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Neutrino mixings and right-handed currents in τ_{M2} decays

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Abstract. The $\tau \to M\nu_\tau$ decays are revisited in the framework of an effective weak interaction Hamiltonian with neutrino mixings and right-handed currents. Hierarchical and Kobayashi-Maskawa neutrino mixings are considered in the evaluation of the ratio $R = \sum_i w(\tau \to M\nu_i)/w(\tau \to M\nu_\tau)$, and manifest left-right symmetry is assumed in our calculations.

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1. Introduction

With the advent of new τ-lepton factories, the decays and properties of this particle will be studied to a great extent, leading to a deep insight in the nature of the weak currents involved in these processes. In particular, the τ_{M2} decays, where M is a meson, are the simplest of the τ-decays to look for massive neutrinos and right-handed currents, since we are in a two-body final state process [1]. Furthermore, some of these decays have high branching ratios, allowing high statistics studies. [For instance if M is a vector (pseudoscalar) particle like the $\rho(\pi)$, we have branching ratio of about 22% (10%)]. The τ_{M2} decays are a subset of all the exclusive decay modes of the tau. This decays have been extensively studied, within an effective $V - A$ theory and massless neutrinos by some authors. We refer the reader to Ref. [2] for a review.

Since, up to now, there is no fundamental principle requiring neutrino zero mass, massive neutrinos are to be considered in any theory of weak interactions. In particular models of weak interactions with right-handed currents must involve a finite neutrino mass. Within the framework of this class of models, the $\tau \to M\nu_\tau$ decays must be considered as an incoherent sum of decay modes $\tau \to M\nu_i$, where ν_i denotes a neutrino mass eigenstate of mass m_i distinct from the weak eigenstate ν_τ which is a sum of the ν_i times a neutrino mass-mixing matrix factor $U_{\tau i}$. The index i runs from one to three for a 3-generation model. The masses of the neutrino mass eigenstates are commonly supposed to be in ascending order of values, *i.e.* $m_1 < m_2 < m_3$. This is the situation in the nondegenerated case when ν_1, ν_2, and ν_3 are, respectively, ν_e, ν_μ, and ν_τ [3].

In this paper we devote ourselves to the study of these decays, looking for effects due to neutrino mixings and right-handed currents. In Sect. 2 we give the amplitude for the four Feynman diagrams and some details of the model under consideration. In Sect. 3 we give the results for the ratio $R = \sum_i w(\tau \to M\nu_i)/w(\tau \to M\nu_\tau)$, with M a pseudoscalar (π, K) or vector meson (ρ, K^*), for the Hierarchical and the Kobayashi-Maskawa mixings, and for manifest left-right symmetry. In Sect. 4 we present our conclusions.

2. Amplitude for $\tau \to M\nu_i$

In considering neutrino mixing and right-handed currents we have four diagrams contributing to the amplitude for $\tau \to M\nu_\tau$. These are

$$M_{(a)} = \frac{G}{2\sqrt{2}}\bar{u}_{\nu_\tau}\gamma_\mu(1-\gamma_5)u_\tau\langle M|J^\mu_{W\,L}(0)|0\rangle \tag{1}$$

mediated by W_L,

$$M_{(b)} = \frac{G}{2\sqrt{2}}\eta\bar{u}_{\nu_\tau}\gamma_\mu(1+\gamma_5)u_\tau\langle M|J^\mu_{W\,L}(0)|0\rangle \tag{2}$$

mediated by $W_R - W_L$ mixing,

$$M_{(c)} = \frac{G}{2\sqrt{2}}K\bar{u}_{\nu_\tau}\gamma_\mu(1-\gamma_5)u_\tau\langle M|J^\mu_{W\,R}(0)|0\rangle \tag{3}$$

mediated by $W_L - W_R$ mixing, and

$$M_{(d)} = \frac{G}{2\sqrt{2}}\lambda\bar{u}_{\nu_\tau}\gamma_\mu(1+\gamma_5)u_\tau\langle M|J^\mu_{W\,R}(0)|0\rangle \tag{4}$$

mediated by W_R.

