Transient Kerr phase conjugation as a pulse propagation problem

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ABSTRACT. We present a general numerical method that introduces the advanced techniques of resonant pulse propagation into the optical phase conjugation (OPC) problem. These techniques were not introduced earlier due to the simultaneous forward and backward propagation of pulses, which are not considered in most propagation codes. The natural consequence is an improvement on the domain of applicability of OPC to shorter pulses and further complex material equations. For simplicity, the numerical procedure is applied to the traditional Kerr materials, in the specific case of a degenerate four wave mixing conjungator. We draw the analogy with known analytical results and extend them to current experimental situations.

RESUMEN. En este artículo presentamos un método numérico que permite incorporar las técnicas numéricas empleadas con éxito en el estudio de la propagación resonante de pulsos al problema de la conjugación de fase de pulsos. Dicho método sobrepasa las limitaciones de las técnicas usualmente empleadas para simular procesos de propagación, como lo es la propagación simultánea de pulsos en direcciones opuestas, y permite el análisis exhaustivo del proceso de conjugación de fase de pulsos ópticos muy cortos, así como la consideración de las ecuaciones dinámicas propias de los materiales conjugadores más complejos. Por simplicidad, el método propuesto es explicado a través de la conjugación de fase en materiales tipo Kerr, en el caso específico de la geometría de mezcla degenerada de cuatro ondas. Se enfatiza la analogía con resultados analíticos previos y se extienden hacia las situaciones experimentales actuales.

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1. INTRODUCTION

To succeed, optical communications, as any other field of communications, must solve the problems of coding, transmission and processing of information on a better and more

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competitive basis that the currently existing information systems. The coding and transmission problems are rather well defined by the recent development of optical information technologies [1]. It has become increasingly clear that the transmission connecting elements for optical communications must be nonlinear optical fibers, and that the basic signal elements of modern optical or optoelectronic communications are ultra short light pulses. The reason is very closely related to overcoming the need of regenerating the optical signal on a short distance, implied by the linear propagation in optical fibers, making unnecessary the conversion of optical to electronic and back to optical signals. Also, the idoneous light sources, the soliton laser, is already a reality [2] and it is based on analogous nonlinear optical features. Those characteristics make the transmission mode naturally digitized into a pulsed fashion, easily linking into current electronic systems. Therefore, optical communications are increasingly relying on nonlinear doped optical fibers, which now are openly available in the communication market, for long distance transmission. On the other hand, the short distance or local communication problem has been mostly solved based on linear characteristics of fibers. There, digitalization is just a practical convenience, not easily visible to users of local optical linking but spelled back a new life by its linking to optical fiber telephony, hence to an all optical, or being rather specific and accord with the evolution of the optoelectronic term, to an all photonic system.

Those features also define the characteristics expected from the processing of optical information into pulsed signals. The dimensions of the currently available ultra short pulses ($c\tau \ll 1$ mm) has the convenient sensible physical dimensions for the expected processing units, but it is again the nonlinear dynamics the one that makes feasible a straightforward optoelectronic or photonic analogy to electronics. Therein the ongoing merging of the research on very short pulses phenomena and processes in nonlinear materials. The time scales mentioned above require the accounting of the deailed intimate material dynamics and its characteristic times, in search of the photonic analogues of electronic switches, logic gates, active filters, amplifiers, etc.

Along these lines, optical phase conjugation (OPC) has received a great deal of attention since its basic and early applied possibilities become known [3]. Among them, and of key current relevance, is realizing their use as an optical processor along the lines discussed above. By itself the OPC problem is a nonlinear propagation case of electromagnetic (EM) waves in a dielectric media with a complex and still incomplete analytical solution, that however, has not been dealt with the same sophisticated numerical techniques available to other propagation problems such as resonant pulse propagation (RPP) [4]. The better known OPC is the one that occurs in Kerr materials, characterized by an instantaneous nonlinear response time. For practical applications, Kerr materials are not convenient because of their scarcity, being more favored in this case the photorefractive materials [5]. However for the purposes of this paper, where we intend to explain the power of numerical techniques, we will deal with Kerr materials since the photrefractive modeling is less transparent but along the same analytical and numerical lines, and we will leave them for a successive note.

The most favored experimental array for OPC, for practical reasons, is the four wave mixing geometry (FWM) [3]. There, two of the waves are intense pumps that for our purposes will be considered continues waves (CW). A third wave, the probe pulse signal,

will be processed by the conjugator into a conjugated pulse signal, the fourth wave, and a transmitted pulse. Amplification, oscillation, integration and other phenomena are known to occur for the conjugated signal [3], but still today little is known about the transmitted signal. Because of its controlable double output, such a device can be thought as a fine photonic candidate for transistor or logic gates electronic analogues.

The OPC by FWM in Kerr materials, as most of the OPC geometries, is usually dealt with the macroscopic nonlinearities and the finite dimensions of the conjugating medium, but the detailed microscopic dynamics between the atomic medium and the pulse is superficially or rarely considered. This procedure leaves out some of the spectral features expected from such device when processing very short pulses [6]. Moreover, the standard OPC theoretical treatments are based on the stationary filtering theory, even in the case of transient phase conjutation [7]. There, the key concept is the conjugator response function, which has been analytically determined and its singularities studied [7–9] under conditions that only apply to long duration (spectrally narrow) pulses, constant coupling and linear absorption coefficient. Under these conditions, one can derive the conjugate wave, the transmitted signal and their transient behavior. However, the analytical form of the response function is far from being simple, because of the nonlinear medium resonances. Such complexity has required from approximations and numerical displays to be partially explored, specially for nonsteady solutions [10–11]. Obviously, this methodology is simple but lacks versatility to deal with the numerical integration of the propagation equations in more realistic conditions such as short pulses, transverse effects, polarization sensitive process and frequency-dependant coupling coefficients or losses.

