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The price of simple unification

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Abstract. We present the results obtained from our systematic search of a simple Lie group G that unifies weak and electromagnetic interactions in a single truly unified theory. We work with fractionally charged quarks and allow for particles and antiparticles to belong to the same irreducible representation. We find that models based on $\mathrm{SU}(N)$, $N = 6, 7, 8, 9$ and 10 are the only viable candidates for simple unification.

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I. Introduction

The standard electroweak model [1] unifies weak and electromagnetic interactions using the group $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. Since this group is not simple (nor semisimple) [2], two different coupling constants are needed. Therefore the mixing angle, θ_{W}, between the two factors is not fixed by the model. Moreover, the electric charge is not quantized as a consequence of the U(1) factor.

At the present time, it is widely believed that as one probes matter with higher and higher energies, bigger and bigger unification symmetries will uncover. Jut above $M_{e\omega} \sim 100$ Gev nature should look explicitly $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ symmetric. At $M_{\mathrm{GUT}} \sim 10^{14}$–$10^{15}$ Gev the grand unification group, the symmetry underlying the unification of strong and electroweak interactions, should become fully visible. Between these two scales, $M_{e\omega}$ and M_{GUT}, several partial unifications may take place. There is therefore ample room for the question: Could the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ theory be embedded in one which is based on a local group G which is simple but which does not include yet the strong interactions? If so, at what price? And what would then be the low energy signatures of such *Simple Unification?*

In this work we derive the set of simple gauge groups that may be considered as candidates for electroweak unification. Our derivation rests on certain assumptions among which we may underline the hypothesis that the gauge group does not perform horizontal transformations and that particles and their antiparticles may belong to the same irreducible representation. We also impose the condition that known quarks and leptons should transform under $SU(2)_L \otimes U(1)_Y$ in the standard way which is now firmly established [3]; in particular we assume that ordinary quarks have fractional electric charge and not integer ones [4].

Previous analysis of the possibility of simple unification were performed at a time when not all the presently known quarks and leptons have yet been discovered and when their now standard transformation under $SU(2)_L \otimes U(1)_Y$ was not so well established. Some of these models led then to predictions for the weak neutral current that are now ruled out experimentally [8–10]. We should also mention the general investigation of S. Okubo [11] of the possibility that G is semi-simple under the assumption that quarks have only charges $2/3$ and $-1/3$ with the conclusion that for hadrons G is essentially a product of $SU(3l)$ groups. With semisimple groups that are products of isomorphic groups G'

$$[G']^n = G' \otimes G' \otimes \cdots \otimes G'$$

(for which reason they are also called pseudosimple [12]) it is possible to have a theory with a single gauge coupling constant if the group G is enlarged by means of the discrete cyclic group Z_n: $G = Z_n \times [G']^n$, where the indicated product is a semidirect one [13]. The extended group is then simple although the Lie algebra is not. We will not deal with this type of groups in this work.

The paper is organized as follows: In Section 2 we present our working hypothesis and their justification. In Section 3 we show that the groups $SU(n)$, $3 \leq n \leq 10$, are the only ones compatible with our working hypothesis. Conclusions are presented in Section 4.

All the candidates for simple unification necessarily contain extra gauge bosons (necessarily heavier than the $SU(2)_L \otimes U(1)_Y$ ones) not present in $SU(2)_L \otimes U(1)_Y$ and they also contain additional fermions. The fermion content of each model is chosen in such a way so as to make the model free of triangular anomalies [16] and the charge of unknown fermions is then fixed by the quantization of charge.

Since the group is simple, $\sin^2 \theta_W$ may be computed at the simple unification scale. If in a particular model its value is too large then the simple unification scale has to be also too large and we are lead to discard that model. We find that models based on $SU(N)$, $N = 6, 7, 8, 9$, and 10 are the only viable candidates for simple unification. We have discussed in detail the $SU(7)$ model elsewhere [17].

2. Hypotheses

We look for a simple unification scenario of the electroweak interactions, with a scheme based on a gauge group G such that:

1. G is a simple Lie group with complex irreducible representation (ireps).
2. G contains $SU(2)_L \otimes U(1)_Y$ and known quarks and leptons transform in the standard way.

