Some issues on spontaneous symmetry breaking

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Abstract. In gauge theories, when using a Higgs representation and aligning the "vacuum expectation value" (v.e.v.), we show that the unbroken symmetry is a subgroup of the initial gauge group G.

Resumen. En teorías de norma, usando una representación de Higgs y alineando el "valor de expectación del vacío" (v.e.v.), mostramos que la simetría que se preserva es un subgrupo del grupo de norma inicial G.

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1. Introduction

One of the most important progresses in particle physics in the last two decades has been the development of the so called Standard Model [1] (SM) with a gauge symmetry $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_Y$, which combines the color gauge group $\mathrm{SU}(3)_C$ of strong interactions (QCD), and the $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ gauge group of weak and electromagnetic interactions. The SM is compatible with extraordinary accuracy with present experimental data. From the theoretical point of view the SM is known to be consistent and renormalizable. As is characteristic of gauge theories, the interactions are generated by demanding the gauge invariance of the theory, which implies that couplings of vectorial gauge fields to known fermions are given by the gauge symmetry. Even though the SM describes with excellent accuracy the physics below energies typically of 100 GeV, it does not give an answer to some questions, such as the origin and values of fermion masses, the number of families, CP violation, etc. For these reasons physicists believe that the SM is not the last theory in particle physics.

There are many attempts which try to give an answer to the above questions, which in a generic form can be named as "models beyond the standard model". All of these

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models imply the existence of new particles, either fermions, gauge bosons or scalars, additional to those introduced in the SM. In this letter we comment on some features of those models which imply the embedding of the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ in a larger group G. The group G may be simple [2] or not. The former case corresponds to Gran Unification Theories (GUT). At present there are many of these models, most of them proposed at the end of 70's. The most usual gauge groups used in GUT are: SU(5), SO(10), E_6, $SU(6)_L \otimes SU(6)_C \otimes SU(6)_R$, etc. [3]. Some extensions where G is not simple are those that unify only families [4].

The renormalizability of a theory requires the lagrangian to be invariant under gauge transformations. Since vector boson mass terms are not gauge invariant, it appears that the gauge boson are exactly massless. However, it is possible for the symmetries of the equations of motion of a theory to be broken by stable solutions, which can pick out a specific direction in the symmetry space. This situation is known as "Spontaneous Symmetry Breaking" (SSB).

SSB occurs when the lowest energy state, the vacuum, of a theory possesses a nonzero distribution of the charge associated with a symmetry generator. A gauge boson propagating through this vacuum state will constantly interact with this charge and will develop an effective mass proportional to the "vacuum expectation value" (v.e.v) of the charge.

The Higgs mechanism is a simple model for implementing SSB. One introduces a set of spin-0 fields into the theory which transform in a nontrivial way under the gauge symmetry. If the v.e.v of one of these fields is nonzero, then all of the symmetry generators for which this field has a nonzero charge will be spontaneously broken and the associated gauge bosons will be massive.

By choosing a Higgs representation and aligning the v.e.v properly, we can break spontaneously the symmetry down, according to the physical requirements at each step of symmetry breaking.

At the energy scale of 10^2 GeV, the symmetry of the universe $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ [1], which might be a subgroup of a possible larger symmetry. In the present letter, we prove that when breaking spontaneously a gauge group, the unbroken symmetry is a subgroup of the original gauge group G.

2. PROOF

We define the unbroken symmetry as the set of the elements of the group G which leave invariant the v.e.v,

$$g\langle\phi\rangle_0 = \langle\phi\rangle_0, \tag{1}$$

For compact Lie groups, the elements can be represented by [5]

$$g = \exp(i\theta_j T_j), \tag{2}$$

where θ_j are real and the T_j are the hermitian matrix generators of the group. Using (2), the Eq. (1) is equivalent to the expression

$$T_j\langle\phi\rangle_0 = 0, \tag{3}$$

and we say that the generator T_j annihilates the vacuum or equivalently that it is an unbroken generator.

We assume that when the spontaneous symmetry breaking of G is achieved, the unbroken symmetry is a set of elements H, and we proof that H is a subgroup of G.

If g_j, $j = 1, \ldots, n$, are elements of H, then

$$g_j \langle \phi \rangle_0 = \langle \phi \rangle_0, \tag{4}$$

$$g_j^{-1} g_j \langle \phi \rangle_0 = g_j^{-1} \langle \phi \rangle_0 = \langle \phi \rangle_0. \tag{5}$$

Therefore, if g_j is element of H, then g_j^{-1} is also an element of H.

Now, if we proof the set H is closed, then H is subgroup of G [6]. Let g_i and g_j be two elements of H, then

$$(g_i g_j) \langle \phi \rangle_0 = g_i \langle \phi \rangle_0 = \langle \phi \rangle_0, \tag{6}$$

therefore, $(g_i g_j)$ is also an element of H.

3. Examples

Let us consider the group $G = \mathrm{SU}(3)$ and the Higgs representation of dimensions 6 and $\bar{3}$ with the v.e.v.

$$\langle 6 \rangle = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad \langle \bar{3} \rangle = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}. \tag{7}$$

i) For the representation 6, the only unbroken generator is associated to the matrix generator

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{8}$$

which is associated with an abelian subgroup U(1) of SU(3).

ii) While for the representation $\bar{3}$ the unbroken matrix generators are

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \tag{9}$$

which are the generators of a SU(2) subgroup of SU(3).

The above examples correspond to cases not related with the physical world because the v.e.v were chosen arbitrarily. In a physical model the unbroken symmetries are constrained by the experiments. For example, the photon is massless and then the v.e.v have to be chosen in such a way as to satisfy this physical condition.

iii) Let us illustrate a more realistic example:

$$\mathrm{SU}(5) \to \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y. \tag{10}$$

To get this breaking, the introduction of a 24 dimensional Higgs representation is needed, with the v.e.v. in the direction

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}. \tag{11}$$

We conclude that when a particular direction of the v.e.v. is chosen, the unbroken symmetry is a subgroup, but in realistic models the aligning of vacuum has to leave invariant certain quantum numbers according to the scale of energy.

In example iii), the v.e.v. is invariant under the symmetry $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$, which is the invariant symmetry up to 10^2 GeV.

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