Experimental evidence of discrete Strouhal numbers for the flow past a sphere in a confined geometry

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ABSTRACT. An experiment to study the motion of a sphere moving along a circular cylinder filled with water is reported. The motion shows a periodic behavior when the Reynolds number exceeds the value for which the trailing ring-eddy structure becomes unstable. The oscillatory motion becomes more apparent when the diameters of the sphere and the cylinder are of the same order. It is found that the Strouhal number seems to take discrete values, supporting a universal behavior proposed by Levi.

RESUMEN. El movimiento oscilatorio se observa de manera más clara cuando se estudia experimentalmente el movimiento de una esfera a lo largo de un cilindro circular lleno de agua. El movimiento muestra un comportamiento periódico cuando el número de Reynolds excede el valor para el cual los vórtices asociados a la parte posterior de la esfera son inestables. El movimiento oscilatorio es más claro cuando los diámetros del cilindro y la esfera son casi iguales. Aparentemente, el número de Strouhal toma valores discretos que apoyan la propuesta de Levi.

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1. INTRODUCTION

Perhaps the most studied problem in fluid dynamics is the viscous flow past a sphere. Consequently, the expression for the drag force on a sphere that moves steadily in an unbounded newtonian fluid is one of the most famous results in hydrodynamics. It was obtained, almost two centuries ago, by George Stokes; it is known as Stokes' law. After Reynolds' seminal contributions, a dimensionless quantity (the Reynolds number, R) was found to be the only relevant parameter characterizing the different stages of this flow. In what follows we use a Reynolds number based on the radius of the sphere (a); $R = au/\nu$, where u is the velocity of the sphere, and ν is the kinematic viscosity of the fluid.

Experimentally, it is a well established fact that as R increases the flow passes through a sequence of different regimes [1]. For small values of R ($\ll 1$), the flow has axial and fore-and-aft symmetry, it is steady, and it is well described by Stokes' analysis. At higher values, the fore-and-aft symmetry breaks down and at values of R around 12, a recirculating region begins to develop in the downstream side of the sphere. This standing ring-eddy grows (logarithmically) with R, increasing the wake's width. The next instability arises when R is close to 65; the wake begins to oscillate gently. With increasing amplitude of the oscillation a state is reached in which some parts of the attached or recirculating fluid break away and are carried downstream. No regular structure is known to exist in the wake, as opposed to the von Kármán vortex street (trailing vortices) that appears behind a circular cylinder; non-symmetric distorted vortex loops do seem to appear in the wake. This flow is rather unstable, and quickly the wake becomes turbulent.

On the theoretical side most things are still to be done. The first improvement on Stokes' results was done in 1910 by Oseen [2]. This extended the description to values of R of order one. Kaplun's method of matched asymptotic solutions [3], a mathematical tour de force, provided a systematic procedure to solve the problem [4]. Yet, it was later realized that the scheme was useless from the practical point of view [5]; the logarithmic expansion in R required an infinite number of terms to provide an experimentally meaningful correction to Oseen's prescription. In a somewhat approximate way the best one could get was a qualitative description for the attached recirculating region [6].

The numerical part of the theory, with its unavoidable case by case restriction and its inherent approximate limitation, was successful from the beginning [7]. From small to moderate values of R, numerical results are in very good agreement with experimental observations [8,9]. As with the experiments, the numerical values of the critical Reynolds numbers, at which the transitions from one regime to the next take place, can only be inferred from extrapolations. An alternative and practical theory is still lacking.

How the phenomenology is modified due to the presence of solid boundaries or free surfaces is still an open question. On both the theoretical and experimental sides, some results for small Reynolds numbers are known, and this in a very limited way. For example, extensions of Stokes' law for special cases, such as the ones in which the sphere approaches a wall, translates parallel to it or moves parallel to the axis of an infinitely long cylinder [10].

While carrying out experiments to determine the drag force on a sphere moving along the axis of a cylinder, we came across to what appeared to be a surprising feature. In the next section we describe the experimental setup and report a particular finding of the experiment. In the last section we provide a qualitative explanation and its relevance to a possible universal behavior.

2. EXPERIMENT AND RESULTS

The experimental setup was as follows (see Fig. 1). A long glass cylinder of length L and diameter D was filled with tap water. A smooth glass sphere of diameter d, initially positioned at the bottom of the cylinder, was attached to a long and thin thread (50 μm diameter), which came out at the upper surface, and went over a set of pulleys to a suitably selected counterweight. When the latter was released, the upward motion of the sphere was registered with a video camera (Sony CCD, V90) around the middle section (20 cm) of the cylinder's length.

