

Decaying domain walls in an extended gravity model and cosmology

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ABSTRACT. We investigate cosmological consequences of an extended gravity model which belongs to the same class studied by Accetta and Steinhardt in an extended inflationary scenario. But we do not worry about inflation in our model; instead, we focus on a topological object formed during cosmological phase transitions. Although domain walls appear during first-order phase transitions such as QCD transition, they decay at the end of the phase transition. Therefore the “domain wall problem” does not exist in the suitable range of parameters and, on the contrary, the “fragments” of walls may become seeds of dark matter. A possible connection to “oscillating universe” model offered by Morikawa *et al.* is also discussed.

RESUMEN. Investigamos las consecuencias cosmológicas de un modelo gravitatorio extendido perteneciente a la clase estudiada por Accetta y Steinhardt en un escenario inflacionario extendido. No consideramos la inflación en nuestro modelo; en cambio nos concentramos en un objeto topológico que se forma durante las transiciones de fase cosmológicas. Aunque aparecen paredes de dominio durante transiciones de fase de primer orden como la transición QCD, decaen al final de la transición. Por lo tanto, el “problema de paredes de dominio” no existe en la región adecuada de parámetros y, por el contrario, los “fragmentos” de paredes pueden devenir semillas de materia oscura. Se discute asimismo una posible conexión con el modelo de “universo oscilante” propuesto por Morikawa *et al.*

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In recent years, many ideas on theory of gravitation have been produced by lots of authors. From the viewpoint of experimental physics, the existence of the “fifth force” has been examined in these days [1]. The tests for gravity theory have been managed in various styles, by means of newly-developed instruments and methods [2]. From the theoretical viewpoint, many authors have studied possible modifications and corrections to Einstein’s gravity theory, in consideration of an attempt to unify gravitation and quantum mechanics. According to superstring theory [3] which has attracted particle-physicists’ attention for several years, Einstein gravity takes some corrections at very high energy [4].

Of course, we cannot discuss gravitation theory apart from evolution of the universe [5]. Various modifications of gravity theory have been tried in order to solve cosmological problems. Observational and theoretical constraints on the amount of time-variation of

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Newton constant have been investigated and the upper limit is given [6]. For planning precise tests of gravity, it is meaningful to study possible consequences of models which admit spatial and temporal variation of the effective Newton “constant”. We can express results of experimental check for models of gravitation by putting observational constraints on a few parameters.

A recent idea of “extended inflation” scenario [7] includes an interesting modification of Einstein gravity. In the “extended inflation” scenario, difficulties in “old” inflation [8] may be avoided by introducing a variable Newton “constant” into the theory. In some models of this type, theories of scalar fields coupled non-minimally to gravity are considered, and they are very similar to Brans-Dicke’s gravity theory [9]. Many variants are also considered in the literature [10,11,12].

As a variant of extended inflation model a “hyperextended inflation” [10] model is known. In this model, self-interaction of the scalar field which is coupled non-minimally to gravity, is postulated and a crucial difference from Brans-Dicke theory is that the scalar field has an equilibrium point of motion. Owing to this property, one can take a wide range of values of parameters keeping away from conflicts with observations.

In the present article, we consider a model of this type which permits variations of an effective Newton constant in space and time. This model is a general, and probably simplest model, which can describe the variation of the strength of gravity. We do not consider inflation in this paper, however. The reason is that there are too many problems left unsolved in inflationary models to examine here; they are for example, the problem of the origin of the very large structure of galaxies recently discovered [13], etc.

Throughout this paper, we choose so-called natural units, *i.e.*, we set $h/(2\pi) = c = k_B = 1$.

Our starting point is to consider the following lagrangian

$$\mathcal{L} = -\frac{1}{2\kappa^2}f(\phi)r + \frac{1}{2}(\nabla_\mu\phi)^2 + \lambda. \quad (1)$$

In this equation, ϕ is a neutral scalar field coupled non-minimally to gravity and f is a function of ϕ and gives the manner of non-linear coupling to gravity; κ^2 is a constant which has the same dimensions and order of magnitude as the Newton constant, *i.e.*, $\kappa \approx 2.4 \times 10^{18} \text{ GeV}^{-1}$; λ stands for an effective cosmological term which comes from the contribution of the other matter fields. This term arises, for example, during a phase transition of a matter field.