3. Particles and their antiparticles may belong to the same irreducible representation.

4. G does not contain the strong interaction group. The strong interactions are described by a separate theory, $SU(3)_c$ of color, which is asymptotically free.

5. The elements of G do not perform horizontal transformations between different families of quarks and leptons nor between quarks and leptons.

6. The structure must be free of triangle anomalies [16].

1) The demand that G is simple excludes semisimple groups. We also exclude from our analysis pseudosimple groups. That G must have complex irreps is a criteria for unification suggested by Georgi and by Gell-Mann, Ramond and Slanky [18]. These criteria stem from the fact that at low energies the standard $SU(2) \otimes U(1)$ model is flavor chiral; that is, the left and right handed components behave differently at low energies.

2) Langacker [3] has shown that the constraints from high precision charged and neutral current experiments are enough to directly establish the canonical (left handed doublet, right handed singlet) assignments for all the known fermions implying in particular the existence of the top quark and of ν_τ. We also assume that quarks have fractional electric charge (2/3 and $-1/3$). The charge of fermions outside the set of ordinary ones is not restricted (except by charge quantization). Therefore conclusions based on the assumption that *all* quarks have either charges $\pm -1/3+$ integers [11] or 2/3 and $-1/3$ [11], [12], [19] do not apply.

Instead of dealing with right handed fermion fields ψ_R, we will work with their charge conjugated objects ψ_L^c which are left-handed. Each family of G should then contain at least the following fermions, all left handed and with the indicated $SU(2)_L \otimes U(1)_Y$ content:

$$\begin{array}{lll} (u,d)^T = \{2(1/3)\} & u^c = \{1(-4/3)\} & d^c = \{1(2/3)\} \\ (\nu,e)^T = \{2(-1)\} & e^c = \{1(2)\} & \nu^c = \{1(0)\} \end{array} \qquad (1)$$

although the last one, the right handed neutrino, may be absent in some models.

Turning back to question of complex representations, we may say that if an ordianry doublet, say $(u,d)_L$, belongs to a real representation of G, then a second doublet of the type $(U^c, D^c)_L$ should also belong to the same representation. Then either $(U^c, D^c)_L = (u^c, d^c)_L$ or u^c mixes with U^c and d^c with D^c. Both possibilities have been shown to be very unlikely [3],[20].

3) The assumption that antiparticles may belong to the same irreducible representation as the corresponding particles limits the proliferation of fermions. In such a situation, however, the hadronic weak currents have a color nonsinglet part (which causes no problem because their matrix elements between color singlet states vanish).

4) Quarks (ordinary and exotics) belong to the $\{3\}$ and $\{\overline{3}\}$ irreps of $SU(3)_c$.

The total number of quark flavors, n_f, has to satisfy the relation $n_f < 33/2$ in order to keep $SU(3)_c$ asymptotically free [21].

3. Analysis

Since the generators of G do not connect quarks to leptons, we must assign quarks and leptons to different irreps of G. Since known quarks carry fractional charge and leptons integer one, the irreps of G which contain quarks must be in general of lower dimension than those to which leptons belong.

The complex irreps of the simple Lie algebras have been classified by Mehta and Srivastava [22]. The result is that only $SU(n)$, $n > 2$, $SO(4n+2)$ and E_6 have complex irreps. The constraint of asymptotic freedom eliminates the $SO(4n+2)$ and E_6 groups. The reason is the following: $SO(6)$ is equivalent to $SU(4)$. For $SO(10)$, its lowest dimensional complex irrep is the 16 dimensional one [23], which can accommodate at least eight quark flavors. If the number of families is three, then $n_f \geq 24$ in conflict with hypothesis 4. For the remaining groups $SO(14), SO(18), \ldots$, n_f is even larger. For the same reason E_6 is eliminated since its lowest dimensional complex irrep is the $\{27\}$.

For $SU(n)$ the lowest dimensional complex irrep is the fundamental one, of dimension n, which may contain at least $n/2$ flavors. If the number of families is three (four), then $n \leq 10(8)$. Therefore we will consider in what follows only the complex irreps of $SU(n)$ for $3 \leq n \leq 10$.

