# Limits to the adiabatical coherent amplification of solitons in active doped optical fibers

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ABSTRACT. We make a numerical study of the initial stage of the coherent soliton adiabatic amplification in active doped optical fibers in the general case of finite inhomogeneous decay time. We are interested in adiabatic amplification because it can recover and reshape the original input pulse, in opposition to fast amplification. Our results show that it is possible to obtain closer adiabatic amplification of optical solitons by increasing the width of the inhomogeneous atomic line, as expected, and define its limits for finite linewidth. For short enough propagation distances, we give an analytical description of the amplification process based on the assumption that the general dynamics can be separated into two parts: the pure amplification due to the resonant atoms and the pure self-phase modulation due to the fiber itself. Such description is useful for obtaining a quantitative estimation of the propagation distance after which the amplification process will deviate from the adiabaticity condition. In other words, it may represent the physical length of the amplifier, avoiding lay in the fast amplification region.

RESUMEN. Se analiza numéricamente la etapa inicial de la amplificación coherente de solitones en fibras ópticas activas en el caso general en que el tiempo de decaimiento inhomogéneo de los átomos resonantes es finito. Nuestro interés se centra en la amplificación adiabática de pulsos, dado que nos permite recuperar tanto su amplitud como su forma. Como era de esperarse, nuestros resultados muestran que conforme se aumenta la anchura de la línea inhomogénea el proceso de amplificación se acerca más a la deseada amplificación adiabática. Para distancias de propagación suficientemente cortas, damos una descripción analítica del proceso de amplificación, suponiendo una separación entre la dinámica de amplificación debida a los átomos resonantes y la automodulación de fase generada por la propia fibra. Tal descripción permite estimar cuantitativamente la distancia de propagación a la cual la condición de adiabaticidad no se cumple. En otras palabras, podría fijar la longitud física del amplificador, evitando caer en la región de amplificación rápida.

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From the practical point of view of realistic long-distance optical communications, solitons in optical fibers are not only attractive because of their capability of overcoming the group velocity dispersion inherent to silica fibers, but also because the loss always present in the fiber affects their amplitude and width in a proportional way only, leaving their secant hyperbolic shapes unaffected [1]. Thus, after some propagation distance, when an initial soliton has lost a certain portion of its energy, one needs to introduce an amplification process in order to ideally recover the original pulse. Following this idea, a considerable amount of both theoretical and experimental work has been done in order to explore the technical problems that could occur in optical repeaters when installed in all-optical telecommunication systems based on solitons.

At present we can identify three fundamental types of optical amplifiers: i) semiconductor laser amplifiers [2], ii) Raman amplifiers [3], and more recently iii) rare-earth doped optical fibers amplifiers (EDFA) [4]. The second type of amplifiers has been successfully tested in the laboratory [5], and theoretical studies have demonstrated that it is possible to propagate solitons at a rate of 2.5 Gb/s over 10,000 Km. or more [6]. However there is an increasing interest on the rare-earth doped optical fiber amplifiers [7], mainly because their gain lines coincide with the spectral regions where the silica fibers have their low-loss windows, and also because it should be possible to integrate them with the new fiber-laser technology [8].

The amplification in a doped fiber can be afforded in two opposite ways. The first one is known as sudden amplification, where the concentration of dopants is very high, which mean an exponential gain in very short distances. On the other hand it is the adiabatic amplification, which allows a smooth recovering and reshaping of the pulse. The importance of this process resides in long optical communications because it can support soliton propagation.

The most promising doped fiber amplifiers are those in which the resonant dopants consist of the trivalent ion  $Er^{3+}$ , whose gain spectrum has a maximum around 1.55  $\mu$ m, because it allows solitons propagation. Recently, the first experimental work on soliton optical amplification in such doped fiber has been reported [9-12], but the corresponding theoretical and numerical studies are still incomplete, mainly due to the difficulty of simultaneously treating both nonlinearities of the fiber and those of the resonant atoms; moreover it seems that other nonlinear effects, such as intrapulse Raman scattering, are simultaneously present [13]. Even neglecting such others nonlinear effects, the only know theoretical results at the present for the optical amplification of solitons in optical fibers are those based on perturbative treatments of the gain source, which apply only to constant or parabolic gain profiles. For the former case, the results indicate that for small enough gain coefficients it is possible to obtain a progressive adiabatic amplification of solitons, but for the parabolic gain profile case, a saturation in the shortening of the pulse width occurs at some distance within the optical amplifier [14]. However, in the specific case of doped optical fiber amplifiers Mel'nikov et al. [15] numerically showed that the coherent nature of the interaction between the optical pulse and the resonant atoms significantly contribute to the amplification process. In fact, they found that the presence of the homogeneous decay time  $(T_2)$  of the resonant atoms will eventually break down the adiabatical amplification process, but they did not give any estimation of how far a given soliton can be adiabatically amplified. On the other hand, for the actual ultrashort pulses