A Weyl transformation of the metric ($g_{\mu\nu} \rightarrow f^{-1}g_{\mu\nu}$) converts the lagrangian to lagrangian including conventional Einstein-Hilbert term [14]

$$\tilde{\mathcal{L}} = -\frac{1}{2\kappa^2}R + \frac{1}{2} \left\{ \frac{1}{f}(\nabla_\mu\phi)^2 + \frac{3}{2\kappa^2} \left(\frac{\nabla_\mu f}{f} \right)^2 \right\} + \frac{1}{f^2}\lambda. \quad (2)$$

In the following, we investigate the property of the model based on this lagrangian (2); whenever we can describe a phenomenon both in terms of the lagrangian (1) and that of (2).

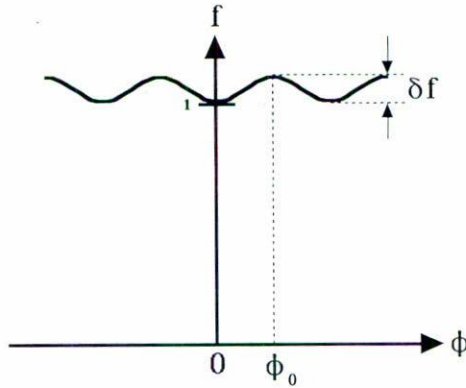


FIGURE 1. A schematic view of the function f .

If the effective cosmological term λ is present, nonlinear self coupling of the scalar field ϕ appears in the lagrangian (2). On account of the self-interaction, topological object in which energy of the scalar field concentrate can form. Such objects take the form of “domain walls” [15]. We study the properties of the object and the role in cosmology.

First we have to specify the functional form of f . We do not expect drastic changes in the effective strength of gravity in the course of evolution of the universe. Accordingly, it is suitable to choose a periodic continuous function, as a trigonometric function (Fig. 1). The difference between the maximum and minimum of f is denoted by δf , for later use. First of all, we study domain walls; thus f has only to take the following “piecewise-linear” shape (Fig. 2):

$$f(\phi) = 1 + a \left(\phi - \left[\frac{\phi}{\phi_0} \right] \phi_0 \right), \quad \text{if } \left[\frac{\phi}{\phi_0} \right] \text{ is an even integer,} \quad (3a)$$

$$f(\phi) = 1 - a \left(\phi - \left[\frac{\phi}{\phi_0} \right] \phi_0 - \phi_0 \right), \quad \text{if } \left[\frac{\phi}{\phi_0} \right] \text{ is an odd integer,} \quad (3b)$$

where $[x]$ denotes an integer part of x . This f involves two parameters, a and ϕ_0 . In this approximation, δf equals to $a\phi_0$. In general, f is characterized by (δf) and $(\kappa^2 \phi_0^2)$.

Suppose that some matter field undergoes first-order phase transition. During the transition, an effective cosmological term λ arises. Then the neutral scalar field ϕ can form a configuration of “kink” in one direction, say, in the z -axis. In three dimensions, this kink seems as an infinitely-spread wall of high energy density, parallel to x - y plane [15]. The (domain) wall of this type appears in general theories with spontaneously broken discrete symmetry [15]. In our model we assume symmetry with respect to $\phi \leftrightarrow -\phi$.

We want to know qualitative features of the object. We also wish to avoid a complicated situation where background curvature cannot be ignored.

To this end, let us suppose that a dimensionless combination $(\delta f)^2/(\kappa^2 \phi_0^2)$ is small in comparison with unity. Then the following static configuration can be obtained as an

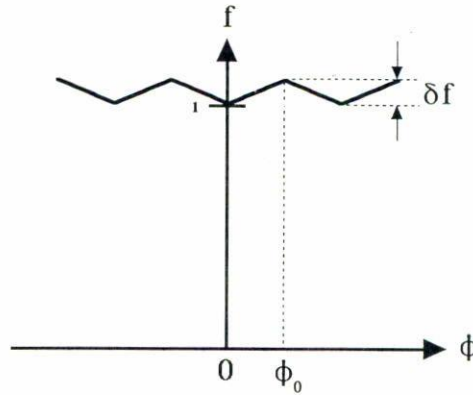


FIGURE 2. The “piecewise-linear” approximation of f .