On the other hand, there is a large experience in RPP dealing with microscopic dynamics, and the object of this work is the introduction of a numerical technique that takes into account such experience and, in addition, going beyond the analytical troublesome intermediate analytical stage by providing terminal information; this is, the conjugated and transmitted pulses. In that sense, the numerical techniques is complementary to the viable solutions, at the same time that it makes more easy the understanding of the solutions in the unstable region by becoming a suitable numerical experiment.

The aim of this paper is to introduce a general numerical method to deal in an unified manner with both the OPC and the RPP features, without unnecessary restrictive conditions. As it will be shown, the method overcomes the traditional difficulties of the simultaneously counter-propagating pulses and its applicability is not restricted to the specific case of OPC by FWM in Kerr media. In the following section we describe the OPC model in which we are interested, and in Sect. 3 we present the numerical procedure which allows the propagation modeling of the pulse phase conjugation process, even in the presence of resonant atomic polarizations. In some instances, the fundamental equations of our method can be analytically solved and compared with the standard time resolved method. At the end of Sect. 3, we present the solution of two ideal but important solvable cases: an infinitely thin and an infinitely thick conjugators, that model an OPC mirror and OPC integrator, respectively. These results reproduce previously known analytical results and generalize them beyond the Beer domain. Finally, in Sect. 4, we apply the numerical method to not so ideal cases, where the presence of frequency dependent atomic polarizations causes severe reshaping over the otherwise conjugate pulses.

2. FOUR WAVE MIXING PHASE CONJUGATING MODEL

We consider the conjugator as a bar of length L filled with a Kerr nonlinear medium and with a resonant coherent non saturable absorption. This medium is pumped by two intense undepleted counter-propagating continous waves of carrier frequency ω_c. The transient phase conjugation problem then reduces to the propagation of two pulses, a probe pulse traveling in the $+z$ direction with a slowly varying complex envelope $E(t,z)$ and a likewise conjugate pulse traveling in the $-z$ direction, $R(t,z)$. Both pulses have carries frequency ω_c and wave number $\kappa = \omega_c/c$. The standard spectral OPC scale is given by the length of the conjugating material, playing the pulse duration a rather passive role. However the experience in RPP, even in the linear regime, is that very short pulses can suffer substantial changes due to the relative width of the spectral distribution of the resonant atoms, leading to unusual oscillations such as in the anomalous absorption process [12]. Therefore, both features should be brought at the same level. The pulse duration should be compared with the inverse spectral width of the medium and with the time of flight of the light within the conjugating medium, whose length, usually less than a few centimeters, corresponds to times below the nanosecond range [13]. This time scale is the partial reason of the reduced initial need for propagation techniques, because optical pulses of tens of nanoseconds could be approximately treated as continuous waves. However, the main technical difficulty resides on the characteristics of the standard RPP codes [14], which have not been designed to deal with propagating and counter-propagating pulses, and just in a few cases, they take into account the medium nonlinearities. For the purposes of this paper, a distinction is made between the well known standard features of the RPP in a dielectric and the OPC nonlinear features.

The relation between the spectral width of the two level atom (TLA) medium and the pulse duration τ in RPP is clearly evident in the so-called anomalous absorption process, that became evident when very short pulses were available. It consists of the spectral filtering of a weak and short pulse of duration τ and carrier frequency ω_c, by the spectral width of the medium, characterized by its absorption coefficient $\alpha(\nu)$ of width Δ_α. Then, the spectral intensity of the pulse, $I(\omega,z)$, is given as a function of the propagation distance, (Fig. 1.a), by [4,12]

$$I(\omega-\omega_c,z) = I(\omega-\omega_c,0)e^{-\alpha(\omega-\omega_c)z}. \tag{1}$$

If the pulse is long, $\Delta_\alpha\tau \gg 1$, we may consider α as a constant and the resulting temporal pulse behavior is the exponential decay predicted by Beer's Law. This is the single RPP contribution considered in previous OPC treatments [8]. However, if the pulse is very short, $\Delta_\alpha\tau < 1$, the filtering process established at Eq. (1) results into temporal oscillations of the pulse envelope, Fig. 1.b, and in a non-exponential decay of the pulse energy. Such process is referred as anomalous absorption, and here we will consider it in order to show the advantages of the numerical procedure.

In general terms, the RPP in an OPC material corresponds to a case of RPP in the presence of strong nonlinearities, but if the pulse is very short and weak, we can still use a RPP linear propagation model and the standard OPC treatment. In addition, it is the relation between τ and the corresponding thickness of the conjugator, c/L, what it

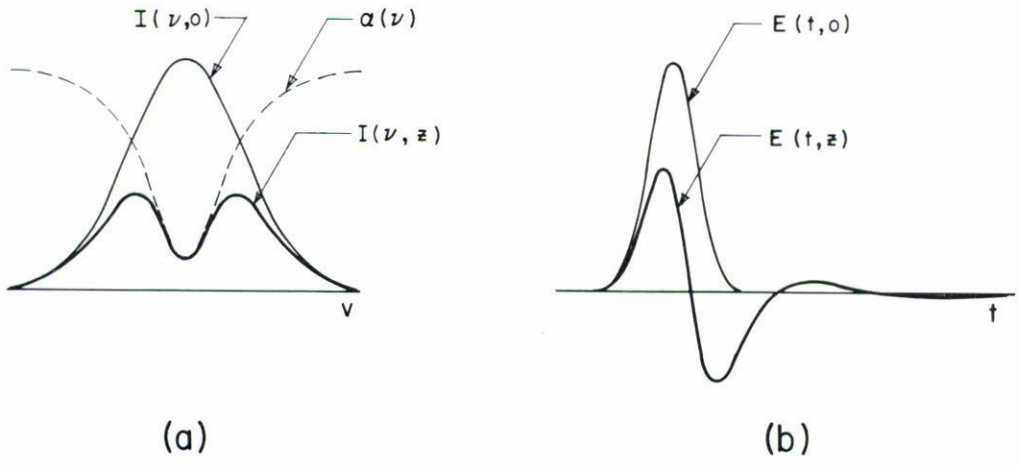

FIGURE 1. The anomaluos absorption process of a weak pulse in the RPP. In (a) the input and output spectral intensity distributions of the pulse. The later is obtained from Eq. (1) and corresponds to the filtering of the input by the absorption function $\alpha(\nu)$ (dashed line). In (b) the respective temporal pulse envelopes obtained from the Fourier antitransform of curves showed in (a). Note that the output pulse exhibits temporal oscillations due to the spectral modulation caused in (a) by the filtering process.

