Reflecting Ronchi rulings with uneven-strip widths for image subtraction: a remark

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ABSTRACT. The image formation analysis of a reflecting Ronchi ruling for image subtraction having unequal reflecting and transparent strip widths is presented. It is shown that, in this case, each one of the diffraction orders (whether odd or even) of the composite image but the zeroth one carries the difference of the encoded images. The analysis is also extended to the case of polychromatic partially-coherent illumination and some potential applications are briefly reviewed.

RESUMEN. Se presenta el análisis del proceso de formación de imágenes propio del intercalamiento de los muestreos periódicos de dos imágenes, al codificarlas (para obtener la sustracción correspondiente) mediante una rejilla de Ronchi reflectora con sus anchos de franjas reflectoras y transparentes desiguales. Se muestra que la diferencia de las imágenes se puede encontrar en todos los órdenes de difracción con excepción del orden cero. El análisis se extiende al caso de iluminación policromática parcialmente coherente y se esbozan las posibles aplicaciones de la técnica.

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1. INTRODUCTION

The use of a reflecting Ronchi ruling to achieve composite, interlaced images for image subtraction after Pennington's technique [1] is a well-known encoding technique since first proposed by Dashiell *et al.* for image difference applications [2]. With a reflecting Ronchi ruling with reflecting strips as wide as the transparent ones, the resulting composite picture for image subtraction consists of two interlaced grids with equal widths, thus giving rise to missing orders (even orders) [3]. Because such a device can found a number of applications in image processing operations such as image subtraction, image correlation and pattern recognition, it is worthy to know how much does it tolerate deviations of equal widths for proper performance. A discussion is presented about the effects appearing in a composite image when using a reflecting Ronchi ruling with unequal reflecting and transparent strip widths (uneven-strip reflecting Ronchi ruling). The use of a polychromatic illuminating source is also considered.

2. ENCODED IMAGES WITH RONCHI RULINGS

As a result of using Pennington's technique [1] with a Ronchi ruling for image subtraction, the interlaced periodic samplings of spatial period $2b$ of two images is achieved. The

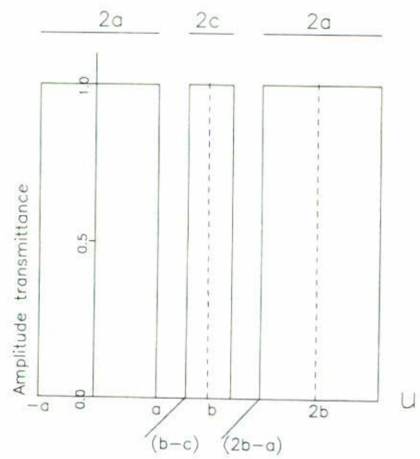

FIGURE 1. Interlaced gratings of period $2b$ and strip-widths of $2a$ and $2c$.

resulting interlaced gratings have a relative shift of half a period (*i. e.*, b), and each grating results, in general, with a stripe width $2a$ and $2c$ respectively ($b > a, c$, Fig. 1). Thus, between two neighboring stripes of width $2a$, there can be a region of width $2(b-a)$ to house the strip of width $2c$. In such a case, between any two whatever adjacent stripes, there is a region of width $b-(c+a)$ with no information. We define the parameter Δ through $\Delta - a = b-(c+a)$, so that c can be written in terms of b as $c = b - \Delta$. Note that $a \leq \Delta \leq b$ means $b - a \geq c \geq 0$ respectively. Also, $a \leq c$ implies $2a \leq b$.

According to the particular method employed for encoding two images into one composite image, it is possible to achieve different kinds of composite images. For example, by using reflecting Ronchi rulings having equal transparent and reflecting strip widths [2], it is possible to get encoded pictures with $\Delta = a$ (meaning only stripes with information), $c = a$ and, as a consequence, $b = 2a$. The use of uneven reflecting Ronchi rulings, the main subject of this note, gives rise to encoded images also with $\Delta = a$ and then, $c = b - a$, but $b \neq 2a$. For comparison, it will be of interest the case of equal strip widths ($c = a$) having "empty" stripes (*i.e.*, $\Delta \neq a$), which can be produced with other techniques [4].