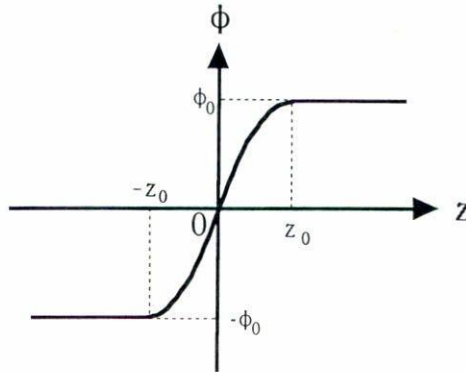


FIGURE 3. The “kink” as a static configuration of ϕ in the presence of Λ .

approximate solution (Fig. 3)

$$\begin{aligned} \phi(z) = h(z) &\equiv \lambda az(2x_0 - z), & (0 \leq z \leq z_0), \\ &\equiv \phi_0, & (z \geq z_0), \end{aligned} \tag{4}$$

where $z_0 \equiv \sqrt{\phi_0/\lambda a} = \sqrt{\phi_0^2/\lambda(\delta f)}$, and it should be read as $h(-z) = -h(z)$.

This kink solution is lying in the z -direction; thus this configuration is understood as infinitely stretching domain wall in the x - y plane.

An effective cosmological term entirely vanishes when the phase transition of the matter field is over. Then, the scalar field loses its self-interaction term and only has the non-minimal kinetic term. The domain wall generated during the phase transition is no longer

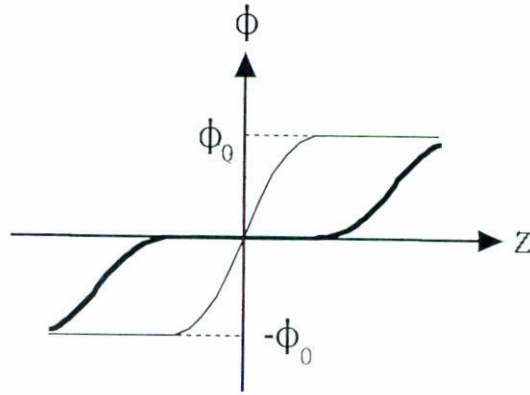


FIGURE 4. The breakup of plane-symmetric walls in the true vacuum. The thick line indicates the configuration of ϕ at $t = 2z_0$.

stable when the phase transition finishes. Now then, how does the wall decay? We assume plane-symmetric walls here again.

There being no potential term, we have only to solve substantially a wave equation with respect to an initial condition. We take the static solution (4) as an initial condition at the time $t = 0$. Furthermore we assume that the time-derivative of ϕ is zero everywhere, at $t = 0$. If the assumption is taken, the effective lagrangian (2) for ϕ reduces to a dominant term (now $\lambda = 0$)

$$\tilde{\mathcal{L}} \approx \frac{1}{2} \frac{1}{f} (\nabla_\mu \phi)^2.$$

Therefore the wave equation we must solve is approximated as

$$\nabla^\mu \left(\frac{1}{f} \nabla_\mu \phi \right) = 0. \tag{5}$$

Using the kink (4) as an initial configuration, we can solve the equation as

$$\phi(z, t) = \frac{1}{a} \left[\frac{1}{4} \{g(z-t) + g(z+t) + 2\}^2 - 1 \right], \quad (t > 0, z > 0), \tag{6}$$

and $\phi(-z, t) = -\phi(z, t)$, where

$$g(y) = \sqrt{f(h(y))} - 1, \quad \text{for } y > 0, \tag{7}$$

and $g(-y) = -g(+y)$ (see Fig. 4).