matters in OPC. Then, we should relate both the atomic width and the thickness of the conjugating medium to the pulse duration.

The method which we are presenting in this paper takes in account the fact that RPP and the OPC pulse propagation raise from two distinctive and different nonlinear polarizations [3,4], which should drive simultaneously the propagation Maxwell equation. The resonant polarization, $P_r^E(t,z)$, is the response of the TLA medium to the EM pulse, $E(t,z)$, and can be written in the generalized form [15]

$$\tilde{P}_r^E(\omega,z) = \chi_r(\omega,z)\tilde{E}(\omega,z), \tag{2}$$

where $\tilde{}$ stands for the time Fourier Transform of the involved variable and $\chi_r(\omega,z)$ is the local atomic susceptibility, which in general depends functionally on $\tilde{E}$. This polarization drives the corresponding propagation equation, and it is obtained from the basic medium atomic equations. For weak pulses $\chi_r(\omega,z)$ is independent of $E(\omega,z)$ and z, and its imaginary (absorptive) part coincides with the Beer's absorption coefficient $\alpha(\omega)/2$ in Eq. (1) [4]. On the other hand, the OPC raises from an intensity dependant nonlinearity [16] that associates the input pulse E and its conjugate pulse R, through the susceptibility χ_c of the polarizations in the Kerr medium,

$$\tilde{P}_c^E(\omega) = \chi_c\tilde{R}(\omega), \tag{3.a}$$

and symmetrically

$$\tilde{P}_c^R(\omega) = \chi_c^*\tilde{E}(\omega). \tag{3.b}$$

Similar, but not identical expressions are valid in other OPC media, such as the photorefractive materials [17]. In order to relate χ_c with the OPC parameters we consider phase conjugation by degenerate four wave mixing (DFWM) in a Kerr-type media [18,19]. In such cases, χ_c is typically given by $\chi^{(3)}I_p$, where $\chi^{(3)}$ is the third order susceptibility of the medium and I_p is the intensity of the pumping waves assumed to be continuous. For simplicity, we will use a scalar OPC model, in spite of being able to handle vectorial features such as the state of polarization [20].

Technically, the difficulty in applying the pulse propagation techniques to the transient phase conjugation problem resides precisely in the OPC dielectric nonlinear polarizations, Eqs. (3). They originate the simultaneous propagation of forward (probe) and backward (conjugate) pulses, a feature that prevents us from parameterizing the equations on local variables to carry on the direct integration on z (propagation distance) and t (time evolution). Moreover, this also means to have initial conditions on both sides of the nonlinear propagating medium. Both of these conditions disqualify most of the codes in use in pulse propagation studies, which in most cases consider just a single direction of propagation [14,21].

3. THE NUMERICAL PROCEDURE

Let $\mathcal{R} = \mathcal{R}(t,z)$ and $\mathcal{E} = \mathcal{E}(t,z)$ denote the polarizations due to conjugation, [Eqs. (3)] and $P_E(t,z)$ and $P_R^*(t,z)$ the polarizations due to the medium atomic susceptibility [Eq. (2)] for the pulse E and the conjugate pulse R^*, respectively. Then, the equations governing the temporal and spatial evolution of the pulse envelopes during the OPC process are generalized to [8]

$$\frac{\partial E}{\partial z} + \frac{1}{c}\frac{\partial E}{\partial t} = -iU(z)[\mathcal{R}^* + P_E], \tag{4.a}$$

and

$$\frac{\partial R^*}{\partial z} - \frac{1}{c}\frac{\partial R^*}{\partial t} = -iU(z)[\mathcal{E} + P_R^*], \tag{4.b}$$

where c is the speed of light, $U(z)$ is the rect (z/L) function, defined as 1 if $-1/2 \leq z/L \leq 1/2$ and 0 elsewhere. If the medium is not a conjugator, the polarization $\mathcal{R}$ and $\mathcal{E}$ are both zero and Eqs. (4) reduce to two independent equations which describe two uncoupled pulses being absorbed according to Eq. (1). On the other hand, with negligible absorption, the polarizations P_E and P_R^* are zero and the Eqs. (4) reduce to the standard phase conjugation equations [7]. In most cases of experimental interest, where the intensity of the probe and conjugated pulses are much lower than the pump intensities, the polarizations responsible of RPP, P_E and P_R^*, and the polarizations responsible of OPC, $\mathcal{R}$ and $\mathcal{E}$, can be theoretically associated with the first and the third term, respectively, in the series expansion of the total polarization of the material [3]. However, such separation on the OPC dynamics, which has been taken in account in writing Eqs. (4), can also be applied to cases where the corresponding susceptibilities are experimentally determined. The OPC in the presence of frequency dependent losses [3] is a good example of these cases.