2.1. Coherent case

To perform the Fourier analysis, it is assumed a telecentric optical system for image formation with object and image coordinates given by U and U'. The corresponding normalized coordinates are

$$u = \frac{RU}{\lambda f}, \quad u' = \frac{RU'}{\lambda f}, \tag{1}$$

where R is the pupil's radius, λ the mean wavelength of the quasi-monochromatic light and f the focal length of both the transforming lens and the inverse transform one. To the spatial frequency coordinate f_u is associated the following change of variable:

$$\mu = \frac{f_u}{R} \tag{2}$$

After encoding two images of amplitude transmittances $\tau(u)$ and $\tau'(u)$ in a composite image (whether $\Delta = a$ or $\Delta \neq a$), its transmittance $\tau_c(u)$ can be written as (assuming linear recording)

$$\tau_c(u) = \tau(u)\left\{\sum_{n=-\infty}^{\infty} \delta(u - 2nb) * \mathrm{rect}\left(\frac{u}{2a}\right)\right\} + \tau'(u)\left\{\sum_{n=-\infty}^{\infty} \delta([u - b] - 2nb) * \mathrm{rect}\left(\frac{u}{2c}\right)\right\}, \tag{3}$$

where δ denotes the Dirac's delta function, $*$ means convolution and

$$\mathrm{rect}(u) = \begin{cases} 1 & \text{for } |u| \leq \frac{1}{2}, \\ 0 & \text{otherwise.} \end{cases}$$

The amplitude $\tau_c(u)$ redistributes itself at the frequency plane of the optical system according to $\hat{\tau}_c(u) = \mathcal{F}\{\tau_c(u)\}$ and leads to

$$\hat{\tau}_c(\mu) = \hat{\tau}(\mu) * \left\{4ab \sum_{n=-\infty}^{\infty} \delta(\mu - n/2b)\,\mathrm{sinc}[2a\mu]\right\} + \hat{\tau}'(\mu) * \left\{4cb \sum_{n=-\infty}^{\infty} \delta(\mu - n/2b)\exp[-i2\pi\mu b]\,\mathrm{sinc}[2c\mu]\right\}, \tag{4}$$

with $\mathrm{sinc}[\theta] = \sin(\pi\theta)/\pi\theta$ and $\mathcal{F}\{\cdots\}$ denotes the Fourier transform operation. Eq. (4) can be also written as

$$\hat{\tau}_c(\mu) = \sum_{n=-\infty}^{\infty} \left\{4ab\hat{\tau}(\mu - n/2b)\,\mathrm{sinc}[na/b] + (-1)^n 4cb\hat{\tau}'(\mu - n/2b)\,\mathrm{sinc}[nc/b]\right\}. \tag{5}$$

Then, for the special case of $c = a$ (equal stripe widths), the nth contribution $\hat{\tau}_n(\mu)$ yields (cases $\Delta \neq a$ and $\Delta = a$ included)

$$\hat{\tau}_n(\mu) = 4ab\,\mathrm{sinc}[na/b]\left\{\hat{\tau}(\mu - n/2b) + (-1)^n\hat{\tau}'(\mu - n/2b)\right\}, \tag{6}$$

which means alternation of sum and difference of the Fourier transform of transmittances. Of course, whenever $b/a = b/c = q$ (q a non-zero integer), Eq. (6) gives rise to the missing orders of $n = pq$ and no alteration can be observed. In connection with image subtraction applications, a reflecting ruling with equal stripe widths produces a composite image having $q = 2$, $\Delta = a$, all of the even orders ($n = 2p$, $p = \pm 1, \pm 2, \ldots$) except the zeroth are absent and

$$\hat{\tau}_{2m+1}(\mu) = 8a^2 \operatorname{sinc}[(2m+1)/2]\Big\{\hat{\tau}(\mu - (2m+1)/2b) - \hat{\tau}'(\mu - (2m+1)/2b)\Big\}, \quad (7)$$

the zeroth order containing the sum information. Here, $m = 0, \pm 1, \pm 2, \ldots$, and this is the case usually described in the literature.

