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Entropy production by neutrinos in the early universe

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Abstract. In this paper we calculate the bulk viscosity of a mixture of electrons, positrons, neutrinos and antineutrinos, at temperatures and densities corresponding to those of the early universe, using a fully relativistic formalism and the scattering cross sections of the Weinberg-Salam theory. We also calculate the entropy generated by the expansion of the Universe in these early stages and show that it is quite negligible in comparison with the total entropy of the Universe.

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1. Introduction

The standard Big Bang theory has been the canonical description of the early universe for many years. One of the main assumptions of this model is that the entropy per baryon was approximately conserved throughout the evolution [1,2] (at least after a possible period of inflation). However, this assumption is only an approximation, since processes out of equilibrium did necessarily occur during the expansion. In particular, it is a well known fact that a mixture of several components can be driven out of thermodynamic equilibrium by an adiabatic expansion if the adiabatic exponents of the components are different, since then each component follows a different cooling law. This was certainly the case in the early universe, because the adiabatic exponents of hadrons, leptons, neutrinos and photons were different at temperatures of the order of or below the corresponding rest mass energy of the particles. It can be shown, however, that thermal equilibrium was restored with a timescale of a few orders of magnitude shorter than the expansion time of the universe, and that, therefore, it is an excellent approximation to assume that the early universe evolved along equilibrium stages [3]. However, a return to thermal equilibrium necessarily generates a certain amount of entropy; therefore it must be shown that this amount is negligible when compared to the entropy of the universe, since otherwise the assumption of constant entropy would not be valid. This problem was studied for the particular case of a mixture of electrons, positrons and radiation, and it was shown that the generated entropy is indeed negligible for a universe with such a composition [3] (hereafter, paper II).

At temperatures around 10^{11} K, however, neutrinos and antineutrinos were as abundant as electrons and positrons, and their contribution to the generation of entropy should also be considered. However, the bulk viscosity and, therefore, the rate of entropy generation, depend on the departure of the actual equation of state of the system from the simple ultrarelativistic equation *pressure* $= \frac{1}{3}$ (*energy density*) (see Eq. 5 of paper II) and, therefore, a simple calculation of this effect is not possible. In the present paper, we use the wholly relativistic treatment developed in Ref. [4] (hereafter, paper I) to a calculate the amount of entropy generated during the expansion of a mixture of electrons, positrons, neutrinos and antineutrinos. In Sec. 2, we present the relevant aspects and formulae of the theory and in Sec. 3 we give our results and conclusions.

2. Theory

a) Description of the system

Consider a mixture of electrons, positrons, neutrinos and antineutrinos, and assume that the time evolution of each type of particle can be described by a Lorentz invariant thermal distribution function $f(x_i, p_i, T_i)$, where the subscript i ($i = 1, \ldots, 4$) refers to the type of particle, x is the position in Minkowski space, p is the four-momentum and T_i is the (time dependent) temperature of particles i. The thermodynamic state of the system is described by its total energy-momentum tensor

$$T^{\mu\nu} = \sum_i T_i^{\mu\nu}, \tag{1}$$

where the energy momentum tensor of particles i is

$$T_i^{\mu\nu} = \left(\frac{g_i c}{h^3}\right) \int d\Gamma_i \, p_i{}^{\mu} p_i{}^{\nu} f_i, \tag{2}$$

where g_i is the occupation number in phase space and $d\Gamma_i$ the invariant momentum space element of particles i. The time evolution of the system is given by Boltzmann's relativistic equation (see Ref. [5])

$$p_i{}^{\mu} f_{i,\mu} = \sum_j C_{ij}, \tag{3}$$

where the comma represents the partial derivative with respect to x and C_{ij} are the collision terms (for more details and explicit expressions of the collision terms, see paper I).

We now make the following simplifying assumptions (for a physical justification see Sec. 2 of paper II):

i) The number densities of electrons and positrons are equal, which implies that their chemical potentials are both equal to zero.

ii) The number densities of neutrinos and antineutrinos are equal, which also implies that their chemical potentials are both equal to zero.

iii) The electron and positron temperatures are equal.

iv) The neutrino and antineutrino temperatures are also equal.