In each case we will compute $\sin^2\theta_W$ at the simple unification scale, M_U, according to the following formula [24],

$$\sin^2\theta_W(M_U) = \frac{\mathrm{tr}\, I_3^2}{\mathrm{tr}\, Q^2} \tag{2}$$

where the traces can be computed in any representation of G and where I_3 and Q are the generators of G related to the third component of weak isospin and electric charge respectively. The value of $\sin^2\theta_W(M_U)$ is related to $\sin^2\theta_W(M_W)$ by the renormalization group equations [24]

$$\sin^2\theta_W(M_W) = \sin^2\theta_W(M_U) - \alpha(M_W) \times \frac{109}{48\pi}\ln\frac{M_U}{M_W} \tag{3}$$

$$\frac{1}{\alpha(M_W)} = \frac{1}{\alpha(0)} - \frac{2}{3\pi}\sum_f Q_f^2 \ln\frac{M_W}{m_f} + \frac{1}{6\pi} \tag{4}$$

where α is the fine structure constant, M_W is the mass of the W, and the sum runs over all fermions of mass m_f. These equations, together with the measured values of $\alpha(0)$ and $\sin^2\theta_W(M_W)$, fix the scale M_U. These equations are valid only if the breaking of G down to $SU(2)_L \otimes U(1)_Y$ is made in a single step [25].

SU(3)

With $n_f < 33/2$ and three families, quarks may belong only to irreps $\{3\}$, $\{6\}$, and $\{10\}$ (irrep $\{8\}$ is real). If the number of families is 4, then irrep $\{10\}$ is excluded. The $SU(2) \otimes U(1)$ content of these representations is

$$\begin{aligned}\{3\} &= \{2(a/3)\} + \{1(-2a/3)\},\\ \{6\} &= \{3(2a/3)\} + \{2(-a/3)\} + \{1(-4a/3)\},\\ \{10\} &= \{4(a)\} + \{3(0)\} + \{2(-a)\} + \{1(-2a)\},\end{aligned}$$

where "a" is a U(1) normalization factor whose value is chosen depending on the SU(3) representation to which the ordinary quark doublet $(u,d)_L$ belongs.

To compute $\sin^2\theta_W$ we may use for simplicity the $\{3\}$ where

$$\begin{aligned}I_3 &= \mathrm{diag}(1,-1,0)/2,\\ Q &= \mathrm{diag}(1+a/3,-1+a/3,-2a/3)/2,\end{aligned}$$

which gives

$$\sin^2\theta_W(M_U) = (1+a^2/3)^{-1}. \tag{5}$$

Therefore, if $(u,d)_L$ belongs to the $\{3\}$ $(a=1)$ or to the $\{6\}$ $(a=-1)$ then $\sin^2\theta_W = 3/4$, which is too large. Besides, with $a=1$, the ordinary fermion content, Eq. (1), demands that at least the $\{3\}$, $\{\bar{3}\}$ and $\{6\}$ should contain quarks giving $n_f \geq 18$ and ruining asymptotic freedom. For $a=-1$ the quark representations are at least $\{6\}$, $\{\bar{6}\}$, and $\{3\}$ and $n_f \geq 23$. If $(u,d)_L$ belongs to the $\{10\}$, then $a=-1/3$, $\sin^2\theta_W = 27/28$ and the known quarks can not be accommodated in representations of dimension $\{10\}$, $\{6\}$ and $\{3\}$. Thus, we have to discared SU(3) as a candidate for simple unification.

SU(4)

With $n_f < 33/2$ and three families, quarks may belong only to irreps $\{4\}$ or $\{10\}$ (irrep $\{6\}$ is real). If the number of families is 4, then irrep $\{10\}$ is excluded. The

$SU(2) \otimes SU(2) \otimes U(1)$ and $SU(2) \otimes U(1)$ content of these representations are

$$\begin{aligned} \{4\} &= \{(2,1)(b/3)\} + \{(1,2)(-b/3)\} \\ &= \{2(b/3)\} + \{1(a-b/3)\} + \{1(-a-b/3)\}, \\ \{10\} &= \{(3,1)(2b/3)\} + \{(1,3)(-2b/3)\} + \{(2,2)(0)\} \\ &= \{3(2b/3)\} + \{1(2a-2b/3)\} + \{1(-2b/3)\} + \{1(-2a-2b/3)\} \\ &\quad + \{2(a)\} + \{2(-a)\}. \end{aligned} \tag{6}$$

