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Stream lines for a pure multipole current distribution

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Abstract. We give an equation describing the electric current streamlines on the surface of a sphere that generate a magnetic field which contains a single multipole component. The equation shows how to wind a coil in order to produce a pure multipole field and helps to give an intuitive grasp of how well existing atomic traps approximate multipoles.

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1. Introduction

One subject that has attracted much attention recently is the trapping of neutral atoms. The traps can be loosely grouped into two categories; optical traps [1] that use counter-propagating laser beams to cool and/or confine the atoms, and magnetic traps [2–4] that use static magnetic fields to hold the atoms. We concentrate here on the latter case. Various trap configurations of experimental interest are shown in Fig. 1.

In a recent paper [5] it was shown how to express an arbitrary current density, confined on the surface of a sphere, as multipolar expansion in terms of vector spherical harmonics. The magnetic field produced could then be expanded in a similar form. The expansion is proposed in order to facilitate the analysis of the fields, the sources and finally the possible trajectories of the trapped particles.

Such a multipole expansion has the advantage that it has a straightforward physical interpretation. Most traps are constructed out of basic components that correspond closely to one or other multipole component. As a first attempt, at gaining an intuitive understanding of the motion of atoms in magnetic traps, it seems

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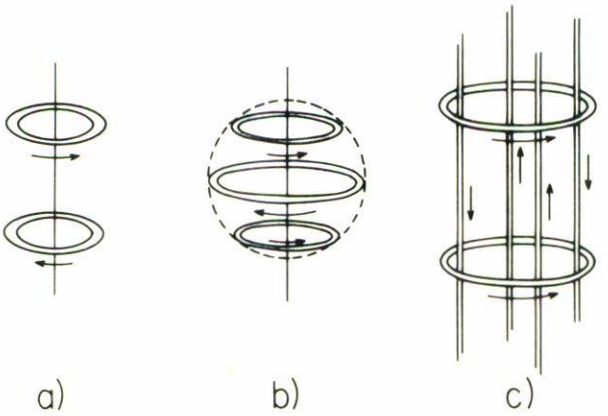

FIGURE 1. Various configurations of magnetic traps for neutral atoms; **a)** quadrupole, **b)** octupole and **c)** hybrid, which is a mixture of a uniform axial field, and octupole and a transverse quadrupole.

reasonable to calculate the orbit of an atom moving in a pure, single component multipole field.

In the present paper we find equations for the unique single electric current streamlines corresponding to a single multipole component. These equations teach us how to wind a coil on the surface of a sphere in order to produce a pure multipole of any order. By inspection of the streamlines it becomes clear how closely particular experimental setups approximate multipole fields.

2. Streamlines

In Ref. [5] it was shown that an arbitrary current distribution on the surface of a sphere can be represented as

$$\vec{J}(\vec{r}) = \frac{\delta(\vec{r}' - \vec{a})}{a} \sum_{\lambda\mu=1} K_{\lambda\mu} \left(-i\vec{l}Y_{\lambda\mu}\right), \tag{1}$$

where $\vec{r}$ is the point at which we are observing the current, a is the radius of the sphere, $\vec{r}'$ is the position where the currents are, $Y_{\lambda\mu}$ are the spherical harmonics contributing with intensity $K_{\lambda\mu}$, and $\vec{l}$ is the usual angular momentum operator, $\vec{l} = -i\vec{r} \times \vec{\nabla}$. After applying Maxwell equations to (1) one obtains the corresponding

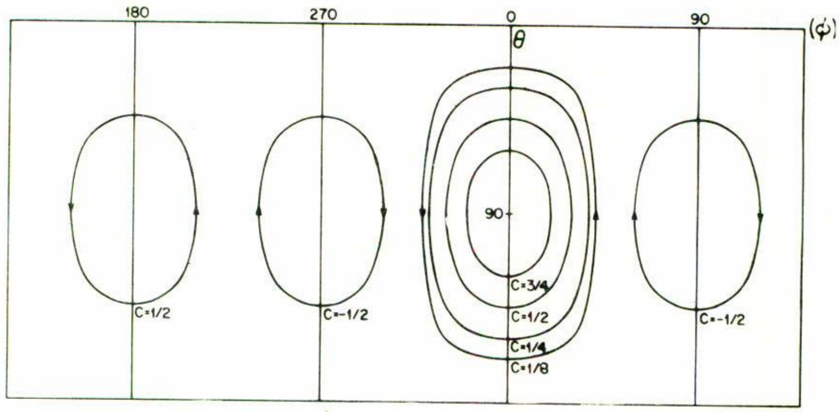

FIGURE 2. Projection of some of the streamlines on the surface of the sphere generated by $Y_{22}(\theta, \phi) = c$.

magnetic fields,

$$\vec{B}^e(\vec{r}) = \sum_{\lambda\mu} \frac{4\pi}{(2\lambda+1)c} \frac{K_{\lambda\mu} a^{\lambda+1}}{r^{\lambda+2}} \left[\hat{r}\lambda(\lambda+1)Y_{\lambda\mu}(\hat{r}) - i\hat{r} \times \vec{l}Y_{\lambda\mu}(\hat{r})(1-\lambda-1)\right], \quad (2a)$$

$$\vec{B}^i(\vec{r}) = \sum_{\lambda\mu} \frac{4\pi}{(2\lambda+1)c} \frac{K_{\lambda\mu} r^{\lambda-1}}{a^{\lambda}} \left[\hat{r}\lambda(\lambda+1)Y_{\lambda\mu}(\hat{r}) - i\hat{r} \times \vec{l}Y_{\lambda\mu}(\hat{r})(1+\lambda)\right]. \qquad (2b)$$

Note the continuity of the normal components of the magnetic field in passing from one side to the other of the spherical surface, and the discontinuity of the tangential components,

$$\left(\vec{B}^e_{\lambda\mu} - \vec{B}^i_{\lambda\mu}\right) = \frac{4\pi}{(2\lambda+1)c} \frac{K_{\lambda\mu}}{a} i\hat{r} \times \vec{l}Y_{\lambda\mu}(\hat{r})(\lambda+1+\lambda), \qquad (3)$$

that are proportional and transverse to the multipole component corresponding to the surface current Eq. (1).