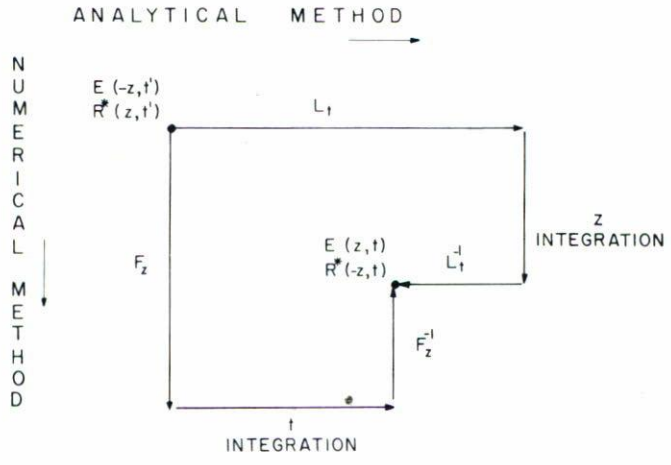

FIGURE 2. The paths of solution of the transient OPC for the previous analytical method and the proposed numerical method. $E(-z,t')$ and $R^*(z,t)$ are the initial pulses falling into the conjugator while $E(z,t)$ and $R^*(-z,t)$ are the final pulses emerging of the conjugator. L_t and L_t^{-1} represent the direct and the inverse time Laplace transform operations while F_z and F_z^{-1} stand for the direct and the inverse spatial Fourier transform operations, respectively.

In general conditions, we can not expect and analytic solution for the system (4). As we mentioned before, the few known results have been obtained in the case of constant coupling, negligible dispersion and constant absorption coefficients, χ_c and $\chi_r = i\alpha_0/2$ respectively, which reduce the polarization functions to: $\mathcal{R}(t,z) = \chi_c R(t,z)$, $\mathcal{E}(t,z) = \chi_c E(t,z)$, $P_E(t,z) = i\alpha_0 E(t,z)/2$ and $P_R(t,z) = i\alpha_0 R(t,z)/2$. The analytical procedure of solution, or time-resolved technique, followed in this case solves the linearized resulting equations by taking Laplace transform in time and then integrating in z [7,8]. However, the complete numerical solution along these lines would have a great difficulty in dealing with the space discontinuity represented by the conjugating medium as well as including the material resonant polarization.

In the numerical code, we have inverted the analytical solution sequence by taking the spatial Fourier transform of the general propagation equations and then carrying their time integration as shown in Fig. 2. Such procedure is closer to our daily experience because it describes the time evolution of the pulse propagation within the conjugating medium as it is penetrated by the forward and backward pulses, just as in a movie mode. At the same time, it allows the inclusion of initial conditions at both entering faces of the conjugator instead of boundary conditions, Fig. 3, and the parallel integration of the medium dynamics atomic equations, as it is indicated in the algorithm sketched in Fig. 4. Moreover, as the $U(z)$ function limits the interaction to occur in a space window located in the conjugating medium, the space aliasing problem involved in the inverse-transform operations [8,9] becomes a minor difficulty since we can arbitrarily vary the space integration window by taking into account the propagation in free space.

A short enough pulse has its spectral components distributed over an interval comparable to the spectral response of the conjugator. Therefore, in order to be able to consider the OPC of short pulses is necessary to take into account the spectral dependence of the

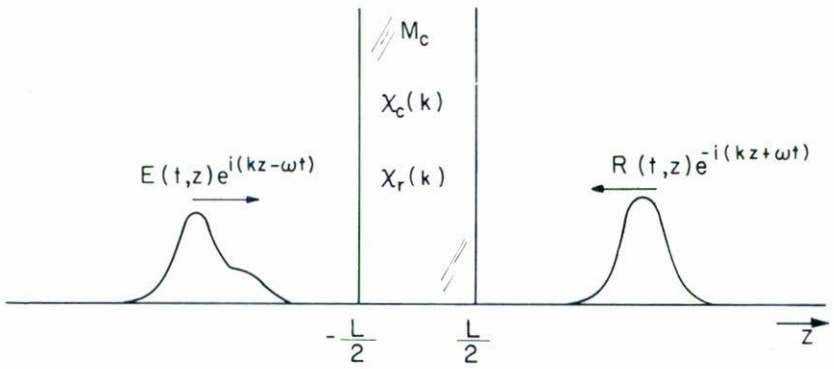

FIGURE 3. Initial conditions for the numerical solution of the pulse phase conjugation in a general case. The aliasing problem is easily avoided by choosing arbitrarily the free space window.

polarization functions. These functions drive the time and spatial evolutions of the probe and conjugate pulses in Eqs. (4), in such way that once they have been space-Fourier transformed, they have the following form:

$$r(t,k) = \chi_c(k) r(t,k), \tag{5.a}$$

$$e(t,k) = \chi_c(k) e(t,k), \tag{5.b}$$

$$p_E(t,k) = \chi_r(k) e(t,k), \tag{6.a}$$

$$p_R^*(t,k) = \chi_r^*(k) r(t,k), \tag{6.b}$$

where k is the Fourier conjugate variable of z centered at the wave number $\kappa = \omega_c/c$; e and r are the space Fourier transform of E and R respectively. According to Eqs. (3) and (5), the function $\chi_c(k)$ is the spectral susceptibility due to the Kerr-type nonlinearity of the DFWM conjugator. $\chi_c(k)$ also models the physical characteristics of the conjugating medium as a function of the spectral distribution and the orientation of the pumping signal or materials that modify the medium characteristics such as thin films, etc. When the pulse has long duration, its value at resonance, $\chi_c = \chi_c(0)$, is the standard OPC coupling constant [7] and does not provide any information on the spectral width or duration of the pulse.