We find that the walls separated from each other soon become to move at nearly light speed.

Two essential characteristics of the behaviour of the wall after λ disappears are: (i) the energy stored at the wall cannot dissipate suddenly; (ii) the bulk of energy cannot stay still, according to the equation of motion.

Now, we consider the significance of the domain wall in our model in cosmology.

The only first-order phase transition in the universe ever known is the (confinement) transition of quantum chromodynamics (QCD) [16]. Although the order of the phase transition is yet debatable, we presume the first-order transition. The temperature of the phase transition is considered as $T = \Lambda_{\text{QCD}} \approx 200$ MeV. Consequently, the energy density of the false vacuum, that is, the effective cosmological term λ in our context, is the order of Λ_{QCD}^4 .

When the temperature of the universe decreases below $T_C = \Lambda_{\text{QCD}} \approx 200$ MeV, the false vacuum energy dominates over the thermal energy of radiations and particles. At the same time the scalar field ϕ takes the form of the wall due to non-linear self-coupling, since the thermal fluctuations of the field become sufficiently small. The domain walls are initially generated by the Kibble mechanism [17]. we expect at least one wall in each horizon-size volume.

We must check the consistency of calling the object as “wall”. If the thickness of the “wall” is wider than the horizon scale of the universe at that time, we can no longer call it as a “wall”. Moreover, the process of formation of the object becomes closely dependent on the growth of the fluctuation of the field. The ratio of the thickness of the “wall” to the horizon length is

$$\frac{\text{thickness of the “wall”}}{\text{horizon length}} = \frac{z_0}{R_H} = \frac{\sqrt{\frac{\phi_0^2}{\lambda(\delta f)}}}{\sqrt{\frac{1}{\kappa^2 \lambda}}} = \sqrt{\frac{\kappa^2 \phi_0^2}{\delta f}}. \tag{8}$$

We find there is no contradiction as long as the following inequalities are satisfied

$$\kappa^2 \phi_0^2 \ll \delta f \ll \kappa \phi_0 \ll 1, \tag{9}$$

which we obtain by combining the condition adopted when we led the approximate solution of the “wall”. Note that the conditions are independent of the value of λ . At the same time, we can safely ignore the effect of the expansion of the universe.

The mass density of the wall η is given by

$$\eta \cong \sqrt{\lambda(\delta f)} \cdot \phi_0 \cong (\delta f)^{1/2} \Lambda_{\text{QCD}}^2 \phi_0. \tag{10}$$

Along with progress of the phase transition, bubbles of the true vacuum are born and expand [18]. The phase transition is over when all the spatial regions come to belong to the true vacuum as a result of the percolation of the bubbles. In the true vacuum where the effective cosmological constant is nearly zero, the domain wall made of the configuration of ϕ decays. Therefore the wall begins to break inside the bubbles. Consequently, the evolution of fragments of walls depends on the manner of bubble nucleations. Even if the

domain walls are initially plane-symmetric, they take random shape at the end of the phase transition because of randomness in nucleation of bubbles.

If the wall formed during QCD phase transition remains to be static after the transition, the existence of wall conflicts with cosmological observation [19] as in the case of axionic domain walls [20]. The wall in our model, however, completely splits off after phase transition and the remnants move away by almost light speed. Thus we can avoid the above-mentioned problem; it can be said that the universe is filled with the relativistic gas of walls [21]. Let us estimate the present mass density of the relics of the wall. At the present time, the density ρ is given as

$$\rho_w \geq \frac{\eta R_H^2}{R_H^3} \left(\frac{T_{\text{present}}}{T_c} \right)^\alpha, \tag{11}$$

where $T_{\text{present}} \approx 2.75 \text{ K} \approx 0.24 \text{ meV}$ and $T_c \approx \Lambda_{\text{QCD}}$.