Here a and b are free parameters coming from the U(1) factors. Irrep $\{10\}$ is ruled out because there are no a and b values such that it contains $\{2(1/3)\} + \{1(2/3)\} + \{1(-4/3)\}$. We can accommodate quarks and antiquarks in irrep $\{4\}$ if we choose $a = b = 1$. Then $\{4\} = (u, d, u^c, d^c)$ and $\sin^2\theta_W(M_U) = 0.45$, a value too large to be taken seriously (it would imply a simple unification scale larger than the Planck scale). We are then led to discared SU(4).

SU(5)

For three families in SU(5), quarks may belong only to irreps $\{5\}$ or $\{10\}$. If the number of families is 4, then irrep $\{10\}$ is excluded. Ordinary quarks $(u,d)_L$, u_L^c, d_L^c may belong either to the $\{5\}$ or to $\{5\} + \{\bar{5}\}$ or to the $\{\overline{10}\}$. In the first case there should be, by charge quantization, a neutral quark in the same multiplet. Except for this neutral the situation is similar to that in SU(4) and we obtain the same value for $\sin^2\theta_W$, that is 0.45, which is unacceptable.

Our hypothesis does not allow to consider the possibility that ordinary quarks belong to $\{5\} + \{\bar{5}\}$. We may however mention that if that were the case one could obtain a reasonable value for $\sin^2\theta_W$. Indeed, in this case the $SU(2)_L \otimes U(1)_Y$ decomposition of $\{5\}$ may be either

$$\{5\} = \{2(1/3)\} + \{1(-4/3)\} + \{1(-2/3)\} + \{1(4/3)\}$$

or

$$\{5\} = \{2(1/3)\} + \{1(4/3)\} + \{1(2/3)\} + \{1(-8/3)\}.$$

This implies either $\sin^2\theta_W(M_U) = 0.33$ or 0.18 respectively.

If ordinary quarks belong to the $\{\overline{10}\}$, then

$$\begin{aligned} \{\overline{10}\} &= \{2(1/3)\} + \{2(1)\} + \{2(-1)\} + \{1(-4/3)\} + \{1(2/3\} + \{1(2/3\} \\ &\quad + \{1(-2/3\} \end{aligned} \tag{7}$$

and $\sin^2\theta_W(M_U) = 0.45$ again. Therefore SU(5) is also excluded.

SU(6)

Here, and for larger SU(n) groups, quarks may belong only to the fundamental representation whose $SU(2)_L \otimes U(1)_Y$ decomposition should be $\{2(1/3)\}+\{1(-4/3)\}+\{1(2/3)\}$ plus more $SU(2)_L$ singlets. If the charge of the additional (Weyl) singlets is q_i then

$$\sin^2\theta_W(M_U) = \frac{1/2}{10/9+\sum_i q_i^2}, \tag{8}$$

which means that the charge of the additional singlets cannot vanish, since otherwise we would have $\sin^2\theta_W(M_U) = 0.45$ again. They also cannot be too large. But there are certainly possibilities to obtain reasonable values for $\sin^2\theta_W$.

The SU(n) models are therefore viable for $6 \leq n \leq 10$ provided leptons belong to such representations that triangle anomalies are canceled. The $SU(2)_L \otimes U(1)_Y$ content of the leptonic representations depend on the charge assignments of the additional quarks in the fundamental representation.