The integral of a pulse envelope on its time duration (the area of the pulse) plays a critical role in defining the RPP in a TLA medium. Its susceptibility $\chi_r(k)$ is obtained by solving the medium atomic equations and it has been thoroughly studied for small area pulses, either for long pulses (Beer's Law) or for very short pulses (anomalous absorption). For large area pulses there is a complex dynamics and $\chi_r(k)$ is not standard [22]. However

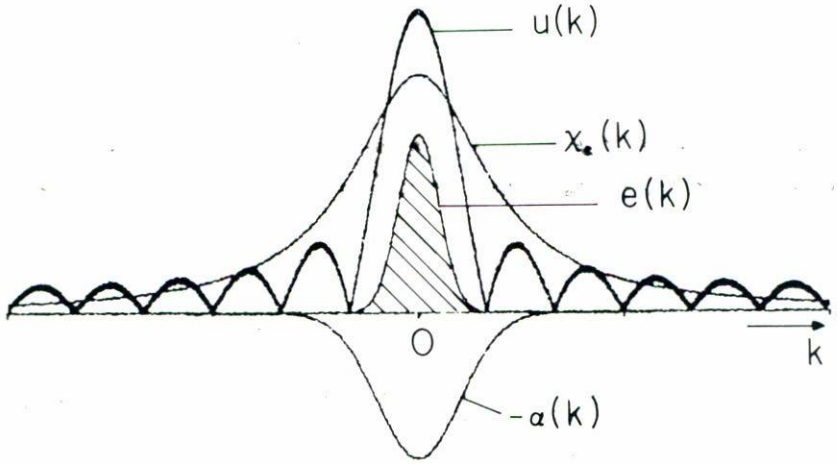

FIGURE 4. Basic algorithm derived from Eqs. (7) for the numerical modeling of the transient OPC process. The versatility of the algorithm allows the inclusion of experimental or theoretical resonant susceptibilities, $\chi_r(k)$, as well as the simultaneous solution of Eqs. (7) with the appropriate resonant dynamical equations when $\chi_r(k)$ is unknown. This last case is indicated by the dashed steps. The algorithm must be applied N times in order to find the pulses E and R at the time $ct = cN\Delta t$.

its numerical determination from a TLA medium constitute an efficient computational tool. Previous OPC studies [7] have considered only small area cases and long enough pulses to make unnecessary any spectral considertions in χ_r. In order to show how the shape of a short enough pulse may be modified by a frequency dependant susceptibility, in this paper we will extend those results to anomalous absorption in OPC by propagating short pulses of small area. In such cases we sill make $\chi_r(k) = i\alpha(k)/2$, and we will neglect the real (dispersive) part of the atomic response of the medium because it affects the phase but not the shape of the propagating pulse [12].

The numerical procedure is started by taking space Fourier Transformer of Eqs. (4) and making use of Eqs. (5) and (6) to obtain

$$\frac{1}{c}\frac{\partial e}{\partial t} + ike = [-i\chi_c(k)r^* - \alpha(k)e] \otimes u, \tag{7.a}$$

$$\frac{1}{c}\frac{\partial r^*}{\partial t} - ikr^* = [i\chi_c^*(k)e - \alpha^*(k)r^*] \otimes u, \tag{7.b}$$

where $\otimes$ stands for the convolution operation and $u = u(k)$ is the space Fourier transform of $U(z)$. The system of Eqs. (7) describes the conjugation process as a competitive pulse

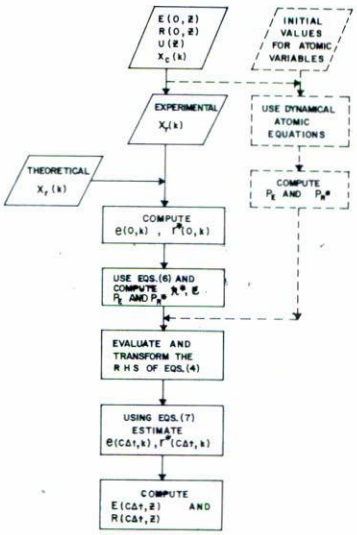

FIGURE 5. The OPC of a pulse as a filtering process of the spectral distribution of the probe pulse, $e(k)$. The spectrum of the reflected pulse will be formed by the gain profile of the conjugating susceptibility, $\chi_c(k)$, but then it will be filtered by the absorptive part of the resonant atomic susceptibility, $\alpha(k)$, and by the spectral function of the conjugator, $u(k)$. Note that we have drawn the modulus of $u(k)$ in order to emphasize that we can modify the characteristics of the conjugation process by varying the amplitude and the relative width of the spectral functions.

filtering, displayed in Fig. 5, with spectral windows created by the conjugator thickness, $\chi_c(k)$ and $\alpha(k)$. Figure 4 shows the basic steps executed by the numerical code for obtaining the propagated pulses $E(c\Delta t, z)$ and $R(c\Delta t, z)$ from the initial pulses $E(0, z)$ and $R(0, z)$. Then the pulses at any time can be obtained from iterative application of these steps. Moreover, the general form of Eqs. (7) makes them suitable to be solved simultanously with the time-dependent resonant atomic Bloch equations [4], or with predetermined resonant atomic susceptibilities [6]. In the first case we can obtain the resonant polarization from the Bloch equations and then substitute it back in the second term of the RHS of Eqs. (7). We will explore the practical applications of this case elsewhere.

In order to show that the method implied by Eqs. (7) reproduces and generalizes analytical known results, we shall consider two important extreme cases. These cases will be compared with their realistic analogous in the following section. The first one corresponds to an infinitely thin conjugator or optical phase mirror (OPM) at $z = 0$. This case has been considered only for long pulses where the response is constant for all frecuencies. However, the wide spectral bandwidth that corresponds to a very short pulse makes necessary to reconsider such case (see appendix A). In Eq. ($A3$), we choose constant coupling $\chi_c(k) = \chi_c$ and obtain

$$E(T, z) = -i\chi_c R^*(T - z, 0) + E(0, z + T) - \int \alpha(k')e(T - z, k')\,dk', \qquad (8.a)$$

$$R^*(T,z) = i\chi_c^* E(T+z,0) + R^*(0,z-t) - \int \alpha^*(k')r^*(T+z,k')\,dk'. \qquad (8.b)$$

The first term in the RHS of Eqs. (8) represents the respective conjugated pulse, which is counter-propagating and in the same spatial semiplane that the corresponding initial pulse. Thus, the conjugator reflects at both faces the conjugated replica of the incident pulses. The other two terms in the RHS of Eqs. (8) are the transmitted pulses as a function of the loss at the OPM. If the pulses are long, $e(k) = e_0\delta(k)$ and $r^*(k) = r_0^*\delta(k)$, the Eqs. (8) reproduce the basic known results for an infinitely thin conjugator operating in steady state conditions [19].

