

Sigmoidal creep transients and the second law of Newton

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Abstract. The main features of sigmoidal transients, not only for viscous glide but also for Power-Law Creep are explained in a physical way. Also the experimental data for the average internal stress, $\langle\sigma_i\rangle$, and the mobile dislocation density, ρ_m , at the inflection point of sigmoidal creep curves for Germanium and Cu-16at%Al are fully described. Finally, the stress and temperature dependence of the total plastic deformation at the inflection point for sigmoidal creep curves in Cu-11.5at%Al are also catered for.

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1. Introduction

Usually, for the theoretical analysis of plastic deformation during transient creep stages, the acceleration of dislocations is taken into account. In fact, there are many microscopic and phenomenological models which consider such acceleration in order to explain some aspects of the plastic deformation process [1-16]. One of these models as discussed in Ref. [16], leads to the Fuchs and Ilschner equation [9], namely

$$\frac{d\langle\dot{\epsilon}\rangle}{dt} = \alpha b \left(\langle v \rangle \frac{d\rho_m}{dt} + \rho_m \frac{d\langle v \rangle}{dt} \right), \quad (1)$$

where $\langle\dot{\epsilon}\rangle$ is the strain rate, α is the average geometrical factor relating the tensile deformation to the shear deformation for polycrystalline samples, b is the magnitude of the Burgers' vector, ρ_m is the mobile dislocation density and, $\langle v \rangle$ is the average glide velocity of mobile dislocations. This equation was obtained by developing some elements of the Statistical Mechanics of Mobile Dislocations. The physical meaning of Eq. (1) in this framework is related to the volumetric net force which describes the dynamical behavior of the center of mass of the mobile dislocation system, $\langle f \rangle$.

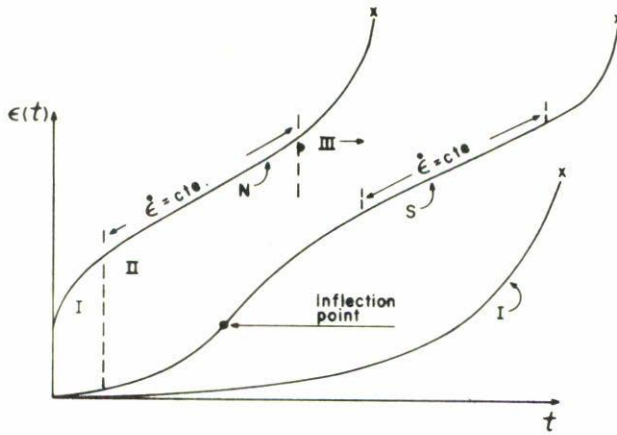


FIGURE 1. Types of curves in a tension creep test. The curve “N” is named a normal creep curve, the second curve “S”, named a sigmoidal creep curve, and finally the curve “I”, named inverted creep transient. The normal creep curve have three stages: I, primary transient stage; II, the steady state stage where the strain rate, $\langle \dot{\epsilon} \rangle$ is constant, and, the tertiary creep stage, III, a situation where creation and evolution of cavities control the sample deformation giving place eventually to rupture.

Further, $\langle f \rangle$ is given by,

$$\langle f \rangle = \frac{d}{dt}(m\rho_m\langle v \rangle). \tag{2}$$

A key element for this interpretation is that the volumetric linear momentum, p , which appears in the Second Law of Newton, is related to the strain rate as given by the Orowan equation [17]. That is

$$\langle \dot{\epsilon} \rangle = \alpha b\rho_m\langle v \rangle. \tag{3}$$

From our point of view, this statistical mechanical approach provides a more intuitive insight to the physical content of the Orowan and the Fuchs and IIschner equations than the one previously considered.

Our main purpose in this paper is to apply the theoretical framework just referred to in order to analyze some transient situations in dislocation creep. We will restrict ourselves to the study of the inflection point in sigmoidal creep curves not only for viscous glide but also for Power-Law Creep. The theoretical results will be used to analyze some creep data in Ge [7] Cu-16at%Al [18].

2. Theory

In this section, we develop a model to study the dynamical behavior of the mobile dislocation system at the inflection point of sigmoidal creep curves [Fig. (1)]. Our model considers that in the early primary transient stage of deformation the only difference between viscous glide and Power-Law Creep is due to the dependence of the glide velocity on the average effective stress, $\langle\sigma_e\rangle$. Here $\langle\sigma_e\rangle$ is given by $\sigma - \langle\sigma_i\rangle$ with σ the applied stress and $\langle\sigma_i\rangle$ the average of the internal stress, taken as usual to be [19],

$$\langle\sigma_i\rangle = \alpha\mu b\sqrt{\rho_m}, \quad (4)$$

where μ is the shear modulus of the material.