For SU(6) if we write

$$\{6\} = \{2(1/3)\} + \{1(-4/3)\} + \{1(2/3)\} + \{1(2a)\} + \{1(-2a)\},$$

then

$$\begin{aligned}\{15\} = {} & \{2(1)\} + \{2(-1)\} + \{1(0)\} + \{1(2/3)\} + \{1(-2/3)\} \\ & + \{2(2a+1/3)\} + \{2(-2a+1/3)\} + \{1(2a+2/3)\} \\ & + \{1(-2a+2/3)\} + \{1(2a-4/3)\} + \{1(-2a-4/3)\}.\end{aligned} \tag{9}$$

If $a = 2/3$ then the charge of the additional quark is $2/3$ or $-2/3$, $\sin^2\theta_W(M_U) = 1/4$ and a set of leptons that cancel the triangle anomalies is one $\{15\}$ (anomaly $= 2$) plus five $\{\overline{6}\}$ (anomaly $= -1$) [26]. The price of simple unfication includes in this case the proliferation of additional [to those of Eq. (1)] fermions: one quark of charge 2/3 and six leptons of charge 2/3, five of 1/3, one of 4/3, one of 1, and three Weyl states of charge 0 per family.

If $a = 1/3$ then $\sin^2\theta_W(M_U) = 3/8$ and a set of leptons that cancel the anomalies is one $\{\overline{15}\}$ plus a $\{\overline{6}\}$. Although this case contains also many additional fermions, it contains less than the previous one (with $a = 2/3$): one quark of charge 1/3, four leptons of 1/3, two of 2/3, one of 1 and three Weyl states of charge 0 (per family).

SU(7)

In this case it is possible to cancel the anomaly of the quark representation, the $\{7\}$, with a single irreducible representation of leptons (and which contains the ordinary ones), the $\{\overline{21}\}$ (the second rank antisymmetric one whose anomaly is -3). Indeed, if we write the SU(6) ⊗ U(1) decompostion of SU(7) as

$$\{7\} = \{1(6)\} + \{6(-1)\},$$

then

$$\{21\} = \{6(5)\} + \{15(-2)\}$$

and the particle content is the same as the $a = 1/3$ SU(6) model (plus one neutral quark), leading therefore to $\sin^2\theta_W(M_U) = 3/8$. A detailed description of this model has been given in Ref. [17]. Here we shall only add that one helicity state o' the neutral quark is in the {7} and the other in a SU(7) singlet.

SU(8)

Writing the $SU(2)_L \otimes U(1)_Y$ decomposition of the fundamental representation of SU(8)in terms of two free parameters a and b as

$$\begin{aligned}\{8\} = {} & \{2(1/3)\} + \{1(-4/3)\} + \{1(2/3)\} + \{1(2a)\} + \{1(-2a)\} \\ & + \{1(2b)\} + \{1(-2b)\},\end{aligned} \tag{10}$$

the second rank antisymmetric tensor representation is decomposed as

$$\begin{aligned}\{28\} = {} & \{1(2/3)\} + \{2(-1)\} + \{2(1)\} + \{2(1/3+2a)\} + \{2(1/3-2a)\} \\ & + \{2(1/3+2b)\} + \{2(1/3-2b)\} + \{1(-2/3)\} + \{1(-4/3+2a)\} \\ & + \{1(-4/3-2a)\} + \{1(-4/3+2b)\} + \{1(-4/3-2b)\} + \{1(2/3+2a)\} \\ & + \{1(2/3-2a)\} + \{1(2/3+2b)\} + \{1(2/3-2b)\} \\ & + \{1(0)\} + \{1(2a+2b)\} + \{1(2a-2b)\} + \{1(-2a+2b)\} \\ & + \{1(-2a-2b)\} + \{1(0)\}.\end{aligned} \tag{11}$$

a and b represent the value of the charge of the additional quarks. Some interesting solutions are the following (the anomaly of irrep {28} is 4):

a	b	$\sin^2\theta_W(M_U)$	minimal leptonic representation
2/3	0	1/4	$\{28\} + 7\{\overline{8}\}$
1/3	1/3	9/28	$\{\overline{28}\} + \{8\}$
1/3	0	3/8	$\{\overline{28}\} + \{8\}$

The number of additional fermions is in this case bigger than in the previous ones.

SU(9)

For four families SU(9) and SU(10) are still viable candidates. Known leptons can be fitted in irrep $\{36\}$ and quarks in $\{9\}+\{1\}$ with the same quarks as in SU(8) plus one extra neutral one. The anomaly of the $\{36\}$ is 5, therefore many exotics must be added in order to have an anomaly free model.