The second case corresponds to an infinitely thick conjugator, where the only known result is the integrator behavior for a weak coupling coefficient [19]. As it is shown in the appendix B, the solutions of the Eqs. (7) in this case can be obtained by using time Laplace transform. In the specific case of null losses and that the constant coupling, $\chi_c(k) = \chi_c$ is a small parameter, we can neglect the RHS of Eq. $(B.2)$ to obtain

$$R^*(ct,z) = i\chi_c \int_{-\infty}^{\infty} e(0,k)\frac{\sin(kct)}{k}e^{ikz}\,dk$$

$$= \frac{i\chi_c}{2}e(0,z) \otimes \mathrm{rect}\,(z/2ct). \qquad (9)$$

Equation (9) represents the well known result of the integrator capability of a thick but weak conjugating medium.

4. NUMERICAL RESULTS

In this section we compare the previous ideal results with their realistic analog to show the quality of the numerical modeling. We leave for a further note the specific analysis of problems of further complexity and of special physical interest such as vectorial features [20] and large area pulses. The numerical analysis is based in the comparisons of the first zeros of the sinc function $u(k)$ given by the material window with the spectral width of the probe pulse, the conjugate pulse, the conjugating coupling $\chi_c(k)$ and the absorptive atomic response $\alpha(k)$, Fig. 5. The last two functions behave as filters for the pulses, gain and loss respectively, and they represent the behavior of the response of the material to EM excitation.

In the thin conjugator there is little sense in considering absorption. But, as in a standard mirror, it is convenient to consider the "spectral reflectivity" given by $\chi_c(k)$ [23]. In Fig. 6 we show an example of a standard thin conjugator obtained by solving Eqs. (4) with a null initial condition for the pulse R^*. The numerical simulation can be associated with the realistic conjugation of a pulse of 80 psec by conjugator with a thickness of 2 cm. The coupling strength has been chosen as constant, $\chi_c(k) = \chi_c$, such that $\chi_c L = 0.35\pi$, thus the conjugator is in stable operation with near unity on-resonance amplification [7]. This means that the spectral distribution of the pulse is located within the central lobe of the medium spectral window and can fully conjugate the incoming pulse. For $R^*(0,z) = 0$,

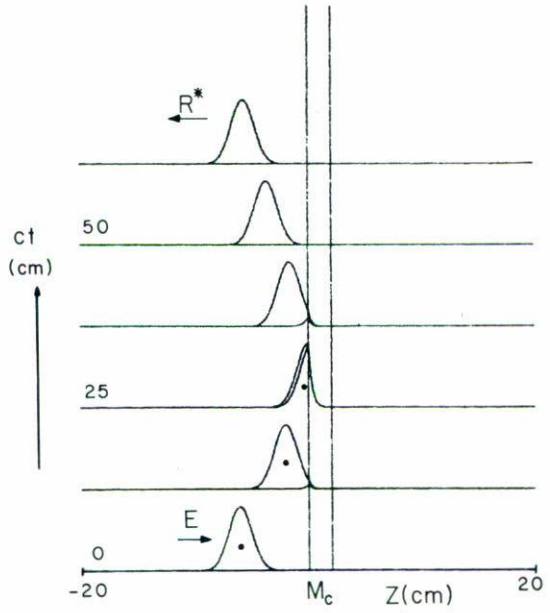

FIGURE 6. A probe pulse with gaussian profile and 80 psec duration is perfectly reflected by a 2 cm width conjugator. The dot is used to distinguish the probe pulse during the reflection. Note that there is no transmitting signal because the stability operation condition $\chi_c L < .5\pi$ is satisfied.

Eqs. (8) predict the complete reflection of the probe pulse, and though the case shown in Fig. 6 is far from the ideal infinitely thin conjugator considered by Eqs. (8), it exhibits a very similar behavior; *i.e* the input pulse generates the reflected (conjugated) pulse as it enters to the conjugator and there is not substantial signal to the right of the conjugator. As it is expected, the behavior of a thin but finite conjugator largely depends on the strength of the coupling coefficient. Our numerical simulations show that for smaller values of χ_c, such that $\chi_c L < 0.25\pi$, when the on-resonance amplification is below unity, the reflected pulse R^* is not the conjugated replica of the input pulse and it is smaller and broader. On the contrary, if we overcome the stability limit, $\chi_c L > 0.5\pi$, the reflected pulse is amplified but its trailing edge exhibits periodic oscillations and it is not the conjugated replica of the input pulse. A considerable transmitted pulse will also appear in this condition and it will show an oscillatory envelope similar to the reflected pulse. All these results reproduce the OPC characteristics reported in Refs. [7], [8] and [24].

