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The dependence on the choice of a basic set of form factors of the Low amplitude for hyperon semileptonic decays

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Abstract. We discuss two questions that arise from the Low theorem for spin 1/2 particles when different basic sets of form factors are chosen in the non-radiative amplitude. At first glance, it seems as if this theorem were not independent of the choice of form factors. We show that one is really free to choose whatever basic set, except that in intermediate steps of the calculation of the Low amplitude one looses this freedom and one must instead follow a careful procedure.

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1. Introduction

When dealing with radiative amplitudes (M_B) involving the emission of a real photon, the Low theorem [1] represents a very important help —specially at small photon momentum (k). This theorem guarantees that M_B will depend up to order $(k)^0$ only on the form factors and the electromagnetic static parameters involved in the non-radiative amplitude (M_0) and that other new form factors —which are most often unknown— will contribute only to order k and higher. Low's very general proof of this theorem left many detail questions unanswered. Chew addressed them —in particular, for the case when spin 1/2 particles are involved— and gave a very thorough proof [2]. In this case, the insertion of a real photon in M_0 invalidates the free use of the Dirac equation and the Gordon relation, which are so effective in reducing the number of independent covariants in M_0. Chew's analysis pays special attention to this point and shows in great detail that, starting from a general basic set of independent covariants and corresponding form factors of M_0, only these latter and no more form factors are allowed in M_B to orders $1/k$ and $(k)^0$; thus verifying the Low theorem.

Despite all of this, the application of the Low theorem is still not straightforward and may lead to the appearance of spurious terms as illustrated recently in Ref. [3]. In the case when spin 1/2 particles are involved there remain some questions which merit a detailed discussion. We have two specific ones in mind. In practical calculations it is convenient to work with two or more sets of basic form factors (SBFF) simultaneously. The SBFF are equivalent at the level of M_0 and are related to one another by linear unambiguous relations. The question then arises: will the corresponding expressions for the Low part of M_B (M_B^L) obtained with each SBFF be unambiguously related to one another by the same linear relations that connect the SBFF? If the answer were negative then, although each form of M_B^L would fulfill the Low theorem in the sense that it only depends on the SBFF from which it was derived, an inherent ambiguity not resolved at the level of M_0 would affect that part of M_B. Even if the answer is affirmative, one may still ask a second question: to what extent is one free to combine in intermediate steps of the calculation of M_B^L two or more SBFF?. If one is not completely free, then especial care must be exercised in order to ensure that no contributions are missed or spurious terms are introduced.

In this paper, we shall address ourselves to answer these two questions. We shall limit ourselves to the case when only spin 1/2 particles are involved and also to the case of practical interest of the effective $V-A$ theory decay amplitudes —in which the SBFF appear in the hadronic covariant only, while the leptonic covariant remains point-like [4]. Nevertheless, our discussion will be illustrative enough as to serve as guidance for more complicated cases. In Section 2 we discuss three SBFF. In Section 3 we obtain M_B^L for each one of these sets and state our questions in practical terms. In Section 4 we show that the answer to the first question is affirmative and we study the limitations in combining different SBFF in M_B^L. Finally, in Section 5 we summarize our conclusions.

2. Sets of basic form factors

The transition amplitude M_0 for the decay of a hyperon A into another one B and an electron-neutrino pair is

$$M_0 = \frac{G_v}{\sqrt{2}} \langle B|W_\lambda|A\rangle \bar{u}_e O_\lambda v_\nu \tag{1}$$

the four-momenta of A, B, e and ν will be denoted by p_1, p_2, ℓ, and p_ν, and m_1 and m_2 will be the masses of A and B, respectively. W_λ in the hadronic covariant of (1) may be expressed in terms of γ-matrices[1] and the Dirac form factors $f_i(q^2)$

[1]Our γ-matrix and metric conventions are those of J.D. Bjorken and S.D. Drell, "Relativistic Quantum Mechanics", Mc. Graw Hill (1964), except that our γ_5 has opposite sign and $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$.

and $g_i(q^2)$, $i = 1, 2, 3$, as

$$W_\lambda = \gamma_\lambda \hat{f}_1(q^2) + \frac{\hat{f}_2(q^2)}{m_1}\sigma_{\lambda\rho}q_\rho + \frac{\hat{f}_3(q^2)}{m_1}q_\lambda, \tag{2}$$

where for short we have introduced $\hat{f}_i = f_i + g_i\gamma_5$. q is the four-momentum transfer defined as $q = p_1 - p_2 = \ell + p_\nu$. The leptonic covariant in (1) is point-like and $O_\lambda = \gamma_\lambda(1+\gamma_5)$.