(~ 100 fsec), the width of the inhomogeneous line of Er atoms in silica fibers (~ 9 TH) is comparable and can not be further neglected. Therefore, the purpose of this paper is investigate the practical limits of the adiabatical amplification in Er-Doped optical Fibers Amplifiers (EDFA), even when the presence of the inhomogeneous broadening of the resonant atoms is taken into account.

As is well known, the space and time evolution of the complex envelope of an electromagnetic pulse through a doped optical fiber can be described by the nonlinear Schrödinger (NLS) equation [16], properly generalized to take into account the influence of the resonant atoms [17]:

$$i\frac{\partial V}{\partial Z} = \frac{\pi}{4}\frac{\partial^2 V}{\partial T^2} + \frac{\pi}{2}|V|^2 V + \frac{\alpha_0 z_0 t_0}{\gamma \pi g(0)}P_{\text{TLA}}.$$
(1)

Here we have used the standard normalization to soliton variables [17]. V(T, Z) represents the pulse envelope normalized to  $\sqrt{\mathcal{P}_1}$ , where  $\mathcal{P}_1 = 2\eta_0|\beta_2|/\tau_0^2 n_2\beta_0$  is the pulse peak power associated to the first order soliton.  $z_0$  is the soliton period and is given by  $z_0 = \pi^2 c^2 t_0^2 / \lambda |D(\lambda)|$ ;  $\alpha_0 = 4\pi^2 N d^2 k^2 g(0) / \hbar^2 \beta t_0$  is the on resonance low-intensity gain coefficient of the resonant atoms, and  $\gamma = (2dt_0/\hbar)\sqrt{\mathcal{P}_1}$ , with d being the dipole matrix element of the resonant transition, N the resonant atomic density, k the wave number, c is the velocity of light and  $\lambda$  its wavelength,  $t_0$  is the temporal width of the initial pulse ( $\tau_0 = 1.76t_0$ ).  $Z = z/z_0$  and  $T = |t - z/\beta_1|/T_0$  are the normalized propagation distance and local time, respectively,  $\beta_i$  are the i - th derivative of the fiber propagation constant  $\beta(\omega)$ , evaluated at the pulse carrier frequency  $\omega_L$ ;  $D(\lambda) [= 2\pi c^2 \beta_2/\lambda]$  is the Group Velocity Dispersion (GVD) in dimensionless units, and  $\eta_0$  and  $\eta_2$  are the linear and the Kerr nonlinear part of the refraction index of the fiber alone, respectively. Finally,  $n_2 = \eta_2/A_{\rm ef}$  is the so called effective nonlinear coefficient, with  $A_{\rm ef}$  being the effective transversal area of the optical fiber [16].

On the other hand, in Eq. (1)  $P_{\text{TLA}}$  stands for the complex envelope of the macroscopic resonant polarization and, assuming a Two Level Atom (TLA) picture for the resonant dopant atoms, it is given by

$$P_{\text{TLA}} = \langle p \rangle = \int p(\Delta, T, Z) g(\Delta) d\Delta, \qquad (2)$$

where  $\Delta = \omega - \omega_L$  is the atomic detuning, p is the complex envelope of the atomic dipole and

$$g(\Delta) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-\Delta^2/2\sigma^2), \tag{3}$$

with  $\sigma = (2T_2^*\sqrt{\ln 4})^{-1}$ , is the normalized inhomogeneous atomic line shape of width  $\sigma$ , with  $T_2^*$  being the inhomogeneous decay time. We will consider the pulse duration  $(t_0)$  much smaller than the spontaneous  $(T_1)$  or any incoherent decay times. Then, the

microscopic dynamics is given by the Bloch equations [15,17]:

$$\frac{\partial p}{\partial T} = i\Delta t_0 p - \frac{p}{t_2} + i\gamma wV, \tag{4a}$$