SU(10)

As in the previous cases, ordinary quarks of one family are placed in the fundamental representation and the charge of the additional quarks in this representation determines the minimal leptonic content of a particular model. For SU(10) some interesting possibilities are (a, b, and c are the charges of the three additional Dirac quarks in one family)

a	b	c	$\sin^2\theta_W(M_U)$
1/3	1/3	1/3	9/32
1/3	1/3	0	9/28
1/3	0	0	3/8

Since the anomaly of $\{45\}$ is 6, the minimal leptonic representation in the three cases is one $\{\overline{45}\}$ plus three $\{10\}$'s.

4. Conclusions

The price of simple unfication of electroweak interactions is: *1)* the presence of additional charged quarks, maybe some neutrals; *2)* the dimension of the leptonic representation is not the same as that of the quark representation; *3)* many additional leptons, the majority with fractional charge; *4)* and, of course, many additional electroweak gauge fields.

What about the masses of the exotic fermions? In a particular model, SU(7), we have shown [17] that the exotic fermions may acquire masses in the Tev regime or above, in accordance with the so called survival hypothesis [27]. This hypothesis states that if a symmetry G is broken down to G' at the mass scale M, then any fermionic representation which is vectorial with respecto to G' (but not to G) gets a mass of order M. (See also Ref. [28]). The fact that $\sin^2\theta_W(M_U)$ lies typically in the interval $(1/4, 3/8)$ means that the scale where the simple unfication group breaks down lies high above the TeV regime.

The models presented in this paper all share the characteristic that fermions and antifermions are together in the same irrep. Therefore G does not commute with color SU(3), and the hadronic weak currents have a color nonsinglet part. As has been shown elsewhere [5], [6], [7], [17], this color nonsinglet part causes no problem because their matrix elements between color singlet states vanish.

Also, since leptons and antileptons are together int he same irrep, simple unification models lead to lepton number violating processes such as $e^-e^- \to \mu^-\mu^-$; $e^-\mu^+ \to e^+\mu^-$; etc. The amplitudes for these processes should however be suppressed by powers of M_W/M_U.

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Note added in proof. A work related to ours is reported in *J. Math. Phys.* **23** (1982) 2529 (Anomaly-free complex representations in SU(N)) by E. Eichten, K. Kang and I.-G. Koh. There one can find all irreducible and reducible complex representations of simple Lie groups which are anomaly free and satisfy the one loop asymptotic freedom condition. However, the meaning of this condition is not the same as that of our hypothesis 4. In the above Ref. asymptotic freedom is applied to the whole unifying group G, while in our case it is applied to the group $SU(3)_c$ of color which lies outside our unifying group.

When dealing with a grand unifying group, the problem of finding the representations that satisfy a given set of conditions gets more involved as can be appreciated in K. Kang and I.G. Koh, *Phys. Rev. D* **25** (1982) 1700 and in K. Kang, Ch.K. Kim and J.K. Kim, *Phys. Rev. D* **33** (1986) 260. SU(7) is discussed in the first Ref. and $SU(9)$ in the second.

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26. The anomaly of an irrep will always be stated in units of the anomaly of the fundamental representation. The anomaly $A_{m,N}$ associated with an antisymmetric representation of $\mathrm{SU}(N)$ with m boxes is [16]

$$A_{m,N} = \frac{(N-3)!(N-2m)}{(N-m-1)!(m-1)!}.$$

27. H. Georgi, *Nucl. Phys. B* **156** (1979) 126; R. Barbieri *et al.*, *Phys. Letters* **90B** (1980) 91.
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Resumen. Presentamos los resultados que hemos obtenido en la búsqueda sistemática de un grupo G de Lie simple que unifique las interacciones débiles y electromagnéticas en una teoría verdaderamente unificada. Nos limitamos a modelos en los que los quarks tienen carga fraccionaria y permitimos que las antipartículas pertenezcan a la misma representación irreducible donde están las correspondientes partículas. Encontramos que, dados los criterios que se especifican en el texto, los modelos basados en SU(N), con $N = 6, 7, 8, 9$ y 10, son los únicos candidatos para unificación simple.