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Half phase conjugate stimulated scattering optical resonator

V. Aboites*

Optisches Institut, Technische Universität Berlin
Strasse des 17 Juni 135, 1000 Berlin 12, Germany

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Abstract. The case of an ideal half phase conjugate (PC) optical resonator formed by a standard mirror and a stimulated scattering PC mirror is considered. It is found that depending on the value of the radius of curvature of the standard mirror, after one round trip the final size of the spot size can be equal, larger or smaller than its initial value. How large is this difference also depends on the frequency shifting characteristic of the PC mirror.

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Since 1972, when discovered by Zel'dovich *et al.* [1], many theoretical and experimental works dealing with several aspects of the phenomenon of optical phase conjugation (PC) have been published [2,3,4]. In particular, one of the most interesting applications of PC is the study and construction of full or half resonant PC cavities [5,6,7] (*i.e.* cavities where both or one of the mirrors is a PC mirror), where phenomena such as pulse compression or Q-switching may occur [8,9]. In this communication we consider the case of an ideal half PC optical resonator formed by a standard mirror of radius of curvature r and a stimulated scattering PC mirror. As will be shown, depending on the radius of curvature of the standard mirror and the frequency shifting of the PC mirror an increase or reduction of the spot size at the real mirror after each round trip may occur.

The diagram of the resonator studied is showed in Fig. 1. Mirror Mr is an ideal 100% reflection mirror of radius r. At a distance l from Mr, an ideal stimulated scattering PC mirror is placed which, after reflection, will downshift the frequency of any incident wave by a quantity δ characteristic of the PC mirror. A Gaussian field E_i of frequency ω propagating along the positive z axis can be written as

$$E_i = E(r)\exp\left(i\left(\omega t - kz - \frac{kr^2}{2q_i}\right)\right) \tag{1}$$

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*On leave from C.I.O., Apartado Postal 948, 37000 León, Gto. México.

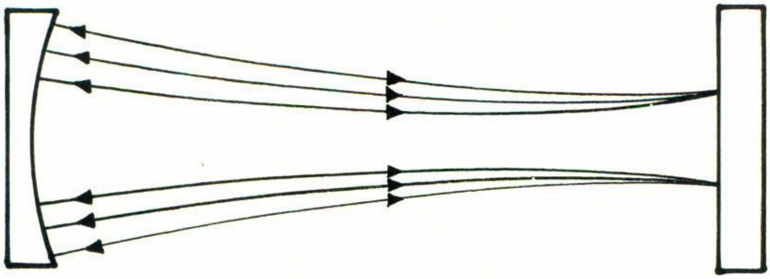

FIGURE 1. Half PC resonator formed by an ideal 100% reflection standard mirror Mr of radius r, and an ideal stimulated scattering PC mirror.

where

$$\frac{1}{q_i} = \frac{1}{R} - i\frac{\lambda}{\pi w^2} = \frac{1}{R} - i\frac{2}{w^2 k} \tag{2}$$

is the usual complex q parameter and $k = 2\pi/\lambda$. As a consequence of reflection on the stimulated scattering PC mirror, the reflected wave E_r has its frequency ω downshifted to

$$\omega - \delta = \omega(1 - \delta/\omega) \tag{3}$$

and E_r is given by

$$E_r \propto E^*(r) \exp\left[i\left(\left(1-\frac{\delta}{\omega}\right)\omega t + \left(1-\frac{\delta}{\omega}\right)kz - \left(1-\frac{\delta}{\omega}\right)\frac{kr^2}{2q_r}\right)\right] \tag{4}$$

where the complex q_r parameter is related to q_i by

$$q_r = -q_i^*\left(1-\frac{\delta}{\omega}\right). \tag{5}$$

Using an ABCD formalism, the effect of the PC mirror can be represented by the matrix

$$\begin{pmatrix} (1-\delta/\omega) & 0 \\ 0 & -1 \end{pmatrix} \tag{6}$$

where the reflected and incident q parameters are related by

$$q_r = \frac{Aq_i^* + B}{Cq_i^* + D} \tag{7}$$

which in general can be written as

$$\frac{1}{q_{n+1}}\left[A + \frac{B}{q_n^*}\right] = C + \frac{D}{q_n^*}. \tag{8}$$

Taking as reference plane the immediate right side of the M_r mirror, we obtain for the total, one round trip matrix

$$\begin{pmatrix} 1 & 0 \\ -2/r & 1 \end{pmatrix}\begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}\begin{pmatrix} (1-\delta/\omega) & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} (1-\delta/\omega) & -\frac{l\delta}{\omega} \\ -\frac{2}{r}(1-\delta/\omega) & \frac{2l\delta}{r\omega} - 1 \end{pmatrix}. \tag{9}$$

To obtain the ABCD matrix for a two, three or n number of round trips, all we need is to multiply matrix (9) by itself n-times. At the reference plane we can take as initial value for the complex parameter

$$\frac{1}{q_0} = \frac{1}{R_0} + i\frac{2}{w_0^2 k}. \tag{10}$$

According to our initial assumptions, after one round trip we obtain for the complex parameter

$$\frac{1}{q_1} = \frac{1}{R_1} + i\frac{2}{w_1^2 k(1-\delta/\omega)}. \tag{11}$$