In Eqs. (1) to (4), $\langle M|J^\mu_{W\,L(R)}(0)|0\rangle$ represents the hadronic matrix element of the left (L) or right (R) handed current associated to the $W_{L(R)}$. The parameters η, κ and λ measure the magnitude of the left-right mixing and right-handed currents. The weak eigenstates neutrino $\nu_{\tau L}$, $\nu_{\tau R}$ are assumed to be superpositions of mass-eigenstate neutrinos N_j with mass m_j [4]

$$u_{\nu_\tau L} = \sum_j U_{\tau j}N_{jL}, \tag{5}$$

$$u_{\nu_\tau R} = \sum_j V_{\tau j}N_{jR}. \tag{6}$$

An appropriate choice of the matrices U and V leads us to the Dirac and Majorana neutrino cases. (No mixing means $U_{\tau j} = V_{\tau j} = \delta_{\tau j}$). The hadronic matrix element is given by

$$\langle M|J^{\mu}_{W\,L}(0)|0\rangle = \begin{cases} f_p U^{*}_{\rm KM} p^{\mu} \\ \\ f_v U^{*}_{\rm KM} \epsilon^{\mu}(p) \end{cases} \tag{7}$$

and

$$\langle M|J^{\mu}_{W\,R}(0)|0\rangle = \begin{cases} f'_p U'^{*}_{\rm KM} p^{\mu} \\ \\ f'_v U'^{*}_{\rm KM} \epsilon^{\mu}(p) \end{cases}, \tag{8}$$

where $f_p(f'_p)$ and $f_v(f'_v)$ are the decay form factors for the case where M is a pseudoscalar meson of 4-momentum p^{μ}, and a vector meson of polarization four-vector $\epsilon^{\mu}(p)$. $U_{\rm KM}$ and $U'_{\rm KM}$ are Kobayashi-Maskawa mixing matrices for the left and right handed hadronic currents, respectively.

Adding Eqs. (1)–(4) and substituting Eqs. (5)–(8) we obtain for the decay amplitude

$$M_j = \frac{G}{2\sqrt{2}} U^{*}_{\rm KM} \begin{Bmatrix} f_p \\ f_v \end{Bmatrix} \left[F_j \bar{N}_{jL} \gamma_{\mu}(1-\gamma_5) u_{\tau} + F'_j \bar{N}_{jR} \gamma_{\mu}(1+\gamma_5) u_{\tau} \right] \begin{Bmatrix} p^{\mu} \\ \epsilon^{\mu}(p) \end{Bmatrix} \tag{9}$$

where

$$F_j = \left(1 + K \begin{Bmatrix} f \\ f' \end{Bmatrix}\right) U_{\tau j}, \tag{10}$$

$$F'_j = \left(\eta + \lambda \begin{Bmatrix} f \\ f' \end{Bmatrix}\right) V_{\tau j}, \tag{11}$$

and

$$f = \frac{f'_p U'_{\rm KM}}{f_p U^{*}_{\rm KM}}, \qquad f' = \frac{f'_v U'_{\rm KM}}{f_v U^{*}_{\rm KM}}. \tag{12}$$

3. Total decay rate

To compute the total decay rate $w(\tau \to M\nu_{\tau})$, we proceed as usual: we sum over final spins (or polarization) and average over the initial one, and integrate over final phase space. The result is

$$w(\tau \to M\nu_\tau) = \left(\frac{G}{\sqrt{2}}\right)^2 \sum_j \frac{1}{4\pi} p_{(j)} \left\{ \begin{matrix} Mf_p \\ f_v \end{matrix} \right\}^2$$

$$\times \left[(|F_j|^2 + |F'_j|^2) \times \left\{ \begin{matrix} (1+\delta_j)(1-\delta+\delta_j) - 4\delta_j \\ \dfrac{(1+\delta_j)(1-\delta+\delta_j) - 4\delta_j}{\delta} + 2(1-\delta+\delta_j) \end{matrix} \right\} \right.$$