The ability of this OPM to reverse the phase [25] of a pulse in the so-called doubled-pass geometry [1] is shown in Fig. 7. With null initial condition for the pulse R^*, the initial probe pulse E_0 first goes through an aberrator consisting of a cell filled with a nonresonant dispersive medium (DM) and acquires an intensity dependent chirping. Then the modulated pulse R^* is produced. At this point, the generation of the pulse R^* occurs under the same conditions and in a similar way than those showed in the Fig. 6. Finally, the reflected pulse passes through the aberrator and the result is an unchirped R_0^* pulse,

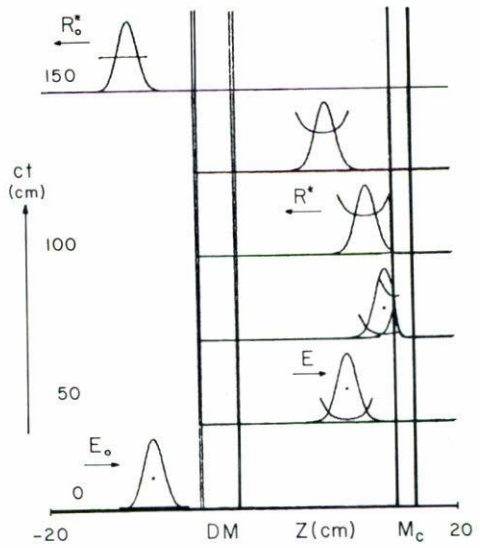

FIGURE 7. The same probe pulse of Fig. 6 is first passed through a dispersive medium, DM, which generates an intensity dependent phase on the pulse. The passage back of the reflected pulse through the DM corrects the original phase distortion. The final constant phase factor of $\pi/2$ was acquired during the reflection on the conjugator, Eq. (8).

which has a constant phase factor of $\pi/2$, which can also be inferred from Eq. (8). With this example we can clearly notice the phase reversal phenomenon that is taking place in the conjugator even in this intensity dependent case, but as we have stated before this process only occurs when the conjugator is in stable operation [24].

In a thick conjugator, we know analytical results only in the case of an infinitely long and weak conjugator where its boundary behaves as an integrator, *i.e.*, it reflects the time-integrated signal of the input pulse [19] Eq. (9). However our numerical method allows the analysis of a realistic case, in which a probe pulse falls into a long but finite weak conjugator. This case can be reached with the use of very short pulses, since their spectral width is larger than both the width of the conjugator spectral window and the width of the absorption function. In order to show this new possibility, we will consider a case where absorption is spectrally narrow, that is when $\alpha(k) = \alpha_0\delta(k)$. In Fig. 8, we present the numerical propagation of an antisymmetric null area input pulse with profile $E(0,z) = \mathrm{sech}(z)\tanh(z)$, ($R^*(0,z)$ is null again), through a long and weak conjugator in the presence of a small α_0 coefficient. There, it can be seen that the input face of the conjugator nearly reflects the integrated signal of the probe pulse; that means that but for a very weak and long tail, the reflected pulse R^* has a $\mathrm{sech}(z)$ profile. The presence of such long tail can be associated with deviations from the original null area input pulse caused by the absorption process. Furthermore, because of the weakness of the conjugator, if

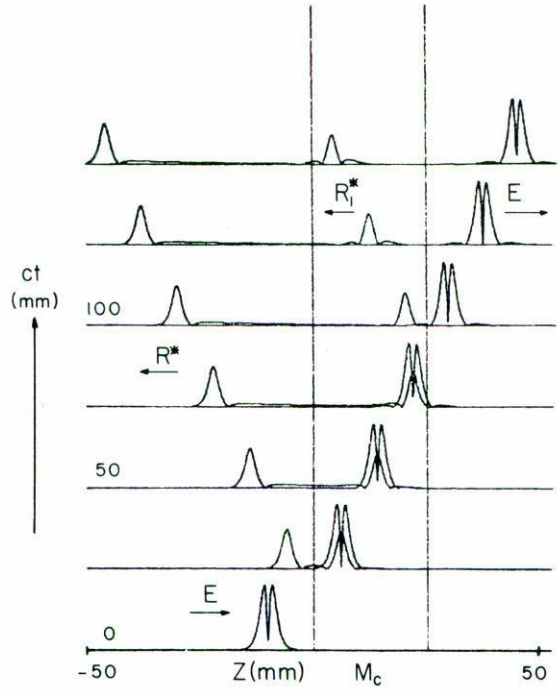

FIGURE 8. A probe pulse with profile $\mathrm{sech}(z)\tanh(z)$ is propagated through a thick and weak conjugator of $L = 25$ mm. The atomic response function is represented by $\alpha(k) = \alpha_0\delta(k)$ and causes a preferential absorption on resonance that lead to anomalous absorption. This effect is clearly noticeable in the wings of the second integrated-reflected pulse R_1^*.

absorption is negligible the input pulse would propagate through the conjugator without any substantial change and would produce another time integrated pulse at the output face of the conjugator. However, Fig. 8 shows that within the conjugator the probe pulse acquires the characteristic wing oscillations of the anomalous absorption effect, though this is a small efect by virtue of the small α_0 used. Nevertherless, the presence of the wing oscillation is more evident in the reflected pulse at the exit face of the conjugator, R_1^*, which is the time-integration of the finally transmitted pulse E_1. We must emphasize that in this case the conjugator does not, strictly speaking, conjugate the probe pulse.

On the other hand, the spectral selective phase conjugate mirrors can be used to create an oscillator. This geometry has been extensively applied and studied in steady state conditions [26], but not in the transient regime. Here, the proposed numerical method can also simulate such problems, providing a good example of its versatility. Fig. 9 shows the evolution of a pulse within the simplest case of an oscillator consisting of two OPM identical to the one used in Fig. 6. Therefore, the pulse is perfectly reflected at both conjugators and it is maintained within the cavity. However, if we consider an amplifier within the phase conjugate mirrors, the problem may reach a wide spectrum of possibilities. Here, with the narrower pulse (< psecs.) we will be in the thick conjugator domain and in the regime where absorption is not constant and where transmission out of the cavity will occur. Out of the competition of loss and gain, we can obtain an original method