$$\frac{\partial w}{\partial T} = \mathcal{I}m[\gamma V p^*],\tag{4b}$$

where  $w(\Delta, T, Z)$  is the atomic population inversion and  $t_2 = T_2/t_0$ ,  $T_2$  being the dipole decay time. The system formed by Eqs. (1-4) has been numerically solved for the passive resonant doped fiber case [17]. However, because d for Erbium is about  $2.5 \times 10^{-20}$  esu, if we use the typical parameters for the fiber:  $\beta_2 = 20 \text{ psec}^2/\text{Km}$  at  $\lambda = 1.55 \ \mu\text{m}$ ,  $\eta_2 = 3 \times 10^{-20} \text{ m}^2/\text{W}$  and  $A_{\text{ef}} = 60 \times 10^{-12} \text{ m}^2$ , then  $\gamma \approx 0.01$ , and the effective pulse area that the TLA sees is very small [18]. Under this condition we can consider that the population inversion is coherently unchanged and then practically constant during the interaction [15,18]. Therefore, we can neglect Eq. (4b) and set w = 1 in Eq. (4a) for an active doped optical fiber. Then, Eq. (4a) can be formally integrated to give  $p(\Delta, T, Z)$  in terms of the time integral  $\int V(T', Z) \exp(-i\Delta T' + T'/t_2) dT'$ . This have been the procedure used to study the coherent amplification of solitons in optical fibers for the homogeneous case [15], that is, when  $\sigma \to 0$  in Eq. (4) or  $g(\Delta) = \delta(\Delta)$ . However, because for the inhomogeneous case such methodology is additionally complicated by the integration in Eq. (2), we have preferred to solve Eq. (4a) taking its time Fourier transform to obtain

$$\tilde{p}(\delta t_0, Z) = \frac{i\gamma t_2}{1 - i[\delta t_0 + \Delta t_0]t_2} \tilde{V}(\delta t_0, Z),$$
(5)

where  $\tilde{}$  denotes the transformed variables and  $\delta t_0$  is the normalized Fourier variable. Substitution of Eq. (5) into the time transform of Eq. (2) gives the familiar result

$$\tilde{P}_{\text{TLA}}(\delta t_0, Z) = \chi(\delta t_0) \tilde{V}(\delta t_0, Z) = \left\langle \frac{i\gamma T_2}{1 - i[\delta t_0 + \Delta t_0]T_2} \right\rangle \tilde{V}(\delta t_0, Z),$$
(6)

where  $\chi(\delta t_0)$  is the local resonant susceptibility [18]. It is necessary to emphasize that this susceptibility is only valid when the area of the pulse remains very small ( $\ll \pi$ ). Finally, we take the time Fourier transform of Eq. (1), and using Eq. (6) we obtain

$$i\frac{\partial V}{\partial Z} = -\frac{\pi}{4}(\delta t_0)^2 \tilde{V} + \frac{\pi}{2}F[|V|^2 V] + \frac{\alpha_0 z_0 t_0}{\gamma \pi g(0)}\chi(\delta t_0)\tilde{V},\tag{7}$$

which can be numerically solved by a technique previously reported [19]. In Eq. (7), F[xy] denotes the time Fourier transform of the xy product. It is worth to emphasize that Eq. (7) is highly nonlinear because it establishes a strong competition between the fiber intensity-dependant refraction index, and the frequency-dependant resonant gain, which increases the pulse intensity. Thus, the resonant atoms alter the NLS equation and the magnitude of their influence is controlled by the parameters  $z_0$ ,  $\alpha_0$  and  $g(0)/t_0$ , which



FIGURE 1. The temporal (a) and spectral (b) behavior of a first order soliton as it propagates in an inhomogeneously broadened EDFA. The graphs were obtained by numerical solution of Eq. (7) with the parameters:  $\gamma = 0.01$ ,  $\alpha_0 z_0 = 0.48$  and  $T_2 = T_2^* = t_0 = 0.1$  psec.

depend on the particular experimental setup, specially on the initial pulse width. Here we will use the typical fiber parameters given above and we will fix the input pulse width at  $t_0 = 100$  fsec., so  $z_0$  will remain fixed at 0.8 m. We will study the influence of the resonant atoms by varying  $\alpha_0$  and  $g(0)/t_0$ , through changes in the dopant density N, and in the inhomogeneous decay time  $T_2^*$ , respectively.