Here the parameter α ($3 \leq \alpha \leq 4$) is required for the reason that we take an account of the effect of transformation of the energy of fragments into that of other massive particle. The constraint that the energy density ρ_w is lower than the critical density of the present universe ($\rho_c \approx 2.6 \times 10^4 T_{\text{present}}^4$) yields

$$\sqrt{(\delta f)\kappa^2\phi_0^2} \left(\frac{T_{\text{present}}}{\Lambda_{\text{QCD}}} \right)^{\alpha-4} \leq 10^4. \tag{12}$$

This constraint can easily be satisfied for a wide range of parameters.

The contribution of ρ_w to the total energy density might explain the missing mass in the universe, which has not yet been identified. It is dependent on a hidden sector coupled to ϕ whether the relics of the wall can provide the dark matter which gathers other matters to generate galaxies. This is because the fragments of wall remain to move fast if there is no interaction between ϕ and other field. In a class of extended inflationary scenario, hidden sectors which violate the weak equivalence principle have been considered [10,11]. In our context, we have no compelling ground to determine the property of the hidden sector. Since the stage of our investigation in this paper is primitive, we will examine the property of other fields coupled to ϕ and gravity in a future work including numerical simulations of walls.

In the rest of the present paper, we investigate the connection to the oscillating universe model [22,23].

In the extended model we treated, $f(\phi)$ was not a smooth function, for a simple analysis. Of course, this is not an essential thing. From now on, we suppose f takes the form of the trigonometric function (Fig. 1), since we want to consider a coherent oscillation of the field ϕ . Our analysis below need only a rough sketch of f ; dimensional analysis can lead to a sufficient estimation.

Suppose that there exists a non-vanishing (positive) cosmological constant in the present universe. Then an effective interaction of the scalar ϕ appears, provided that the value of the cosmological term is of the same order of the thermal energy or more. In this time,

we consider a spatially homogeneous, oscillating field. Using an expansion of the effective interaction with respect to small ϕ , we obtain the period of oscillation as

$$\tau \approx \sqrt{\frac{\phi_0^2}{\lambda_p(\delta f)}}, \quad (13)$$

where λ_p is the present cosmological constant.

Recent astronomical observation revealed very large, periodic structure with a period of about 400 million lightyears in the universe [24]. Morikawa and several authors have claimed that the structure is an illusion induced from an error in estimation of the distance scale by means of measurement of galaxies' redshift [22,23]. An assumption that the expansion rate of the scale factor is oscillating leads to such an artificial pattern. Morikawa took a model in which the oscillation of a coherent scalar field bring about the oscillation of "Hubble constant" through modified Einstein equations. Here we try to identify such a scalar field with ϕ in our model.

The period (13) becomes 400 million years when we take λ_p as

$$\lambda_p = \frac{\kappa^2 \phi^2}{\delta f} \times 10^{-8} \text{ (eV)}^4. \quad (14)$$

According to another group of authors in Ref. [23], a finite cosmological constant of order of $\rho_c \approx (3 \text{ meV})^4$ is demanded by observations of distant galaxies. If we take such an allowed value for λ_p in (14),

$$\frac{\kappa^2 \phi_0^2}{\delta f} = 8 \times 10^{-3}. \quad (15)$$

If we further require a sufficient amplitude of oscillating Hubble parameter which can explain the observation, δf must be larger than 3×10^{-3} , according to Steinhardt in Ref. [25]. If we choose $\delta f = 3 \times 10^{-3}$, we must set $\phi_0 = 10^{15} \text{ GeV}$ according to (15). These values for parameters give rise to no contradiction with the previous constraints (9). The constraint (12) is also cleared if $\alpha > 3.4$. This value of α means that a finite fraction of energy density produced by the domain walls remains in the form of relativistic gas.

It can never be justified that our naive model describes the true nature of the gravity theory. The time variation of the gravitation constant and other constant is strictly constrained by observation; even if the constraints are able to be satisfied, fine-tuning is necessary, for instance, in the phase of oscillations. The important point we want to claim is that a general modification of gravity could explain different problems in cosmology in the same time, such as dark matter problem and problems in large scale structure of the universe. Accordingly, it is interesting and significant to investigate the detail of the consequence of the model and to make more complicated models which have more relevance to cosmological puzzles and no conflicts with observation. We also have to investigate the possibility that the first-order transition should be identified to unknown transition, for instance, in relation to sub-quark dynamics.