Using the Dirac equation and the Gordon relation, W_λ in (1) may be rewritten as

$$W_\lambda = \gamma_\lambda \hat{J}_1 + \hat{J}_2\frac{p_{1\lambda}}{m_1} + \hat{J}_3\frac{p_{2\lambda}}{m_1} \tag{3}$$

where $\hat{J}_i = J_i + H_i\gamma_5$ contain the new SBFF J_i and H_i, $i = 1, 2, 3$. These form factors and the Dirac ones are unambiguously related by the equations

$$\begin{aligned} J_1 &= f_1 + \left(1 + \frac{m_2}{m_1}\right) f_2, & H_1 &= g_1 - \left(1 - \frac{m_2}{m_1}\right) g_2, \\ J_2 &= f_3 - f_2, & H_2 &= g_3 - g_2, \\ J_3 &= -(f_2 + f_3), & H_3 &= -(g_2 + g_3). \end{aligned} \tag{4}$$

Another [5] SBFF closely related to the last one makes W_λ look like

$$W_\lambda = \gamma_\lambda \hat{F}_1 + \hat{F}_2\frac{p_{1\lambda}}{m_1} + \hat{F}_3\frac{q_\lambda}{m_1} \tag{5}$$

where $\hat{F}_i = F_i + G_i\gamma_5$ and the connections with the Dirac form factors, again unambiguous, are

$$\begin{aligned} F_1 &= f_1 + \left(1 + \frac{m_2}{m_1}\right) f_2, & G_1 &= g_1 - \left(1 - \frac{m_2}{m_1}\right) g_2, \\ F_2 &= -2f_2, & G_2 &= -2g_2, \\ F_3 &= f_2 + f_3, & G_3 &= g_2 + g_3. \end{aligned} \tag{6}$$

The connection between the second and the third SBFF is

$$\begin{aligned} J_1 &= F_1 & H_1 &= G_1 \\ J_2 &= F_2 + F_3 & H_2 &= G_2 + G_3 \\ J_3 &= -F_3 & H_3 &= -G_3 \end{aligned} \tag{7}$$

Given Eqs. (2), (3), and (5) we may proceed to obtain three forms for the Low part of the radiative amplitude M_B corresponding to M_0. This will be done in the next section.

3. Low radiative amplitudes

M_B^L can be readily obtained by using Eq. (18) of Chew's paper[2] [2]. All we have to do is to identify the basic covariants and form factors used by Chew in his Eq. (12) with the ones of Section 2, make the substitutions, and perfom the operations indicated in his Eq. (18).

When Eqs. (2), (3), and (5) are used the identifications are (in an obvious compact notation)

$$\begin{aligned} \Gamma_i &= \gamma_\lambda,\ \sigma_{\lambda\nu}q_\nu,\ q_\lambda,\ \gamma_\lambda\gamma_5,\ \sigma_{\lambda\nu}q_\nu\gamma_5,\ q_\lambda\gamma_5, \\ G_i &= f_1,\ f_2,\ f_3,\ g_1,\ g_2,\ g_3, \end{aligned} \tag{8}$$

$$\begin{aligned} \Gamma_i &= \gamma_\lambda,\ p_{1\lambda},\ p_{2\lambda},\ \gamma_\lambda\gamma_5,\ p_{1\lambda}\gamma_5,\ p_{2\lambda}\gamma_5, \\ G_1 &= J_1,\ J_2,\ J_3,\ H_1,\ H_2,\ H_3, \end{aligned} \tag{9}$$

and

$$\begin{aligned} \Gamma_i &= \gamma_\lambda,\ p_{1\lambda},\ q_\lambda, \gamma_\lambda\gamma_5,\ p_{1\lambda}\gamma_5,\ q_\lambda\gamma_5, \\ G_i &= F_1,\ F_2,\ F_3,\ G_1,\ G_2,\ G_3, \end{aligned} \tag{10}$$

respectively, and always $i = 1, 2, \ldots, 6$. In each case $\Gamma_i' = \gamma_\lambda(1+\gamma_5)$ and $X = q^2$. If ϵ_μ is the photon polarization, the result for M_B^L has the general form

$$M_B^L = \frac{eG_v}{\sqrt{2}}\epsilon_\mu\Big\{[1] + [2] + [3]_a + [3]_b + [3]_c + [4] + [5]\Big\}. \tag{11}$$