Usually creep tests are carried out on annealed samples; then we can assume that during the first minutes of the test, the annihilation events are negligible as compared to the creation of dislocations. Therefore, considering that all the dislocations are mobile, the change in ρ_m is due to the creation of new dislocations. And according to Montemayor-Aldrete *et al.* [20] the creation rate of dislocations, $\dot{\rho}_m^+$, is given as,

$$\dot{\rho}_m^+ = \frac{\alpha\sigma\langle\dot{\epsilon}\rangle}{\bar{u}}, \quad (5)$$

where \bar{u} is the mean value for the self-energy of dislocation per unit length.

On the other hand, the mobile dislocations glide with an average glide velocity $\langle v \rangle$. In 1959, in their classical paper Johnston and Gilman [3] reported direct measurements of single dislocation velocities as a function of applied stress, and suggested a relationship between v and σ^n . In 1969, Li [6] gave a generalization for such empirical relation taking into account the average internal stress for samples macroscopically deformed. Generalizing the Li expression, we have that $\langle v \rangle$ is given by

$$\langle v \rangle = \alpha B_n \left(\langle \sigma_e \rangle \right)^{n-2}, \quad (6)$$

where n is the stress exponent which can take a value 3 for viscous glide, or a value 5 for Power-Law Creep. Also B_3 and B_5 are the mobility of dislocations for viscous glide and Power-Law Creep, respectively.

The previous considerations and the use of Eq. (1) lead to the following result

$$\frac{d\langle\dot{\epsilon}\rangle}{dt} = \alpha^2 b B_n \dot{\rho}_m^+ \left(\langle \sigma_e \rangle \right)^{n-3} \left[\sigma - \frac{n}{2} \langle \sigma_i \rangle \right]. \quad (1.a)$$

According to our theoretical scheme, with the use of Eqs. (2) and (3), the volumetric force acting on the mobile dislocation system, $\langle f \rangle$, is given by $\langle f \rangle =$

$\frac{m}{ab} \frac{d}{dt} \langle \dot{\epsilon} \rangle$. Therefore, substituting Eq. (1.a) into this equation allows us to explain and describe different types of primary transient stages (Fig. (1)), depending on the initial value for $\langle f \rangle$ in a given sample.

In other words, different types of transients appear depending on the sign taken by $[\sigma - (n/2)(\alpha\mu b\sqrt{\rho_m})]$. In the following, a sigmoidal creep transient will be analyzed in terms of the sign and qualitative changes on the force acting on the system of mobile dislocations. As it can be seen in Fig. (1), a sigmoidal transient in a creep curve is composed of an inverted transient followed by a normal transient, with these two curves linked by the inflection point. In our scheme, the sigmoidal starts with the inverted transient characterized by a volumetric force greater than zero ($\langle f \rangle > 0$). In this stage $\langle f \rangle$ is decreasing its magnitude with time (because of the raising of ρ_m), and consequently the volumetric linear momentum of mobile dislocations increases its magnitude (*i.e.* $\langle \dot{\epsilon} \rangle$ increases its values) with time, but at a decreasing rate. Eventually a zero value for $\langle f \rangle$ is reached. This is so because with deformation the value for ρ_m grows up and for a given time the term $[\sigma - (n/2)(\alpha\mu b\sqrt{\rho_m})]$ takes momentarily the value zero, before the change of sign on this term takes place. At this precise moment we arrive at the inflection point of the curve, characterized also by a maximum value for the strain rate. Beyond this point $\langle \dot{\epsilon} \rangle$ necessarily diminishes its value because now $\langle f \rangle$ has a negative sign, and also its magnitude begins to rise. In other words the value for $p = m\rho_m \langle v \rangle$ diminishes continuously with time because $\langle f \rangle$ is lower than zero during the normal creep transient.