It is not difficult to see that under these conditions, the net energy gain per unit time of electrons and positrons must be equal. Indeed, consider first the electron gas; due to assumption *iii)*, the only interactions that may result in a net energy transfer to (or from) the gas are: $e + e^+ \to \nu + \bar{\nu}$, $e + \nu \to e + \nu$ and $e + \bar{\nu} \to e + \bar{\nu}$. Analogously, the energy gains (or losses) of the positron gas will result from the interactions $e + e^+ \to \nu + \bar{\nu}$, $e^+ + \nu \to e^+ + \nu$ and $e^+ + \bar{\nu} \to e^+ + \bar{\nu}$. It is evident from assumption *i)* that the inelastic collisions ee^+ will result in equal energy losses on both gases. In addition, the energy gain of the electrons through $e\nu$ collisions will be equal to the energy gain of the positrons by $e^+\bar{\nu}$ collisions, since the interactions are CP invariant, and the same holds for $e\bar{\nu}$ and $e^+\nu$ interactions. It follows that the total energy gains of the electron gas and of the positron gas will be always equal. And since the heat capacities of the two gases are equal (by assumption *i)*), the temperature changes induced in both gases by these energy gains will be also equal, *i.e.* the electron gas and the positron gas will maintain equal temperatures throughout the evolution of the system toward thermal equilibrium. It is easy to see that the same result holds for the neutrinos and the antineutrinos (although their temperatures are different from that of the electrons and positrons). It thus follows that the whole system behaves like a binary mixture, one component being the electron-positron gas and the other the neutrino-antineutrino gas.

b) Entropy generation

The entropy generated during the expansion of a system is usually written in terms of a "bulk viscosity" ζ as

$$nk_BT\dot{s} = \zeta(\dot{n}/n)^2, \tag{4}$$

where n is the (constant) baryon density, k_B is Boltzmann's constant, s is the entropy density and the dot represents the derivative along the four-velocity U^μ (the total time derivative): $\cdot = U^\mu \partial/\partial x^\mu$. In our case, the rate of entropy production $\dot{s}$ should be compared with the total entropy of the system which is given by the first law of thermodynamics (with the chemical potentials equal to zero) as:

$$nk_BTs = P + E, \tag{5}$$

where P and E are the total pressure and energy density of the system. Therefore

$$\frac{\dot{s}}{s} = \left(\frac{\dot{n}}{n}\right)^2 \frac{\zeta}{(P+E)}. \tag{6}$$

Now, as we have shown, the system under consideration behaves like an expanding binary mixture, and the bulk viscosity of such a system was calculated in paper I:

$$\zeta = c_T \tau T \left[\left(\frac{\partial P_2}{\partial E_2}\right)_n - \left(\frac{\partial P_1}{\partial E_1}\right)_n \right]^2, \tag{7}$$

where ${c_T}^{-1} = {c_1}^{-1} + {c_2}^{-1}$ is the "reduced" heat capacity per unit volume and at constant volume of the system, $\tau \equiv (\Delta T/\Delta\dot{T})$ is the relaxation time toward thermal equilibrium of the mixture, and P_i and E_i ($i = 1, 2$) are the partial pressures and the energy densities respectively of particles i (in this formulae the subscripts 1 and 2 refer to the two components of the binary mixture and should not be confused with the subscripts $1, \ldots, 4$ that we have been using for the four components of the system; P_1, for example, will be in our case the combined pressure of electrons and positrons, *i.e.*, the partial pressure of "component 1"). We will now use Eq. (7) to calculate the fractional amount of entropy produced by the interactions between neutrinos, antineutrinos, electrons and positrons during the early expansion of the universe.

c) Calculations

The relaxation time τ of a mixture of electrons, positrons, neutrinos and antineutrinos has been calculated, as a function of the temperature, for the particular case of a small temperature difference between the electron-positron gas and the neutrino-antineutrino gas (paper II). (It should be stressed that the exact cross sections given by the Weinberg-Salam theory, including both charged and neutral currents, were used in this calculation). To make use of this result, we adopt in the following the same assumption, *i.e.*, we take the temperature of the electron-positron gas to be T and the temperature of the neutrino-antineutrino gas to be $T + \Delta T$ with $\Delta T/T \ll 1$. The "reduced" heat capacity c_T may be calculated by noting that the value of the heat capacity of "component 1" will be twice that of the electron gas, c_e, and that of "component 2" will be twice the heat capacity c_ν of the neutrino gas. These, in turn, are given by [Eqs. (15) and (16) of paper III]:

$$c_e = 4\pi \left(\frac{mc}{h}\right)^3 k_B \phi^2 \int_0^\infty \frac{\sinh^2\phi \cosh^3\phi}{\cosh(\phi\cosh\phi) + 1} d\phi, \tag{8}$$

$$c_\nu = 84\pi \left(\frac{mc}{h}\right)^3 k_B \zeta(4) \phi^{-3}, \tag{9}$$

where we have introduced the "temperature parameter" $\phi \equiv mc^2/k_B T$, and where

m is the mass of the electron, c is the velocity of light, h is Planck's constant, and $\zeta(4) = 1.0823$ is Riemann's zeta function of argument 4.