Denoting by λ_1 and λ_2 the anomalous magnetic moments of A and B, respectively, in each of the three cases we get

$$[1] = \left[\frac{\ell_\mu}{\ell\cdot k} - \frac{p_{1\mu}}{p_1\cdot k}\right]\bar{u}_2 W_\lambda u_1 \bar{u}_e O_\lambda v_\nu, \tag{12}$$

$$[2] = \frac{1}{2\ell\cdot k}\bar{u}_2 W_\lambda u_1 \bar{u}_e \gamma_\mu \not{k} O_\lambda v_\nu, \tag{13}$$

$$[3]_a = -\frac{\lambda_1}{e}\frac{1}{2p_1 \cdot k}\bar{u}_2 W_\lambda (\not{p}_1 + m_1)\sigma_{\mu\rho}k_\rho u_1 \bar{u}_e O_\lambda v_\nu, \tag{14}$$

$$[3]_b = \frac{\lambda_2}{e}\frac{1}{2p_2 \cdot k}\bar{u}_2 \sigma_{\mu\rho}k_\rho (\not{p}_2 + m_2) W_\lambda u_1 \bar{u}_e O_\lambda v_\nu, \tag{15}$$

$$[3]_c = \frac{1}{2p_1 \cdot k}\bar{u}_2 W_\lambda \not{k}\gamma_\mu u_1 \bar{u}_e O_\lambda v_\nu, \tag{16}$$

$$[5] = 2\left[\ell_\mu \frac{q \cdot k}{\ell \cdot k} - q_\mu\right]\frac{\partial}{\partial q^2}(\bar{u}_2 W_\lambda u_1 u_e O_\lambda v_\nu). \tag{17}$$

But for the term [4] we get

$$[4]_f = \bar{u}_2\left[\frac{p_{1\mu}k_\rho}{p_1 \cdot k} - g_{\mu\rho}\right]\frac{\left(\hat{f}_2\sigma_{\lambda\rho} + \hat{f}_3 g_{\lambda\rho}\right)}{m_1}u_1\bar{u}_e O_\lambda v_\nu, \tag{18}$$

$$[4]_J = \bar{u}_2\left[\frac{p_{1\mu}k_\rho}{p_1 \cdot k} - g_{\mu\rho}\right]\hat{J}_2\frac{g_{\lambda\rho}}{m_1}u_1\bar{u}_e O_\lambda v_\nu, \tag{19}$$

$$[4]_F = \bar{u}_2\left[\frac{p_{1\mu}k_\rho}{p_1 \cdot k} - g_{\mu\rho}\right]\left[\hat{F}_2 + \hat{F}_3\right]\frac{g_{\lambda\rho}}{m_1}u_1\bar{u}_e O_\lambda v_\nu. \tag{20}$$

Eqs. (12)–(17) have an identical appearance for each of the choices of SBFF that we are considering, while Eqs. (18)–(20) look very different. The choice of the SBFF is indicated by the subindex in [4].

If now one uses the equations that connect the different SBFF and Eqs. (18) and (19), for instance when Eq. (4) is used in Eq. (19) and then subtracted from Eq. (18), one finds,

$$[4]_f = [4]_J + \bar{u}_2\left[\frac{p_{1\mu}k_\rho}{p_1 \cdot k} - g_{\mu\rho}\right]\left[\frac{\hat{f}_2}{m_1}\right](\sigma_{\lambda\rho} + g_{\lambda\rho})u_1\bar{u}_e O_\lambda v_\nu. \tag{21}$$

