + I_2 - 2\sqrt{I_1 I_2}\,\text{sinc}(\Delta/2)\cos(\Phi + 5\pi/4),$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\,\text{sinc}(\Delta/2)[-\cos\Phi + \sin\Phi]/\sqrt{2}, \tag{12}$$

From these relations we may obtain

$$\Phi(x,y) = \tan^{-1}\left\{\frac{I_C(x,y) - I_B(x,y)}{I_A(x,y) - I_B(x,y)}\right\}. \tag{13}$$

c) *Three 120 Degrees Steps.* The three measurements do not necessarily have to be separated 90 degrees. The phase separation between them may be 120 degrees, as shown in Fig. 12.c. The phase may be found as follows

$$I_A = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\text{sinc}(\Delta/2)\cos(\Phi + 120°),$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\,\text{sinc}(\Delta/2)[-.5\cos\Phi - .866\sin\Phi],$$

$$I_B = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\text{sinc}(\Delta/2)\cos(\Phi + 240°),$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\,\text{sinc}(\Delta/2)[-.5\cos\Phi + .866\sin\Phi],$$

$$I_C = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\text{sinc}(\Delta/2)\cos\Phi, \tag{14}$$

which gives the following result for the phase,

$$\Phi = \tan^{-1}\left\{\frac{\sqrt{3}(I_A - I_B)}{I_A + I_B - 2I_C}\right\}, \tag{15}$$

d) *Two Steps Plus One.* This is an interesting Three step method, suitable for systems with vibrations, like in the test of large astronomical mirrors [10]. The phase of one of the beams is rapidly switched between two values, separated by $\pi/2$. This is done fast enough to reduce the effects of the vibration. A third reading is taken any time later, to measure only the sum of the irradiance of the beams, independently of their relative phase, by using an integrating interval $\Delta = 2\pi$. Thus, we may write

$$I_A = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\text{sinc}(\Delta/2)\cos\Phi,$$

$$I_B = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\text{sinc}(\Delta/2)\cos(\Phi - \pi/2),$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\,\text{sinc}(\Delta/2)\sin\Phi,$$

$$I_C = I_1 + I_2, \tag{16}$$

which gives the following result for the phase

$$\Phi = \tan^{-1}\left\{\frac{I_B - I_C}{I_A - I_C}\right\},\tag{17}$$

e) Carré Method. Another phase shifting method, suggested by Carré, takes four equally spaced measurements, two increasing the phase and two decreasing it, as shown in Fig.12.d

$$I_A = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\text{sinc}(\Delta/2)\cos(\Phi - \tfrac{3}{2}\alpha),$$

$$I_B = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\text{sinc}(\Delta/2)\cos(\Phi - \tfrac{1}{2}\alpha),$$

$$I_C = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\text{sinc}(\Delta/2)\cos(\Phi + \tfrac{1}{2}\alpha),$$

$$I_D = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\text{sinc}(\Delta/2)\cos(\Phi + \tfrac{3}{2}\alpha),\tag{18}$$

From these equations, the phase can be obtained as follows

$$\Phi = \tan^{-1}\left\{\tan\frac{\alpha}{2}\frac{(I_B - I_C) + (I_A - I_D)}{(I_B - I_C) - (I_A - I_D)}\right\},\tag{19}$$

4.5. Synchronous detection

The frequency and the wavelength of the function in Fig. 8 are both well known, since we know the speed with which the phase of the reference wave is shifted. hence, it is possible to use a method described in communication theory, called synchronous detection. In this method the signal is correlated with a sinusoidal signal with the same frequency as the signal to be detected. This methods have been used with success for many years in radio communications and can be performed in many ways. As pointed out by Bruning [5,11] a mathematically equivalent method is to make a sampling of the irradiance function and then, a fitting of these data points to a sinusoidal curve with the known frequency. Once the fitting is made, all maxima and minima are determined, Hence, the problem is solved.

With this method N measurements of the intensity are made, for N different phase shifts. The number N is greater than or equal to three. In general, the phase shifts may be of any magnitude, and not necessarily with the same phase interval separations between them, as shown by Greivenkamp [12]. Then, the fitting of the data to the closest sinusoidal function with the known frequency is obtained by means of a least squares procedure on a computer. Let us assume that we have taken N measurements of the irradiance I_i, equally spaced on a period of the irradiance function, with phases α_i added to the initial phase Φ, in such a way that the new

phase in each measured point is $\phi = \Phi + \alpha_i$, where

$$\alpha_i = \frac{2\pi i}{N}, \tag{20}$$

with $i = 1, 2, 3, \ldots, N$. (Figs. 12.e and 12.f). Then, it is possible to show with elementary Fourier theory that the phase Φ to be determined is given by

$$\tan \Phi = -\frac{\sum I_i \sin \alpha_i}{\sum I_i \cos \alpha_i}. \tag{21}$$

Another synchronous detection method, closer to the traditional electronic methods, would be to measure the irradiance function in a continuous manner. then the signal is electronically filtered with a band pass filter, in order to obtain the desired fundamental Fourier component of the signal. The phase is then measured by electronic means. However, the new sampling procedures just described and also called digital methods, are much more accurate.

5. Conclusions

The new electronic phase shifting digital techniques have improved the accuracy of the interferometric measurements by at least one order of magnitude. Besides, with the help of the new imaging devices and microcomputers, not only the precision is better, but also e measurements can be made in real time. Unfortunately the equipment requirements are considerable greater than that for the traditional interferometric methods, making them more expensive. We hope that new research will make phase shifting interferometry cheaper and easier to perform.

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Resumen. Los métodos interferométricos permiten medir la forma de una superficie con muy alta precisión, usando como unidad de medida la longitud de onda de la luz. Por otro lado, las superficies ópticas más comunes son las esféricas, aunque las asféricas, es decir, las que no son esféricas, son cada vez más comunes. Estas superficies asféricas tienen generalmente la forma de una cónica de revolución. El determinar en forma rápida y precisa los errores que pudiera tener una superficie asférica es un problema poco sencillo y cada vez es más necesario resolverlo. La razón es que las superficies asféricas son cada día más necesarias y de alta precisión en los instrumentos ópticos modernos.

La interferometría de corrimiento de fase tiene alrededor de diez años de haber sido introducida, con una ventajas extraordinarias con respecto a los métodos tradicionales, cuando se aplica a la medición de superficies asféricas. Aquí se presenta una revisión de estos avances.