To illustrate the general behavior followed by a pulse within an EDFA, we solve Eq. (7) numerically taking a first order soliton as the initial pulse; *i.e.*  $V(T,0) = \operatorname{sech}(T)$ . In Fig. 1 we display the time and the spectral evolution of such initial pulse at several propagation distances within the doped fiber, for the intermediate case  $T_2 = 100$  fsec  $(t_2 = 1), T_2^* = 100$  fsec  $(t_2^* = 1)$  and a gain of 2.6 dB/m ( $\alpha_0 = 0.6 \text{ m}^{-1}$ ). In Figs. (1a) and (1b), one observes that at the beginning the pulse is progressively amplified at almost adiabatic rate. However, at the propagation distance  $Z \approx 2.5$ , the temporal behavior of the pulse, Fig. (1a) exhibits a noticeable wing modulation, which can also be appreciated as the two central peaks showed by the corresponding spectrum, Fig. (1b). This modulation is associated to fact that the amplification does not occur in an exact adiabatic way [14], and it is expected by the nonlinearity of Eq. (7). The first tendency of the amplifier is first to simple increase the pulse amplitude, causing that the pulse will not be more an exact first-order soliton. Then, the optical fiber nonlinearities reduce the pulse width in order to recover a perfect soliton and the process will star again. In fact, Fig. (1a) shows that the modulation on the pulse wings is again present at  $Z \approx 4$ . Such behavior had been pointed out in the cases of constant gain, parabolic profile, and also when the coherent effects due to the resonant homogeneous atomic line were considered, and it represents the fact that the adiabatic amplification does not imply the occurrence of a monotonous process.



FIGURE 2. Spatial behavior of the pulse area S(Z) for three different inhomogeneous decay times. In (a)  $T_2^* = 0.3$  psec, in (b)  $T_2^* = 0.1$  psec, and in (c)  $T_2^* = 0.0333$  psec. The other parameters were  $T_2 = 0.100$  psec,  $z_0 = 0.8$  m and, in order to isolate the effect of the inhomogeneous broadening,  $\alpha_0 t_0/g(0)$  was fixed at 1.2 m<sup>-1</sup>, corresponding to an TLA density of  $N = 10^{17}$  cm<sup>-3</sup>.

However, the subsequent pulse behavior for longer propagation distances is quite different in all three cases. For the constant gain case, such behavior will repeat indefinitely and will asymptotically produce an exact first-order amplified soliton [14]. For the parabolic gain profile case the oscillating process will continue until the pulse will reach a temporal width limit imposed by the width of the gain profile [14]. However, for the coherent description of the EDFA, it was found that the process will be interrupted because of the finite  $T_2$  [15]. In this case the memory effect of the resonant dipoles will cause the formation of subpulses in the trailing wing of the initial soliton and the pulse itself will be eventually destroyed. Therefore, the results showed in Fig. 1 indicate that such initial behavior will be qualitatively the same even if the presence of the inhomogeneous atomic line is taken into account. However, we can expect quantitative differences and that they should depend on the strength of the gain and on the relative width of the atomic line shape.

In order to investigate such quantitative differences we will center our attention in the spatial behavior of the pulse area, defined as  $S(Z) = \int |V| dT$ . The reason is the following: as S(z) is proportional to the product of the pulse amplitude times the pulse width, the ideal adiabatical amplification implies constant area [1,14], so deviations of the pulse area trace from an horizontal line indicate how far of adiabaticity the EDFA is working. To appreciate the influence of the inhomogeneous broadening on the initial EDFA behavior we have solved Eq. (7) for several values of  $T_2^*$ , and we have displayed the corresponding pulse area curves in Fig. 2. In these runs we have used the same parameters of Fig. 1,  $t_0 = T_2 = 100$  fsec,  $z_0 = 0.8$  m, but the ratio  $\alpha_0 t_0/g(0)$ , which is directly proportional to N [15], has been fixed at 1.2 m<sup>-1</sup>, which corresponds to a density of active atoms of  $N = 10^{17}$  cm<sup>-3</sup>. As it can be seen, all curves have the expected similar qualitative behavior, but the initial rate of growing and the area value at which their first maximum appears are all different. Fig. 2 also shows that as the width of the inhomogeneous atomic line becomes broader  $(T_2^* \to 0)$ , the desired adiabatical amplification can be maintained for



FIGURE 3. Spatial behavior of the pulse area for three different gain coefficients. In (a)  $\alpha_0 = 1.8 \text{ m}^{-1}$ , in (b)  $\alpha_0 = 0.6 \text{ m}^{-1}$  and corresponds to the pulse showed in Fig. 1, and in (c)  $\alpha_0 = 0.2 \text{ m}^{-1}$ . All the other parameters are the same that those used in Fig. 1.