Similarly, if Eqs. (6), (18) and (20) are used one gets

$$[4]_f = [4]_F + \bar{u}_2\left[\frac{p_{1\mu}k_\rho}{p_1 \cdot k} - g_{\mu\rho}\right]\left[\frac{\hat{f}_2}{m_1}\right](\sigma_{\lambda\rho} + g_{\lambda\rho})u_1\bar{u}_e O_\lambda v_\nu. \tag{22}$$

Clearly, $[4]_f \neq [4]_J$ and $[4]_f \neq [4]_F$. In contrast, one sees that using Eqs. (7) in Eq. (19) one obtains Eq. (20). *i.e.*, $[4]_J = [4]_F$. We are then led to the first question we mentioned in the introduction: although the three results obtained above for M_B^L comply with the Low theorem in the sense that only the form factors involved in M_0 appear, are they unambiguously connected one to another through the relations between the different SBFF valid for M_0? Eqs. (21) and (22) above seem to point towards a negative answer.

In the following section we shall show that, despite appearances, the answer to this question is affirmative.

4. Equivalence of the Low radiative amplitudes

Eqs. (21) and (22), along with the fact that Eqs. (12)–(17) have identical appearance in the different SBFF that we have chosen, seem to indicate that the three expressions for M_B^L that we have obtained are not related one to another through the connecting Eqs. (4), (6), (7) valid for M_0. This would imply that M_B^L is not unambiguously determined by the information contained in M_0 solely. Before drawing a conclusion it is necessary to analyze in detail the other terms in M_B^L of Eq. (11).

The term [1] is easily seen to be equivalent in the different SBFF. The same applies to [2] and [5]. The next three terms in Eq. (11) require detailed attention. Let us look at $[3]_a$ of Eq. (14); in it we shall substitute W_λ of Eq. (2), but first we shall rearrange it using only γ-matrix relations into

$$\begin{aligned} W_\lambda = \gamma_\lambda \hat{f}_1 + \hat{f}_2 \left[1 + \frac{m_2}{m_1}\right] \gamma_\lambda - 2\hat{f}_2 \frac{p_{1\lambda}}{m_1} + \left(\hat{f}_2 + \hat{f}_3\right) \frac{q_\lambda}{m_1} \\ + \hat{f}_2 \left[\frac{(\not{p}_2 - m_2)}{m_1}\gamma_\lambda + \gamma_\lambda \frac{(\not{p}_1 - m_1)}{m_1}\right]. \end{aligned} \tag{23}$$

When this form of W_λ is substituted into Eq. (14), the Dirac equation, the fact that $(\not{p}_1 - m_1)(\not{p}_1 + m_1) = 0$, and the connecting Eqs. (6) are used, one obtains $[3]_a$ as if W_λ of Eq. (5) had been used directly in it. Other choices of the SBFF give the same result, therefore $[3]_a$ is equivalent for different SBFF. This procedure may be repeated for $[3]_b$, except that now the fact that $(\not{p}_2 + m_2)(\not{p}_2 - m_2) = 0$ must be used, and one sees that also $[3]_b$ is equivalent for the different SBFF. To analyze $[3]_c$ of Eq. (16) we again use W_λ of Eq. (2) rearranged as in (23). Then, we notice that the second summand in the last parenthesis of (23) when applied to $\not{k}\gamma_\mu u_1$ can be rearranged into

$$\gamma_\lambda(\not{p}_1 - m_1)\not{k}\gamma_\mu u_1 = -2p_1 \cdot k(\sigma_{\lambda\rho} + g_{\lambda\rho}) \left(\frac{p_{1\mu}k_\rho}{p_1 \cdot k} - g_{\mu\rho}\right) u_1. \tag{24}$$

All this makes $[3]_c$ become

$$[3]_c = \frac{1}{2p_1 \cdot k} \bar{u}_2 \left[\gamma_\lambda \hat{F}_1 + \hat{F}_2 \frac{p_{1\lambda}}{m_1} + \hat{F}_3 \frac{q_\lambda}{m_1}\right] \not{k}\gamma_\mu u_1 \bar{u}_e O_\lambda v_\nu$$

$$- \bar{u}_2 \left[\frac{p_{1\mu} k_\rho}{p_1 \cdot k} - g_{\mu\rho}\right] \left[\frac{\hat{f}_2}{m_1}\right] (\sigma_{\lambda\rho} + g_{\lambda\rho}) u_1 \bar{u}_e O_\lambda v_\nu, \tag{25}$$

when Eqs. (6) are used. This form of $[3]_c$ is not equivalent to $[3]_c$ when W_λ of Eq. (5) is substituted directly in Eq. (16), an extra term appears. We shall not go into further details anymore, but it can be shown with similar analyses that $[3]_c$ may not be equivalent when other choices of SBFF are made.