Turning our attention to Eq. (5) again, in the case of $c = b - a \neq a$ ($\Delta = a$, composite image from an uneven reflecting Ronchi ruling), the nth term contributing to the sum can be written as

$$\begin{aligned} \hat{\tau}_n(\mu) &= 4ab \operatorname{sinc}[na/b]\hat{\tau}(\mu - n/2b) \\ &\quad + (-1)^n 4(b-\Delta)b \operatorname{sinc}[n(b-\Delta)/b]\hat{\tau}'(\mu - n/2b) \\ &= \begin{cases} 4ab\hat{\tau}(\mu) + 4(b-a)\hat{\tau}'(\mu), & \text{for } n = 0 \\ 4ab \operatorname{sinc}[na/b]\{\hat{\tau}(\mu - n/2b) - \hat{\tau}'(\mu - n/2b)\}, & n = \pm 1, \pm 2, \ldots \end{cases} \end{aligned} \quad (8)$$

where the following property ($\Delta \neq b$)

$$\begin{aligned} 2(b-\Delta)\operatorname{sinc}[n(b-\Delta)/b] &= \frac{2(b-\Delta)}{n[(b-\Delta)\pi/b]}\sin[n\pi - n\pi\Delta/b] \\ &= -(-1)^n 2\Delta \operatorname{sinc}[n\Delta/b], \quad n = \pm 1, \pm 2, \ldots \end{aligned} \quad (9)$$

has been used with $\Delta = a$ (note that when $b = 2a$, then $b - \Delta = b - a = a$, but Eq. (9) suffers no violation because of the missing orders of $n = 2p$). Eq. (8) implies that all of the diffraction orders except the zeroth carry the amplitude difference of $\hat{\tau}$ and $\hat{\tau}'$, in spite of the fact, that there are no missing orders. There is a compensation of the alternation appearing in Eq. (6). Fig. 2 shows an example of Eq. (9) with $a = 0.9$ and $b = 1.1$.

At this stage it is opportune to point out a remark of practical interest. Many available Ronchi rulings have uneven-strip widths, as can be checked by looking for missing orders at their Fourier plane. According to the Fourier analysis just shown, such gratings could be used as the basis of reflecting rulings for image subtraction.

2.2. Partially coherent case

In order to find the conditions a polychromatic source must fulfill to provide illumination of an optical system able to perform the subtraction operation, a plane wave of spatial frequency given by μ_0 illuminating the composite image is considered. Then, the amplitude

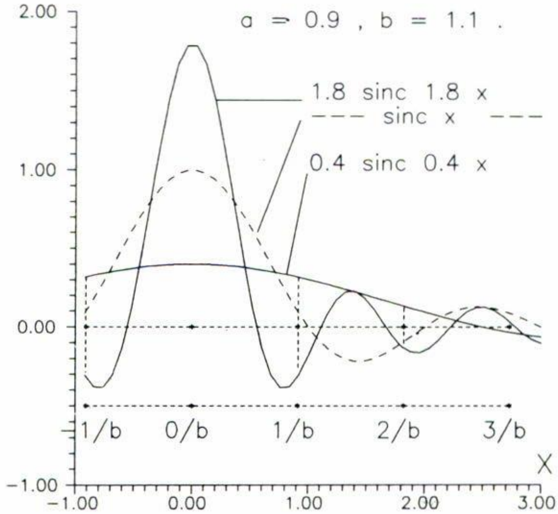

FIGURE 2. Relationship between the Fourier transforms of complementary gratings (each having a stripe width of a and $b-a$ respectively) as a function of the dummy variable X: their envelopes are $2a\,\mathrm{sinc}[2aX]$ and $2(b-a)\,\mathrm{sinc}[2(b-a)X]$ (continuous lines). The spatial period of the gratings is $2b$. The function $\mathrm{sinc}[X]$ is also drawn for comparison (dashed line). The spectra of each sampled object are centered around $X = n/2b$ (indicated in both horizontal dashed lines). At each of these points the value of each envelope equals in magnitude and alternates in sign with n (intersections of both curves for n odd and vertical dashed lines for n even except zero). This alternation comes up in addition to the alternation due to the mutual b-shift and compensates it. Case of $a = 0.9$ and $b = 1.1$.