longer propagation distances, as expected because the effective line width of the resonant atoms,  $1/T_2 + 1/T_2^*$ , is increased in the presence of finite  $T_2^*$ . On the other hand, the homogeneous line case previously reported [15], can be recovered by setting  $T_2^* \to \infty$ , and the spatial behavior of its area could constitute an upper limit for the curves of the Fig. 2. It is important to remark that, as we are considering the coherent nature of the light-resonant atoms interaction, all curves (even the corresponding to  $T_2^* = 10$  fsec) will exhibit more frequently and more higher oscillations for further propagation distances and will completely depart form the adiabatic amplification; that is, EDFA can only sustain adiabaticity for a finite propagation distance.

In addition, Fig. 3 shows the influence of the gain strength on the spatial behavior of the pulse area during the first stage of the amplification process. Here, again, the other parameters are the same that those used in Fig. 1, but curve (a) corresponds to a gain of 0.87 dB/m ( $\alpha_0 = 0.2 \text{ m}^{-1}$ ), curve (b) to a gain of 2.6 dB/m, while curve (c) corresponds to a gain of 7.8 dB/m ( $\alpha_0 = 1.8 \text{ m}^{-1}$ ). As it can be seen, adiabaticity can be maintained for longer propagation distances if one reduces the gain, as expected because the perturbation to the exact NLS equation is smaller. An important feature is that, if we increase the gain we will obtain a exponential amplification for Z greater than 2, as have been reported lately, but having an adiabatic amplification for short propagation distances.

At present no analytical solution to Eq. (7) is known, so the result of the combined effects of the gain and of finite inhomogeneous decay time is impossible to be theoretically described. Nevertheless, whichever estimation of the pulse evolution within the EDFA can be of very practical interest, because it could represent a quantitative limit for the occurrence of the coherent adiabatical amplification of ultrashort pulses and the opportunity to clarify its dependence with the experimental variables. Observing the pulse area curves, Figs. 2 and 3, it seems reasonable to quantitatively characterize them by the occurrence of their first oscillation, as for further propagation distances the coherent nature of the process will dominate. A roughly analytical description of the pulse area evolution can be

obtained for short enough propagation distances if we suppose that the full pulse dynamics established by Eq. (7) can be separated into two parts: i) a growing of the pulse area due to the resonant susceptibility and ii) a decreasing of the pulse area due to the fiber dispersion nonlinearities. To carry out such dynamical separation it is necessary to assume that the pulse remain unchirped and that it approximately maintains its hyperbolic secant shape. Both assumptions are required in order to be able to evaluate the Fourier transform of  $|V|^2V$  in Eq. (7).

For the first step, because of the area of the pulse can be associated with the onresonance ( $\delta t_0 = 0$ ) component of its spectrum, to explain the growing on the pulse amplitude caused by the resonant gain we can consider that the pulse follows the expression  $V_1(T, Z) = v(Z) \operatorname{sech}(T)$ , with v(0) = 1 for our initial first-order soliton, and to ask for the spatial behavior of v(Z) which is just governed by the imaginary part of the resonant susceptibility. Introducing  $V_1$  in Eq. (7) and equating the real parts of the resulting equation we obtain

$$\frac{\partial v}{\partial z} = \frac{\alpha_0 z_0 t_2 t_0}{\pi g(0)} \left\langle \frac{1}{1 + (\Delta T_0)^2 t_2^2} \right\rangle v = Gv,\tag{8}$$

representing an exponential growing for v(Z) (and consequently for the pulse area S(Z)) at a rate G. It is worth to note that the gain coefficient G in Eq. (8) only depends on the ratio  $T_2^*/T_2$  because it can be rewritten in the following form:

$$G = \frac{\alpha_0 z_0 t_2 t_0}{\pi g(0)} \left\langle \frac{1}{1 + (\Delta T_0)^2 t_2^2} \right\rangle v = \frac{\alpha_0 z_0}{\pi} \int \frac{1}{1 + y^2} e^{-y^2/2(\sigma T_2)^2} \, dy, \tag{9}$$

where  $y = \Delta T_2$ .