Analogously, the pressures and the energy densities of the "two components" will be twice the pressure and the energy density of the electron gas and of the neutrino gas, respectively. We thus have

$$P = P_1 + P_2 = 2p_1 + 2p_2 \tag{10}$$

and

$$E = E_1 + E_2 = 2e_1 + 2e_2. \tag{11}$$

These factors of two, however, are not important for our purposes since we are only interested in the derivatives $(\partial P_i/\partial E_i)$. The value of this derivative for the electron gas follows directly from the explicit expressions for its pressure and energy density (see, for instance, [7]), which may be written, for the case of zero chemical potential, as

$$\left(\frac{\partial p_e}{\partial e_e}\right) = \frac{A}{3B}, \tag{12}$$

where

$$A = \int_0^\infty \frac{\sinh^4 \phi}{\exp(\phi \cosh \phi) + 1} d\phi, \tag{13}$$

and

$$B = \int_0^\infty \frac{\sinh^2 \phi \cosh^2 \phi}{\exp(\phi \cosh \phi) + 1} d\phi. \tag{14}$$

As for the neutrinos, we obviously have

$$\frac{\partial p_\nu}{\partial e_\nu} = \frac{1}{3}. \tag{15}$$

Our calculation is complete with the above equations.

3. Results and conclusions

Our results for the bulk viscosity ζ and the fractional entropy generation $\dot{s}/s$ are shown in Figs. 1 and 2 for values of the temperature parameter ϕ between .01 and 0.2, which correspond to "cosmic" times between 0.00029 and 0.114 seconds in the standard Einstein-de Sitter model of the early universe (with 3 families of neutrinos —note that μ and τ neutrinos do not contribute to the generation of entropy, because they had already decoupled at the considered epoch). The calculation cannot be

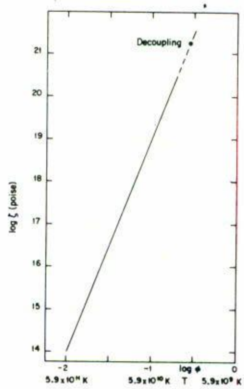

FIGURE 1. Bulk viscosity ζ as a function of the temperature parameter ϕ for an expanding mixture of electrons, positrons, neutrinos and antineutrinos.

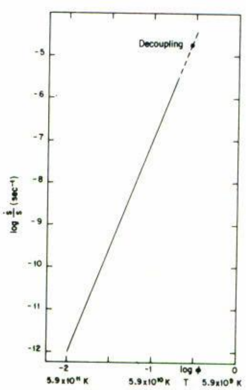

FIGURE 2. Fractional amount of entropy generated per unit time $\dot{s}/s$ during the expansion of a mixture of electrons, positrons, neutrinos and antineutrinos, as a function of the temperature parameter ϕ.

extended to lower temperatures (longer times) because the neutrinos decoupled from matter at $\phi = 0.25$ (see paper II).

It is readily seen in Fig. 1 that the bulk viscosity follows the law $\zeta = 1.1 \times 10^{24} \phi^5$ poise with an excellent approximation.

Finally, we see in Fig. 2 that the amount of entropy generated by the interactions between neutrinos, electrons and their antiparticles during the expansion was indeed negligible up to the moment of decoupling. Since the amount of entropy generated by the interactions between electrons, positrons and radiation was also negligible (see paper I), we conclude that the assumption of constant entropy during the early expansion phases of the universe is an excellent approximation.

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Resumen. En este artículo calculamos la viscosidad volumétrica de una mezcla de electrones, positrones, neutrinos y antineutrinos, a temperaturas y densidades correspondientes a las del Universo Temprano, usando un formalismo relativista y las secciones de dispersión de la teoría de Weinberg-Salam. También calculamos la entropía generalizada por la expansión del Universo en las primeras etapas y demostramos que es despreciable en comparación con la entropía total del Universo.