distribution at the frequency plane is $\hat{\tau}_c(\mu-\mu_0)$. By filtering out the nth contribution using a filter centered at $\mu = \mu_0$ and width L and assuming nonoverlapping-order distribution as given by

$$2(\mu_w + B_\mu) \leq 2L \leq \frac{1}{2b}, \tag{10}$$

the total image irradiance due to the quasi-monochromatic partially-coherent source with distribution $\Gamma(\mu_0)$ [5,6] becomes

$$I_n(u') = \begin{cases} 2\mu_w|4ab\tau(u') + 4(b-a)b\tau'(u')|^2, & n = 0 \\ 32\mu_w a^2 b^2 \,\mathrm{sinc}^2[na/b]|\tau(u') - \tau'(u')|^2, & n = \pm 1, \ldots \end{cases} \tag{11}$$

with $\Gamma(\mu_0) = \mathrm{rect}[\mu_0/2\mu_w]$, B_μ the half bandwidth of the sum $\tau + \tau'$ and

$$I_n(u') = \int_{-\infty}^{\infty} d\mu_0 \Gamma(\mu_0) \Big| \mathcal{F}^{-1}\left\{\hat{\tau}_n(\mu - \mu_0)\right\}\Big|^2. \tag{12}$$

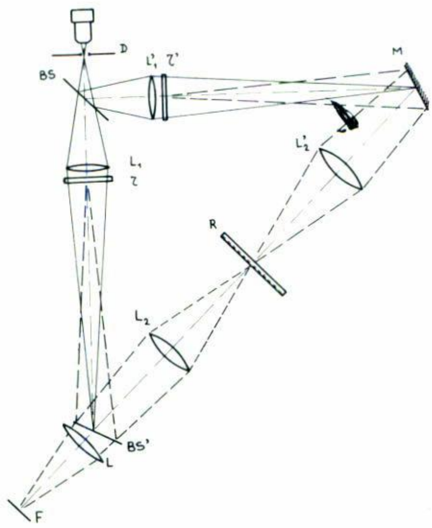

FIGURE 3. Experimental set-up to make color composite images using a polychromatic light source.

3. DISCUSSION

Some early efforts to carry out image subtraction with white light sources do not seem to pay full attention to the benefits offered by Pennington's technique and many of them are aplicable only to gray-tone images [7–11]. However, an extension of Pennington's technique is the use of a polychromatic partially coherent source for illumination [4]. Such a variant seems to be particularly suitable to optically perform the subtraction of color images. Related color image operations such as color image correlation techniques have been already proposed both optically [12] and by computer methods [13].

To see how this possible version of the technique can work, consider the set-up shown in Fig. 3, where D is a diagraphm at the focal plane of a microscope objective, BS and BS' are beam splitters. Two color transparencies τ and τ' are illuminated with a polychromatic partially-coherent light source through collimators L_1 and L_1' and imaged to both sides of an uneven reflecting Ronchi ruling R by using the lenses L_2 and L_2' through the mirror M and BS' respectively. Lens L images R onto the observation plane F.

Thus it becomes possible to make the photograph of the resulting composite picture with a color film slide at F. After suitable development, the resulting color slide is to be placed in a polychromatic optical image processor. Assuming that the color slide does not introduce any phase distortion and provided that the nonoverlapping-order condition is fulfilled [Eq. (10)], then the image subtraction for each wavelength λ [Eq. (11)] from all the color-blurred diffraction orders but the zeroth can be obtained. After filtering the *nth* color-diffraction order, the resulting irradiance at the image plane must be the incoherent superposition of all the subtraction images' irradiances associated with each wavelength λ. Thus, the resulting image is to be interpreted as the additive color superposition on colorimetric terms. The key for color applications is, of course, the polychromatic illumination and not the use of an uneven reflecting Ronchi ruling. This kind of rulings, however, can be used in place of even widths gratings.

Note that the basic set-up can carry out, in principle, the subtraction operation in real time, provided there are no phase distortions relative to the Eqs. (6) and (11) arising from the slides. Moreover, for the color slides subtraction to be performed, appropriate dispersion compensators must be also included.

4. CONCLUSIONS

It has been shown that Pennington's technique for image subtraction tolerates the use of a reflecting Ronchi ruling with uneven stripe widths. Furthermore, because the use of a polychromatic partially-coherent source is also possible (provided some non overlapping-order conditions are satisfied), the method offers also the possibility of color image subtraction as a natural extension.

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