It is the appearance of this extra term in (25) that helps to answer the first question. Comparing (25) with (22) we remark that there is an exact cancellation between the excess terms in both equations and that the combination $[3]_c + [4]_f$ is indeed equivalent to $[3]_c + [4]_F$ under the use of the connecting Eqs. (6). The same result is obtained for other choices of SBFF. We may then conclude that M_B^L is unambiguously determined by the connecting equations between different SBFF, valid at the level of M_0.

The above analysis contains also the answer to our second question: to what extent is one free to combine in intermediate steps of the calculation of M_B^L two or more SBFF? the answer is clearely that one is not free in general. As a matter of fact, one must excercise especial care because appearances may lead to the omission of some contributions, as in the case with $[3]_c$ above.

In the final section we shall summarize the correct procedure to use one or more SBFF simultaneously in M_B^L. Let us close this section by remarking that the excess terms both in $[3]_c$ and in [4] are gauge invariant by themselves, *i.e.*, their presence is not related to a cancellation required by overall gauge invariance.

5. Conclusions

From the foregoing analysis we conclude that the Low part of the radiative amplitude M_B^L when spin 1/2 particles are involved in the non-radiative amplitude is unambiguously determined (due to the approximation $k \to 0$) [4] in different SBFF —thus confirming the analysis of Chew—, and that the result obtained for M_B^L in one set can be translated directly into the result for it in another set by using solely the connecting equations between the two sets established at the non-radiative amplitude level. In this sense one is completely free to choose any SBFF.

In contrast, in intermediate steps of the calculation of M_B^L one looses this freedom, despite appearances, since partial terms of M_B^L are often not equivalent when expressed in terms of different SBFF. Because of this, it is then recommendable to attach an index to W_λ of Eqs. (2), (3), and (5) denoting the particular SBFF being used, *i.e.*, W_λ in (2) should be W_λ^f, in (3) W_λ^J, and in (5) W_λ^F. The discussion in

Section 4 shows then that

$$[1]^f = [1]^J = [1]^F,$$

$$[2]^f = [2]^J = [2]^F,$$

$$[5]^f = [5]^J = [5]^F,$$

and

$$[3]^f_{a,b} = [3]^J_{a,b} = [3]^F_{a,b},$$

while, although $[3]^J_c = [3]^F_c$ and $[4]^J = [4]^F$,

$$[3]^f_c \neq [3]^J_c, \qquad [4]^f \neq [4]^J,$$

$$[3]^f_c \neq [3]^F_c, \qquad [4]^f \neq [4]^F,$$

but always

$$[3]^f_c + [4]^f = [3]^J_c + [4]^J_c = [3]^F_c + [4]^F.$$

If one is willing to use simultaneously two or more SBFF, then, the correct procedure to do it is:

i) to choose W_λ expressed in the original SBFF,

ii) to substitute it in the partial term of M^L_B that has been selected,

iii) then to use in that W_λ the connecting equations to the second SBFF and to perform the necessary algebraic steps to reproduce the W_λ expressed in this second SBFF, and

iv) to keep whichever extra terms may appear.

One should not replace directly in that partial term of M^L_B the W_λ expressed already in the second SBFF, this procedure may lead to the omission of terms necessary to keep the full M^L_B free of the choice of SBFF.

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Resumen. Discutimos dos interrogantes que surgen con el teorema de Low para partículas de spin 1/2, cuando se eligen en la amplitud no radiativa diferentes conjuntos básicos de factores de forma. A primera vista, pareciera que el teorema no es independiente de la elección de los factores de forma. Demostramos que se tiene la libertad de elegir cualquier conjunto básico, excepto que esta libertad se pierde en los pasos intermedios del cálculo de la amplitud de Low, así que uno debe proceder de manera cuidadosa.