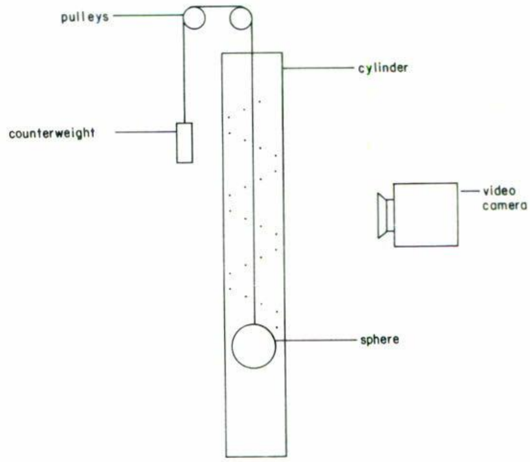

FIGURE 1. Experimental scheme.

The values of the different quantities were $d = 1.40$ cm, 2.50 cm; $L = 155$ cm, 112 cm; $D = 1.60$ cm, 3.66 cm. The video camera provided 30 frames/s.

For a given sphere and cylinder, and a set of counterweights (which determined the average terminal velocity, v), a rattling sound became apparent during the motion. A careful analysis of the video showed that the sphere was bouncing off the glass surface of the cylinder at regular time intervals, and following a sinusoidal path around the cylinder's axis. By changing the counterweights, thereby modifying the average terminal velocities, the regular rattle seemed to change its frequency in a discontinuous way. Using the frequency of either the sound or the oscillatory motion of the sphere, a Strouhal number S $(= \omega\lambda/v)$ was defined; ω is the frequency and λ is the diameter difference $(\lambda = D - d)$.

To obtain each value of the Strouhal number we carried out a large number of experiments. For a given λ, the (average) terminal velocity was measured from the video-tape by carrying out a frame by frame analysis; a millimeter scale was attached to the cylinder side and used as a reference length when determining the distance between successive maxima or minima of the sphere's path. The 1/30 sec time interval between one frame and the next provided the remaining information. The frequency was determined in a similar way. Repeating the same event many times, we obtained an average frequency, and the average Strouhal number was constructed accordingly. Changing the velocity (counterweight) and proceeding in the same way, we also determined the Reynolds number and constructed the corresponding tables.

In Table I we report the mean velocity of the sphere, the Strouhal number, and the corresponding Reynolds number. This is also the content of Table II for a different value of the sphere to cylinder diameter ratio.

The data in Table I correspond to a diameter ratio of 0.875 ($d = 1.40$ cm, $D = 1.60$ cm) and the associated errors lie betwen 4% and 20%. In Table II, the diameter ratio is 0.683 ($d = 2.50$ cm, $D = 3.66$ cm) and the error is less than 1%. In this last case the measurements were made from the photographs taken with a (Cannon EOS) camera and a stroboscope, allowing a higher resolution.

TABLE I. Velocity in cm/sec

u	S	R
2.6	0.14	363.4
2.6	0.13	363.4
2.8	0.12	391.3
3.0	0.15	419.3
3.0	0.14	419.3
3.0	0.15	419.3
3.1	0.15	433.2
3.1	0.15	433.2
3.1	0.14	438.2
3.4	0.13	475.2
4.5	0.28	628.9
4.7	0.26	656.8
4.8	0.27	670.8
4.9	0.25	684.8
4.9	0.26	684.8
5.0	0.28	698.8
5.1	0.27	712.7
5.2	0.25	726.7
5.3	0.27	740.7
5.4	0.33	754.7
5.4	0.34	754.7
7.3	0.29	1020.2
7.5	0.29	1048.1

TABLE II. Velocity in cm/sec

u	S	R
17.8	0.15	4741.4
24.9	0.12	6742.5
28.3	0.16	8050.3
45.5	0.11	12965.2
46.0	0.11	13010.8
46.3	0.13	13130.1
48.3	0.13	14937.1
53.1	0.13	14478.0
53.5	0.13	15513.4
54.6	0.12	16437.9
69.0	0.08	19047.5

Figs. 2 and 3 show the Strouhal number as a function of the Reynolds number. In both plots there is evidence that the frequency (Strouhal number) takes discrete values when the terminal velocity (Reynolds number) varies.

3. DISCUSSION

Two points are relevant and summarize our findings. One is the regularization of the vortex shedding process behind a sphere, and the other is that we have an additional example of Levi's law. We briefly discuss these issues.