As mentioned before, at the inflection point $\langle f \rangle = 0$, and therefore for this point $\langle \dot{\epsilon} \rangle$ has a maximum value. Physically at this point the net force acting on the mobile dislocation system is zero, and then we can expect that the dynamic parameters of the system will be dynamical constants, provided that the applied stress and temperature are constant during a creep test. Therefore based on the above considerations it will be very interesting to find these dynamical constants. Following this path, the only possibility to obtain $(d/dt \langle \dot{\epsilon} \rangle)_{i.p.} = 0$ at the inflection point according to Eqs. (5) and (6) is,

$$(\langle \sigma_i \rangle)_{i.p.} = \frac{2\sigma}{n}, \quad (7)$$

where the subindex, *i.p.*, indicates the inflection point. Also using Eqs. (4) and (7) it is straightforward to show that,

$$(\rho_m)_{i.p.} = \left(\frac{2\sigma}{n\alpha\mu b} \right)^2 \quad (8)$$

and with the aid of Eqs. (3), (6), (7) and (8) an expression for $\langle \dot{\epsilon} \rangle$ many also be obtained, namely,

$$(\langle \sigma_i \rangle)_{i.p.} = \left(1 - \frac{2}{n} \right) \left(\frac{2}{n\mu b^{1/2}} \right)^2 B_n \sigma^n. \quad (9)$$

Parameter	Germanium		Cu-16at%Al (723° K)	
	Exp.	Theory	Exp.	Theory
$(\langle\sigma_i\rangle/\sigma)_{i.p.}$	2/3	2/3	0.37	0.40
$(\rho_m)_{i.p.}/\left(\frac{\sigma}{\mu b}\right)^2$	4	4	0.89	*

TABLE I. The values for $\langle\sigma_i\rangle$ and ρ_m at the inflection point for sigmoidal creep curves in Ge and Cu-16at%Al, according to [3] and [18] respectively; and the comparison of the values for $\langle\sigma_i\rangle$ and ρ_m with theoretical calculations. Note we take $\alpha_{GE} = 1/3$ from [7]. *Fitted value in order to obtain the value of α for the Cu-Al alloy $\alpha_{Cu-Al} = 0.45$.

The value for the total plastic deformation of the sample at the inflection $(\epsilon_p)_{i.p.} - \epsilon_0 \equiv (\Delta\epsilon_p)_{i.p.}$ (with ϵ_0 the deformation at the starting time) can be calculated considering that the value for $(\rho_m)_{i.p.}$ is equal to the original dislocation density $\rho_m(t_0)$ plus the one resulting from all the dislocations created between the starting time and the inflection point, which we denote by $\Delta\rho_m$, (*i.e.* $\Delta\rho_m = (\rho_m)_{i.p.} - \rho_m(t_0)$). Assuming as before that no annihilation events occur in this stage of deformation then from Eqs. (5) and (8): it follows that

$$(\Delta\epsilon_p)_{i.p.} = \frac{\bar{u}}{\alpha\sigma} \left[\left(\frac{2\sigma}{n\alpha\mu b} \right)^2 - \rho_m(t_0) \right]$$

and, if $\rho_m(t_0) \ll (2\sigma/n\alpha\mu b)^2$ then, $(\Delta\epsilon_p)_{i.p.}$ can be evaluated approximately from the following expression

$$(\Delta\epsilon_p)_{i.p.} = \frac{1}{\alpha^3} \left(\frac{2}{n} \right)^2 \left(\frac{\bar{u}}{\mu b^2} \right) \left(\frac{\sigma}{\mu} \right). \quad (10)$$

Equations (7), (8), (9) and (10) are the main results of this section. They will be contrasted with experimental data in the next section.

3. Comparison with experimental results

In this section we compare the theoretical results with the experimental data for Germanium given by Berner and Alexander [7], and also with the experimental data for Cu-16at%Al due to Hasegawa, Ikeuchi and Karashima [18].

For Germanium, which obeys viscous glide creep, one has $n = 3$. And using this value in Eqs. (7) and (8), the obtained values for ρ_m and $\langle\sigma_i\rangle$ at the inflection point shown in Table I. The agreement between theory and experimental data is rather good.

On the other hand, for the Cu-16at%Al alloy, which obeys a Power-Law Creep, $n = 5$. For this case the values for $\langle\sigma_i\rangle/\sigma$ calculated at the inflection point are shown

Alloy	σ (MPa)	T (K)	$(\Delta\epsilon)_{i.p.}$ Exp.	$(\Delta\epsilon_p)_{i.p.}$ Theory	$\frac{\Delta\epsilon_{exp}-\Delta\epsilon_{theo}}{\Delta\epsilon_{exp}}$
Cu-11.5at%Al	39.2	723	$(7.2 \pm 0.7)10^{-3}$	$.4 \times 10^{-3}$	+2.8%
Cu-11.5at%Al	19.6	823	$(3.7 \pm 0.7)10^{-3}$	4.1×10^{-3}	+10.8%
Cu-11.5at%Al	39.2	823	$(8.5 \pm 0.5)10^{-3}$	8.6×10^{-3}	+1.2%
Cu-16at%Al	39.2	723	$(7.8 \pm 0.5)10^{-3}$	*	