These equations, in principle, can be used both in the analysis and synthesis of a given magnetic field configuration from an electric current distribution restricted to the surface of a sphere.

In practice, however, since current distributions are generated windings of conductors, it is more useful to know the path traced out by a given current element or charge carrier. These paths are then the curves that the conductors in the winding should follow. The winding density is clearly given by $|\vec{J}|$ for uniform conductors.

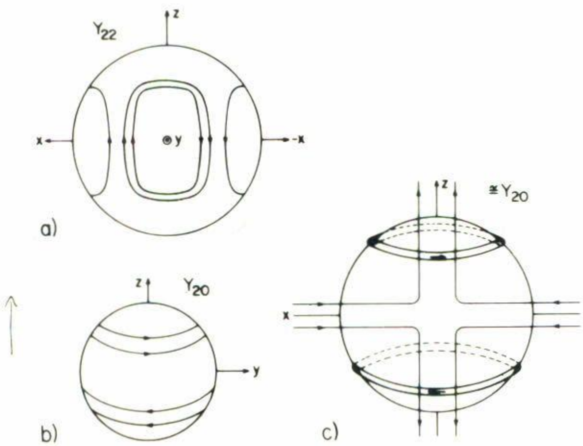

FIGURE 3. Typical streamlines on the surface of a sphere: a) generated by $Y_{22}(\theta,\phi) = c$. b) By $Y_{20}(\theta,\phi) = c$. c) Shows the experimental set up for a quadrupole trap with 2 coils at $\pm 45°$ and currents flowing in opposite directions. The dominant component for this trap is Y_{20}.

The family of current streamlines will be described by an equation of the form

$$\Omega(\theta,\phi) = c, \tag{4}$$

where c is some constant. Different values for c yield different streamlines. The vector field representing the electric current density is everywhere tangent to the streamlines and hence

$$\vec{\nabla}\Omega(\theta,\phi)\cdot\vec{J}(\vec{r}) = 0. \tag{5}$$

For a general $\vec{J}$ the solution to this differential equation must be found numerically but for $\vec{J}(\vec{r}) = -i\vec{l}Y_{\lambda\mu}$ it can be solved analytically and exactly using standard techniques and assuming a separable solution of the form $\Theta(\theta)\Phi(\phi)$. After some work this gives

$$\Omega(\theta,\phi) \propto Y_{\lambda\mu}. \tag{6}$$

Since this result is now to be used in Eq. (4), the constant of proportionality is unimportant. The complete family of streamlines is constructed by allowing c to vary over its full range.

Consider the particular case of the streamlines that generate the pure quadrupole component corresponding to $Y_{22}(\hat{r})$ in Eqs. (2a) and (2b). The spherical harmonic, $Y_{22}(\theta,\phi) = \sin^2\theta\exp^{2i\phi}$ and the current density is given by Eq. (1). The current streamlines found using Eq. (6) are shown for this case, for various values of c, in Fig. (2). Note that each value of c yields two disconnected streamlines because here Eq. (6) is of second order.

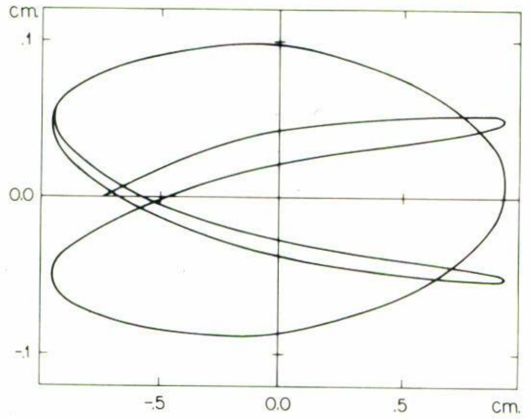

FIGURE 4. A possible trajectory for a 70 mK carbon atom confined in a 1 T quadrupole magnetic trap. Both scales are in centimeters.

In Fig. (3a) we show the streamlines of Fig. (2) on the surface of the sphere. Fig. (3b) shows streamlines generated by the equation Y_{20}. Fig. (3c) shows the experimental set up for a quadrupole trap that has as its dominant component the spherical harmonic Y_{20}. Note however that the trap has a different current distribution than that for a pure Y_{20} and therefore contains other spherical harmonic components.

It has recently become possible to calculate, by computer simulation, the orbit of a neutral atom within a magnetic trap. An example of a possible orbit described by a carbon atom in a 1 Tesla quadrupole trap with initial temperature 70 mK, is shown in Fig. (4). Further details of this aspect will be given in a later publication Ref. [6].

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Resumen. Se obtiene una ecuación que describe las líneas de flujo de corriente en la superficie de una esfera las cuales generan un campo magnético que contiene una sola componente multipolar. La ecuación muestra cómo enrollar un alambre para producir un campo multipolar puro y ayuda a dar un entendimiento intuitivo de que tan buena es la aproximación de las trampas atómicas existentes a multipolos.