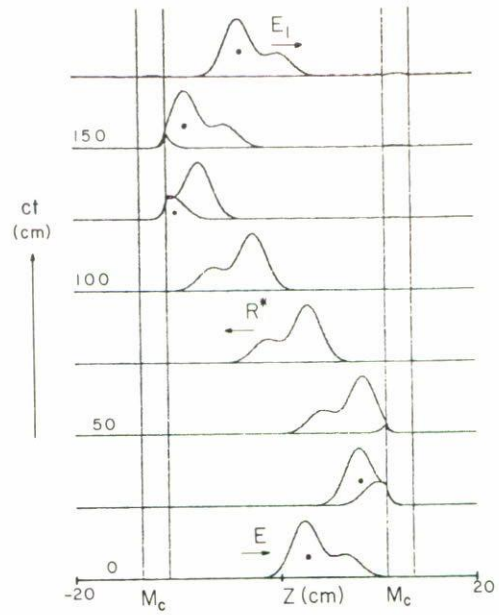

FIGURE 9. Numerical simulation of a pulse in an OPM-OPM scalar resonator. The width of both conjugators is 2.5 cm and their operation condition is the same that in Fig. 6. Note that the two conjugators work like conventional mirror resonator. The introduction of an amplifying medium and transversal dynamics should be included for a realistic OPM-OPM model.

to compress pulses along the lines of spectrally burned holes in the amplifier [15]. This problem will be discussed in detail elsewhere.

5. CONCLUSIONS

We have presented a numerical method to carry on studies of the transient phase conjugation propagation in non ideal cases. Basically, the method allows to consider the spectral characteristics of the pumping, the microscopic dynamics of the conjugating material, and also other features beyond the capability of previous theoretical analysis of OPC. The consideration of transient phase conjugation of very short pulses was introduced, by relating their duration to the atomic response width and the conjugating material thickness.

APPENDIX A: AN INFINITELY THIN PHASE CONJUGATOR

If the medium is very thin, it can be modeled by writing $U(z) = \delta(z)$ and $u(k) = u(0) = 1$. For compactness, we will use $T = ct$ and $\dot{e}$ as the T derivative of e. Then, Eqs. (7) can

be written in the form

$$\dot{e}(T,k) + ike(T,k) = \int_{-\infty}^{\infty} [-i\chi_c(k')r^*(T,k') - \alpha(k')e(T,k')]\,dk', \qquad (A1.a)$$

$$\dot{r}^*(T,k) - ikr^*(T,k) = \int_{-\infty}^{\infty} [i\chi_c^*(k')e(T,k') - \alpha^*(k')\dot{r}(T,k'))]\,dk'. \qquad (A1.b)$$

If we integrate and notice that the integral

$$\frac{1}{2\pi}\int_0^T \int_{\infty}^{\infty} e^{-ik'[z\pm(T'-T)]} f(T')\,dk'\,dT' = \int_0^T \delta[z \pm (T'-T)]f(T')\,dT'$$

$$= \begin{cases} f(T \mp z) & \text{if } \pm z < T < T \pm z, \\ 0 & \text{otherwise,} \end{cases} \qquad (A2)$$

we finally obtain

$$E(T,z) = E(0,z+T) + \int_{-\infty}^{\infty} [-i\chi_c(k')r^*(T,k) - \alpha(k')e(T,k')]\,dk', \qquad (A3.a)$$

$$R^*(T,z) = R^*(0,z-T) + \int_{-\infty}^{\infty} [-i\chi_c^*(k')e(T+z,k') - \alpha^*(k')r^*(T+z,k')]\,dk'. \qquad (A3.b)$$

APPENDIX B: AN INFINITELY THICK PHASE CONJUGATOR

If the conjugating medium is very thick the medium window can be approximated by u(k) = δ(k). We will use the notation $Q(k) = ik + \alpha(k)$, so the Eqs. (7) can be written as

$$\dot{e}(T,k) + Q(k)e(T,k) = -i\chi_c(k)r^*(T,k), \qquad (B1.a)$$

$$\dot{r}^*(T,k) + Q^*(k)r^*(T,k) = i\chi_c^*(k)e(T,k). \qquad (B1.b)$$

These equation can be integrated by substitution into each other. We obtain after changing the order of the integration,

$$r^*(T,k) - r^*(0,k)e^{-Q^*T} - \frac{i\chi_c^*(k)e(0,k)}{Q^*-Q}\left[e^{-QT} - e^{-Q^*T}\right] =$$

$$\frac{|\chi_c(k)|^2}{Q^*-Q}\int_0^T r^*(T',k)\left[e^{-Q(T-T')} - e^{-Q^*(T-T')}\right]dT'. \qquad (B.2)$$

The solution of this equation can be obtained by taking Laplace transform on time. If we denote by s the Laplace variable of t and by $\bar{e}$ the Laplace transform function of e we get

$$\bar{e}(s,k) = \frac{e(0,k)(s+Q^*) - i\chi_c(k)r^*(0,k)}{(s+Q^*)(s+Q) - |\chi_c(k)|^2}, \qquad (B3.a)$$

$$\bar{r}^*(s,k) = \frac{r^*(0,k)(s+Q) + i\chi_c^*(k)e(0,k)}{(s+Q)(s+Q^*) - |\chi_c(k)|^2}. \qquad (B3.b)$$

The poles through which the Eqs. $(B3)$ can be Laplace inverted in order to obtain the pulses $E(t,z)$ and $R(t,z)$ are given by

$$S(k) = \frac{-\left[Q(k)+Q^*(k)\right] \pm \sqrt{4|\chi_c(k)|^2 + \left[Q^*(k)-Q(k)\right]^2}}{2}. \qquad (B4)$$

Again, in the case that $\alpha(k)$ and $\chi_c(k)$ are constant, $S(k)$ will coincide with the results obtained in the analytical analysis [2]. Otherwise, the atomic response function and the coupling parameter are functions of frequency.

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