$$\left. + 2\,\mathrm{Re}(F_j F'^*_j)\sqrt{\delta_j} \left\{ \begin{matrix} \delta \\ -3 \end{matrix} \right\} \right] \tag{13}$$

where $\delta = m^2/M^2$, $\delta_i = m_i^2/M^2$, with m the meson mass and M the τ mass; $p_{(j)}$ is given by

$$p_{(j)} = \frac{M}{2} \lambda^{1/2}(1, \delta, \delta_j) \tag{14}$$

with $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$.

In Eqs. (13) we have absorbed the $|U_{\mathrm{KM}}|^2$ factor into the decay constants f_p and f_v and the upper (lower) line corresponds to the pseudoscalar (vector) meson case. The sum is over the incoherent neutrino mass eigenstates only.

For left-handed currents only and no neutrino mixings we obtain the result [5–6]

$$w^0(\tau \to M\nu_\tau) = \left(\frac{G}{\sqrt{2}}\right)^2 \frac{p}{4\pi} \left\{ \begin{matrix} Mf_p \\ f_v \end{matrix} \right\}^2 \left\{ \begin{matrix} (1-\delta) \\ (\frac{1}{\delta}+2)(1-\delta) \end{matrix} \right\} \tag{15}$$

where $p = \frac{M}{2}(1-\delta)$.

The rate for the mode $\tau \to M\nu_\tau$, Eq. (13), relative to that for the conventional decay $\tau \to M\nu_\tau$, with $m_{\nu\tau} = 0$ and no right-handed currents, is given by

$$R_p \equiv \sum_j \frac{p_{(j)}}{p} \frac{1}{1-\delta} \Big[(|F_j|^2 + |F'_j|^2)\left((1+\delta_j)(1-\delta+\delta_j) - 4\delta_j\right)$$

$$+ 2\,\mathrm{Re}(F_j F'^*_j)\delta\sqrt{\delta_j} \Big] \tag{16}$$

for the case when M is a pseudoscalar meson, and

$$R_v \equiv \sum_j \frac{p_{(j)}}{p} \frac{1}{(1-\delta)(2+1/\delta)} \Big[(|F_j|^2 + |F'_j|^2)$$

$$\left(\frac{(1+\delta_j)(1-\delta+\delta_j) - 4\delta_j}{\delta} + 2(1-\delta+\delta_j) \right) - 6\,\mathrm{Re}(F^*_j F'_j)\sqrt{\delta_j} \Big] \tag{17}$$

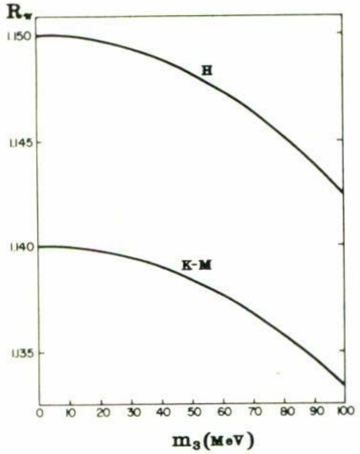

FIGURE 1. Plot of R_π *vs* m_3 for Hierarchical mixing (curve H) and Kobayashi-Maskawa mixing (KM).

for the case when M is a vector meson. We proceed to the study of these results as follows. For manifest left-right symmetry [6] we have $\eta = K$, $V_{\tau j} = U_{\tau j}$ and $f = f' = 1$. Then $F_j = (1+K)U_{\tau j}$ and $F_j = (K+\lambda)U_{\tau j}$. The parameters K and λ are expressed by the gauge coupling constant, the masses of the gauge bosons and the mixing angle between the light and heavy gauge bosons [7]: $K \simeq -1.44 \times 10^{-3}$, $\lambda = 0.4028$ (for $M_{WR} = 400$ GeV). In Fig. 1 we plot R_π *vs* m_3, for hierarchical mixing (H) and Kobayashi-Maskawa mixing (KM) in the neutrino sector. We use, for Hierarchical mixing, the values of $U_{\tau j}$ given in [8]

$$|U_{13}|^2 = 0.0003, \quad |U_{23}|^2 = 0.059 \quad \text{and} \quad |U_{33}|^2 = 0.94.$$