The main consequence of increasing the pulse amplitude is that the dispersion caused by the Kerr nonlinearity will not exactly cancel out the fiber GVD, and the pulse becomes modulated. This can be seen by nothing that  $F[|V_1|^2V_1] = (1+\delta^2 t_0^2)v^2(Z)\tilde{V}_1/2$ . Therefore, if we now neglect the influence of the resonant atoms and use  $F[|V|^2V] \approx F[|V_1|^2V_1] =$  $(1+\delta^2 t_0^2)v^2(Z)\tilde{V}/2$  in Eq. (7), the nonlinearity fiber dispersion for the amplified pulse is given by

$$i\frac{\partial V}{\partial Z} = \frac{\pi}{4}(\delta t_0)^2 [v^2 - 1]\tilde{V} + \frac{\pi}{4}v^2 \tilde{V},$$
(10)

the solution of which is

$$\tilde{V}(\delta t_0, Z) = \tilde{V}(\delta t_0, 0) e^{-i\pi (v^2 - 1)/8G} e^{-i\pi ((v^2 - 1)/2G - Z)\delta t_0^2/4}.$$
(11)

In the time domain the quadratic dispersion showed by the pulse in Eq. (11) represents both a pulse broadening and a decreasing of its amplitude by the same factor. Taking the inverse Fourier transform of Eq. (11), it can be showed that such factor is  $[1 + \pi^2(v^2/2G - Z)^2/4]^{-1/2}$  (Ref. [16]). Therefore, as a first approximation for short enough distances, we



FIGURE 4. Graphs of Eq. (12) for three different values of the parameter G. Curves (a)–(c) correspond to the gain coefficient G obtained from the parameters used in curves (a)–(c) of Fig. 3 using Eq. (9). The reasonable good agreement between this curves and those displayed in Fig. 3 before the occurrence of the first maxima is clearly noticeable. The dashed lines indicate the behavior of Eq. (12) for longer propagation distances where the phase modulation acquired by the pulse has destroyed its initial hyperbolic secant shape. In this region the assumptions made in obtaining Eq. (12) have been highly violated and it cannot be valid.

can conclude that the initial exponential growing of the pulse area implied by Eq. (8) will be decreased by the modulation caused by the fiber, Eq. (11), that is

$$S(Z) \approx S(0) \frac{e^{GZ}}{\sqrt{1 + \frac{\pi^2}{4} \left(\frac{(e^{2GZ} - 1)}{2G - Z}\right)^2}}.$$
 (12)

Equation (12) resumes the competition between the resonant atoms and the fiber nonlinearities, and explain the oscillating behavior of the pulse area curves. Fig. 4 shows the graphs of S(Z) according to Eq. (12) for the same three gain coefficients G used in Fig. 3. As can be seen, the approximate curves of Fig. 4 exhibit a reasonable good agreement with the numerically obtained curves of Fig. 3 during their first oscillations. The slight differences in curves (a-c) of Fig. 3 and 4 in such interval can be a consequence of the dispersion caused by the resonant atoms which has been ignored in obtaining Eq. (12). Obviously, after the pulse area has reached its first local maximum, Figs. 3 and 4 do not coincide, because for longer Z the initial soliton becomes chirped, its profiles depart from the hyperbolic secant shape and therefore the approximations made in obtaining Eq. (12) can not be applied. However, the importance of Eq. (12) is that it gives an approximate value for the distance where the pulse area has its first local maximum, i.e., it may represent the maximum length of amplifier at which is possible to recover and reshape the original pulse. Moreover, the estimation of such value can be easily accomplished: when the parameters and conditions of the experimental setup are given, one only needs to compute G using Eq. (9) and then plot Eq. (12). It is interesting to note that a variation in  $\alpha_0$  will produce more drastic changes in the first maximum of the pulse area curve than

a change of the same order in  $T_2^*$ . That is clearly seen comparing Figs. 2 and 3, an it is also expressed in Eq. (9) because  $\alpha_0$  affects directly the gain area coefficient G, while  $T_2^*$  only affects the average of the lorentzian gain profile over the inhomogeneous atomic line. Thus, the presence of finite  $T_2^*$  just softens the results presented in Ref. [15] where  $T_2^*$  was infinite.

In conclusion, we have found an approximate expression for the initial behavior of the pulse area within an EDFA, using adiabatical amplification, which can serve for estimating the first local maximum of its oscillating characteristic. This point can be used as a quantitative limit for the occurrence of adiabaticity in an EDFA, fixing the physical size of the amplifier, at the time that it allows to characterize the influence of the experimental parameters over the amplified soliton. The expression presented can also be applied in the general case of coherent optical amplification when the finite inhomogeneous decay time is considered.

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