TABLE II. The accumulated plastic deformation at the inflection point for sigmoidal creep curves, $(\Delta\epsilon_p)_{i.p.}$, for Cu-Al alloys, as given by Hasegawa *et al.* [18]; and its comparison with theoretical results. The shear modulus for Cu-11.5at%Al, $\mu_{11.5Al} = 5.25 \times 10^4$ MPa, and for Cu-16at%Al, $\mu_{16Al} = 5.16 \times 10^4$ MPa were calculated using the Quéré expression [21] with μ_{Cu} and μ_{Al} as given by [22]. The temperature dependence on μ was taken into account following Staker and Holt [23]. *Fitted value in order to obtain a mean value for the self-energy of dislocation per unit length, \bar{u} , for Cu-16at%Al alloy, $\bar{u} = 5.52 \mu b^2$. The same value for \bar{u} was used in the calculations of $(\Delta\epsilon_p)_{i.p.}$.

in Table I. The agreement between theoretical and experimental data for $\langle\sigma_i\rangle$ is also satisfactory here. The theoretical value for ρ_m at the inflection point was fitted to the experimental data in order to determine the value taken by α in this alloy. The obtained value $\alpha = 0.45$ will be used in further calculations.

Relative to the total plastic deformation of the sample at the inflection point $(\Delta\epsilon_p)_{i.p.}$ in Table II we present our calculations for two alloys and different test conditions. In order to make the calculations for $(\Delta\epsilon_p)_{i.p.}$ a value for the mean total-energy of dislocation per unit length, $\bar{u} = 5.52\mu b^2$, was used. This value was obtained by fitting the theoretical expression for $(\Delta\epsilon_p)_{i.p.}$ with the experimental data for Cu-16at%Al at $\sigma = 39.2$ MPa and $T = 723^\circ$ K. The theoretical values for $(\Delta\epsilon_p)_{i.p.}$ are in good agreement with the experimental data, with a maximum error of 11%. It is worth pointing out that in the case of Cu16at%Al, using the values of μ , and σ from Table II and the α value from Table I and considering values of ρ_m for the annealed sample before deformation taken from Ref. [18] the validity of Eq. (10) is confirmed in that $\rho_m(t_0) = 0.03(2\sigma/(5\alpha\mu b))^2$ for the smallest stress, while for the highest stress the factor 0.03 is replaced by 0.015. On the other hand, there are not enough data (particularly the values of \bar{u}) in the case of Ge to check the condition leading to Eq. (10). For this reason no attempt was made to use Eq. (10) for Ge.

4. Conclusions

Our model explains in a physical way the main features of sigmoidal transients, not only for viscous glide but also for Power-Law Creep. Specifically the model allowed us to arrive at the following conclusions:

- i) For the first time, a physical explanation about the maximum value for the strain rate at the inflection point in sigmoidal creep curves was given.
- ii) The experimental data for $\langle\sigma_i\rangle$ and ρ_m in Germanium and Cu-16at%Al at the inflection point of a sigmoidal creep transient were fully explained.

- iii) The total plastic deformation of the samples at the inflection point for Cu-16at%Al, was also explained.
- iv) Also our analysis allowed us to make a determination of the Taylor factor and the mean self-energy per unit length of dislocation for Cu-16at%Al.
- v) Finally our results indicate that the right determination of the experimental data for the plastic deformation at the inflection point, gives an additional support to both the Fuchs and Ilschner equation, and also to the equation which describes the creation rate of mobile dislocation as given by Montemayor-Aldrete *et al.*

At this stage, it is necessary to emphasize that the application of the equation of Fuchs and Ilschner here developed is not the only one possible. One can, for instance, make a dynamic determination of the drag coefficient for mobile dislocations, from sigmoidal creep curves in Cu-16at%Al. Work along this line is in progress.

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Resumen. Las características principales de los transitorios sigmoidales son explicadas de una manera Física, no solo para termofluencia por deslizamiento viscoso, sino también para la termofluencia que obedece ley de potencias. También se describen de manera completa los datos experimentales correspondientes al promedio del esfuerzo interno, $\langle \sigma_i \rangle$, y, a la densidad de dislocaciones móviles, ρ_m , para el punto de inflexión de curvas sigmoidales en Germanio y Cu-16%atAl. Finalmente también se describe correctamente la dependencia que con la temperatura y el esfuerzo aplicado presenta la deformación plástica total al punto de inflexión en curvas sigmoidales exhibidas por Cu-11.5%atAl.