To find out the relation between the initial spot size and wave radius w_0, R_0 and its final values w_1, R_1, we can use Eqs. (8), (10) and (11) to obtain

$$\frac{A}{R_1} + \frac{B}{R_1 R_0} + \frac{4B}{w_1^2 w_0^2 k^2 (1-\delta/\omega)} - C - \frac{D}{R_0} = 0, \tag{12.a}$$

$$\frac{2B}{w_0^2 R_1 k} - \frac{2A}{w_1^2 k(1-\delta/\omega)} - \frac{2B}{w_1^2 R_0 k(1-\delta/\omega)} - \frac{2D}{w_0^2 k} = 0. \tag{12.b}$$

As long as the condition

$$\left|\frac{l\delta\lambda^3}{2\pi^3}\right| \ll |w_1^2 w_0^2 c| \tag{13}$$

is fulfilled, we can neglect the third term of equation (12.a). Since c is the speed of light and considering typical dimensions of l, δ, λ, w_1 and w_0, we can see that the above expression will usually be satisfied for any "normal" resonator. Therefore, using (9) and (12.a) we obtain at the reference plane

$$R_0 \simeq R_1 = R \tag{14}$$

and

$$\frac{1}{R} = -\frac{1}{r}, \tag{15}$$

which is a well known feature of this sort of systems and is an expected result.

Using (12.b), (9), (14) and (15) we can obtain for the relation between w_0 and w_1 at the reference plane

$$w_1^2 = \frac{\left(1 + \dfrac{l\delta}{k\omega}\right)}{\left[\left(1 + \dfrac{l\delta}{k\omega}\right) - \dfrac{2l\delta}{r\omega}\right]} w_0^2. \tag{16}$$

Therefore

$$w_1 = (1 + \Delta) w_0 \tag{17}$$

where Δ is given by

$$\Delta \simeq \frac{l\delta}{r\omega}; \tag{18}$$

from this expression we can identify three situations:

i) For $r = \infty$, $\Delta = 0$. After any number of round trips, the spot remains the same size.

ii) For $r > 0$, $\Delta > 0$. The spot size will grow on each round trip.

iii) For $r < 0$, $\Delta < 0$. The spot size will diminish on each round trip.

Case *i)* reproduces after one round trip the initial spot size. However, since the initial and final frequencies are not the same we can not strictly speak of the Fox and Li [10] self-reproductibility criterion.

For cases *ii)* and *iii)* this growing and diminishment of the spot size could at first sight be interpreted as a sort of "positive and negative diffraction phenomena". However, this effect is only due to the presence of the mirror Mr. If we repeat the above calculation for Fig. 1 but *without* including the mirror Mr, we obtain for a reference plane located at a distance l from the PC mirror

$$R_0 \simeq -R_1 \tag{19}$$

and

$$w_1 = \left(\frac{1 + \dfrac{l\delta}{R_0\omega}}{1 - \dfrac{l\delta}{R_1\omega}} \right)^{1/2} w_0, \tag{20}$$

which for $R_0 \neq -R_1$ gives $w_1 > w_0$ and for $R_0 \equiv -R_1$, $w_1 \equiv w_2$, just according to the basic property of any PC mirror. We can see that the change in spot size stated in *ii)* and *iii)* takes place only *after* reflection on mirror Mr.

The continuous shifting of frequency on each round trip means that the operation of such resonator will be possible only as long as $\lambda/2 < l$. Also, as can be seen from expression (13), operation for long wavelengths is not possible. For example, taking $w \simeq w_0 = 0.5$ cm, $l = 100$ cm and $\delta = 5850$ MHz (stimulated Brillouin scattering frequency shift in CS_2), we obtain from (13) $\lambda = 0.583$ cm, which is already in the microwave range.

The experimental observation of cases *ii)* and *iii)* depends on the magnitude of the quantity Δ. For typical substances used as PC mirror the frequency shifting is very small, 250 MHz for stimulated Brillouin scattering process in SF_6, which for $r \simeq l$ and visible light gives a Δ value of $\simeq 5 \times 10^{-7}$. However, for hydrogen the frequency shift of the Raman scattering process is [11] 4155 cm^{-1}. Therefore, using this substance as PC mirror and (for the sake of this calculation) assuming $l \simeq r$, for visible light, Δ has a value of 0.249 which should be experimentally detectable.

In conclusion, the study of an ideal half PC resonator formed by a standard mirror and a stimulated scattering PC mirror was realized. It was found that for $r = \infty$ oscillation is possible as long as $\lambda/2 < l$ and condition (13) is satisfied. For $r \neq \infty$, in addition to the previous two conditions, the change of the spot size at the real mirror after each round trip means that oscillation is possible only as long as the transversal size of the field does not exceed the real mirror size. In any case, this is essentially a non stable resonator.

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Resumen. Se estudia el caso de un resonador óptico ideal de conjugación de fase (CF), formado por un espejo estándar y un espejo de CF por dispersión estimulada. Se encuentra que, dependiendo del valor del radio de curvatura del espejo estándar, después de un viaje redondo la dimensión final de la sección transversal del haz puede ser igual, mayor o menor que su dimensión original. Qué tan grande es esta diferencia también depende del corrimiento en frecuencia característico del espejo de CF.