For KM mixing, we use, as an example, solution (C) from Ref. [9]

$$|U_{13}|^2 = 0.1681, \quad |U_{23}|^2 = 0.0004 \quad \text{and} \quad |U_{33}|^2 = 0.8281.$$

For these values of $U_{\tau j}$ we observe that the dominant contribution comes, as expected, from m_3 only. In Fig. 2 we do the same for R_ρ. We note for R_π that H-mixing is greater that KM-mixing in about 1% for the full range of m_3. For R_ρ we note the same behaviour up to m_3 around 90 GeV. Above this m_3 value KM-mixing is greater that H mixing by 0.1%. Eq. (13) do not incorporate radiative corrections, which depend on m_3 and meson structure functions. For the $\tau \to \pi\nu$ decay, with no neutrino mixing and no right-handed currents, in an effective $V-A$ theory, radiative corrections give a contribution -5.4% to -4.4% for $0 \leq m_{\nu\tau} \leq 100$ (MeV) [5]. For the $\tau \to \rho\nu_\tau$ decay the contribution of the radiative corrections is in the range -0.77% to 0.66% for $0 \leq m_\tau \leq 80$ (MeV) [6]. Then, radiative corrections in $\tau \to \pi\nu_\tau$ are much greater than the contributions arising from right-handed currents. But for

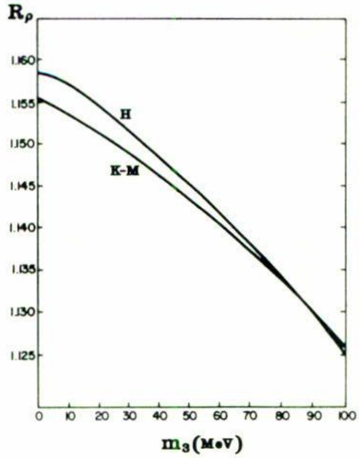

FIGURE 2. Plot of R_ρ *vs* m_3 for hierarchical mixing (curve H) and Kobayashi-Maskawa mixing (KM).

$\tau \rightarrow \rho\nu_\tau$ the contributions coming from right- handed currents amount to around 3%, for both H and KM mixings. We conclude that the $\tau \rightarrow \rho\nu_\tau$ decay mode is a suitable one to look for right-handed currents, without taking into account radiative corrections. Eqs. (16) and (17) are insensitive to radiative corrections, except for radiative corrections coming from diagrams mediated by the heavy right-handed weak boson, which are small for $M_{WR} \geq 400$ GeV.

4. Conclusion

We have calculated the $\tau \rightarrow M\nu_\tau$ decays in the framework of a model of weak interactions with neutrino mixing and right-handed currents. Our results shows that, for manifest left-right symmetry and for $M_{WR} = 400$ GeV, these decays are 1.6% ($\tau_{\pi\nu}$) and 3.4% ($\tau_{\rho\nu}$) greater than the corresponding one in the absence of right handed currents and neutrino mixing. For both cases the experimental result do not exclude the kind of contribution studied here.

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Resumen. En el contexto de un hamiltoniano de interacciones débiles efectivo con mezcla de neutrinos y corrientes derechas se revisan los decaimientos $\tau \to M\nu_\tau$. En la evaluación de la razón $R = \sum_i w(\tau \to M\nu_i)/w(\tau \to M\nu_\tau)$ se consideran la mezcla de neutrinos tipo jerárquica y la tipo Kobayashi-Maskawa. Suponemos simetría manifiesta izquierda-derecha en nuestros cálculos.