The explanation for the wavy motion of the sphere, as it is seen in a plane containing the axis of the cylinder, comes directly from the values of R at which it takes place. In this regime the trailing fluid behind the sphere detaches itself and a vortex shedding process sets in. As each vortex is emitted, a reaction sideways-force acts on the sphere deviating its motion from a straight line. As a result the sphere approaches the walls of the cylinder, bouncing off as it rises. If the mean velocity of the sphere (R) is too small, preventing vortex shedding, or the radius of the cylinder is too large (with respect to the

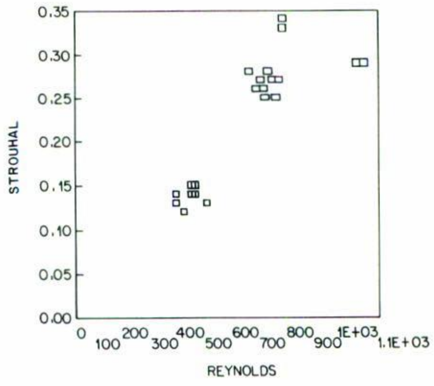

FIGURE 2. Plot of S against R. Data of Table I.

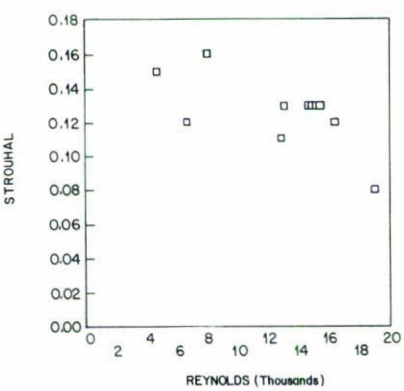

FIGURE 3. Plot of S against R. Data of Table II.

radius of the sphere), this process does not occur; this we confirmed by using either a larger cylinder with $D = 15$ cm or glicerine (larger viscosity) as a working fluid.

One of the noteworthy and interesting features that follows from the experiment is the observation that the vortex shedding from the back of a sphere does have a regular behavior, as opposed to what has been reported before [1,11]. The presence of the cylinder might be partly responsible for this "regularization" of the wake once the trailing ring-eddy becomes unstable. We point out that for a very different range of Reynolds numbers and with a different geometrical arrangement, a similar observation has been made previously [12].

In trying to understand the apparently discrete values of the Strouhal number, as if a sort of quantizing mechanism were present, we learned that a "universal" Strouhal law had been proposed earlier by Roshko [11], and later generalized by Levi [13]. Reviewing some very dissimilar flows such as wakes, jets, cavitation, spillway nappes, or wall bursting, among others, Levi argues that in all these cases the Strouhal number is some integer times $1/2\pi$. Our results agree with his contention, adding support to a possible universal behavior for a wide class of fluid dynamical systems. An important point in our analysis is that the relevant length is the diameter difference λ, which characterizes the region of "blockage", as noticed by Levi.

As more evidence becomes available in support of this universal behavior, the roots of this strikingly general result begin to form a fundamental question whose answer must be sought from the Navier-Stokes equations.

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REFERENCES

1. S. Taneda, *J. Phys. Soc. Japan* **11** (1956) 302; A. Goldburg and B.H. Florsheim, *Phys. Fluids.* **9** (1966) 45; see also G.K. Batchelor, *An Introduction to Fluid Dynamics*, Cambridge University Press (1967).
2. C.W. Oseen, *Ark. f. Mat. Astr. og Fys.* **6** (1910) 29.
3. S. Kaplun, and P.A. Langerstrom, *J. Math. Mech.* **6** (1957) 585.
4. I. Proudman and J.R.A. Pearson, *J. Fluid. Mech.* **2** (1957) 237.
5. L.A. Skinner and J. Quart, *Mech. Appl. Math.* **28** (1975) 333.
6. M. Van Dyke, *Perturbation Methods in Fluid Mechanics*, Parabolic Press, Ca. (1975).
7. A. Thom, *Proc. Roy. Soc. A* **141** (1933) 651.
8. C. Hunter, M. Tajdari and S.D. Boyers, *SIAM J. Appl. Math.* **50** (1990) 48.
9. S.C.R. Dennis and J.D.A. Walker, *J. Fluid Mech.* **48** (1971) 771.
10. H. Brenner, *J. Fluid. Mech.* **18** (1964) 144; O. Sano and H. Hasimoto, *J. Fluid Mech.* **87** (1978) 673; O. Sano, *J. Phys. Soc. Japan* **56** (1987) 2713.
11. A. Roshko, *NACA Techn. Notes* (1954) 3169.
12. R. Richter and E. Naudascher, *J. Fluid Mech.* **78** (1976) 561.
13. E. Levi, *Kinam* **5** (1983) 255.