We wish to study the effect of cosmic expansion and gravity on the production of walls as well as the numerical simulation including the bubble nucleation and the wall intersection.

After completing this work, the present author became aware of the paper [26]. They also considered the wall which disappears after a certain phase transition. Moreover they claimed that the wall can provide the seed for generating galaxies. While their motivation in cosmology is very interesting, their analysis depends crucially on a peculiar model. We think that we must consider models which are related with many cosmological aspects and can be checked by observations.

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REFERENCES

1. E. Fischbach *et al.*, *Phys. Rev. Lett.* **56** (1986) 3. For a review, see: E. Fischbach *et al.*, *Ann. Phys. (N.Y.)* **182** 1; E.G. Adelberger *et al.*, *Phys. Rev.* **D42** (1990) 3267.
2. C.M. Will, *Phys. Rep.* **113** (1984) 345.
3. M.B. Green, J.H. Schwarz and E. Witten, *Superstring theory*, Cambridge University Press, Cambridge (1986).
4. D. Gross and E. Witten, *Nucl. Phys.* **B 277** (1986) 1.
5. See for example, E.W. Kolb and M.S. Turner, *The Early Universe*, Addison-Wesley Pub., New York (1990).
6. E.W. Kolb, M.J. Perry and T.P. Walker, *Phys. Rev.* **D33** (1986) 869; J. Rich, D. Lloyd Owen and M. Spiro, *Phys. Rep.* **151** (1987) 239, and references there in.
7. D. La and P.J. Steinhardt, *Phys. Rev. Lett.* **62** (1989) 376; *Phys. Lett.* **B220** (1989) 375.
8. A. Guth, *Phys. Rev.* **D23** (1981) 347. For a review, see: A.D. Linde, *Rep. Prog. Phys.* **47** (1984) 925; R.H. Brandenberger, *Rev. Mod. Phys.* **57** (1985) 1; K.A. Olive, *Phys. Rep.* **190** (1990) 307, and references there in.
9. C. Brans and R.H. Dicke, *Phys. Rev.* **124** (1961) 925.
10. P.J. Steinhardt and F.S. Accetta, *Phys. Rev. Lett.* **64** (1990) 2740.
11. T. Damour, G.W. Gibbons and C. Gundlach, *Phys. Rev. Lett.* **64** (1990) 123, and references there in.
12. R. Holman, E.W. Kolb and Y. Wang, *Phys. Rev. Lett.* **65** (1990) 17.
13. M.J. Geller and J.P. Huchra, *Science* **246** (1989) 897.
14. K. Maeda, *Phys. Rev.* **D39** (1989) 3159.
15. A. Vilenkin, *Phys. Rep.* **121** (1985) 263.
16. E. Witten, *Phys. Rev.* **D30** (1984) 272, and references there in.
17. T.W.B. Kibble, *J. Phys.* **A9** (1976) 1387.
18. S. Coleman, *Phys. Rev.* **D15** (1977) 2929.
19. Ya B. Zel'dovich, I. Yu. Kobzarev and L.B. Okun, *Sov. Phys. JETP* **40** (1975) 1.
20. P. Sikivie, *Phys. Rev. Lett.* **48** (1982) 1156.
21. See Ref. [5], p. 218.

22. M. Morikawa, *Ap. J.* **362** (1990) L37.
23. C.T. Hill, P.J. Steinhardt and M.S. Turner, preprint FERMI-PUB-90/129-T; P.J. Steinhardt, presented at the IUPAP Conf. on Primordial Nucleosynthesis and Evolution of the Early Universe (Tokyo, Japan, Sept. 4-8, 1990), preprint UPR # 0440T.
24. T.J. Broadhurst, R.S. Ellis, D.C. Koo and A.S. Szalay, *Nature* **343** (1990) 726.
25. M. Fukugita, F. Takahara, K. Yamashita and Y. Yoshii, *Ap. J.* **361** (1990) L1.
26. D. Saez and V.J. Ballester, *Phys. Rev.* **D42** (1990) 3321.