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Salpeter's approach to the relativistic two-body problem

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ABSTRACT. We apply Salpeter's approach to the calculation of relativistic two-body bound states. In addition to the intermediate propagation of two particles, Salpeter's approach allows for the propagation of two antiparticles (Z-graphs). As a consequence, Salpeter's equation becomes identical in structure to a random phase approximation equation familiar from the study of nuclear collective excitations. Thus, for a sufficiently attractive interaction, Salpeter's equation might give rise to imaginary eigenvalues. The occurrence of imaginary eigenvalues is an indication of a pairing instability against the formation of strongly bound particle and antiparticle pairs. We show that this pairing instability develops once the energy gap between two-particle and two-antiparticle states, originally at $4M$, disappears. To further illustrate Salpeter's approach, we calculate the relativistic two-body amplitude and bound state energy for the deuteron in Walecka's scalar-vector model. These results are then compared with those obtained in a calculation that neglects the Z-graphs and with nonrelativistic results.

RESUMEN. Aplicamos la formulación de Salpeter al cálculo relativista de estados ligados de dos cuerpos. Además de la propagación intermedia de dos partículas, la aproximación de Salpeter permite la propagación de dos antipartículas (Z-graphs). Como consecuencia, la ecuación de Salpeter se vuelve idéntica en estructura a una ecuación de fases al azar (RPA) conocida en el estudio de excitaciones nucleares colectivas. Por lo tanto, para una interacción suficientemente fuerte, la ecuación de Salpeter puede dar lugar a eigenvalores imaginarios. La existencia de eigenvalores imaginarios indica la presencia de una inestabilidad en la formación de pares de partículas y antipartículas fuertemente ligadas. Mostramos que la formación de esta inestabilidad ocurre tan pronto como la banda de energías prohibidas entre estados de dos partículas y dos antipartículas, originalmente con valor $4M$, desparece. Para ilustrar la aproximación de Salpeter hemos también calculado la amplitud relativista y la energía del deuterón, usando el modelo de Walecka. Estos resultados son posteriormente comparados con aquéllos obtenidos en la ausencia de Z-graphs y con resultados no-relativistas.

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1. INTRODUCTION

The two-nucleon problem has played a central role in nuclear physics for over half a century. In principle, one could use quantum chromodynamics (QCD) to calculate the nucleon-nucleon (NN) force and, ultimately, to solve the nuclear many-body problem. In practice, however, establishing the quark-gluon content of the NN interaction has proven to be an exceedingly difficult task. Although QCD has made important contributions to our understanding of the NN force, the theory has not yet been developed to a point where it can have a real impact in our understanding of the nuclear many-body problem. In trying to solve the nuclear many-body problem one must therefore rely on phenomenological descriptions of the NN force. These approaches attempt to construct a NN force that, while being sophisticated enough to reproduce a large body of two-nucleon data, is still simple enough to be used as input for many-body calculations.

The commission of more powerful and sophisticated machines (*e.g.*, CEBAF, Mainz, MIT-Bates, NIKHEF) will, more then ever, challenge our understanding of the nuclear many-body problem. The picture of the nucleus as a collection of particles moving independently in a smooth mean-field potential will, clearly, not be sufficient. In fact, there is already ample evidence, based on a large body of experimental data, in support of large deviations from independent-particle motion [1-3]. For example, a (model-dependent) analysis of exclusive $(e,e'p)$ data claims a depletion of single-particle orbits of as much as 30% [2]. It is believed that two-body (short-range) correlations, generated by the strong repulsive NN core, are responsible for these large deviations from independent-particle behavior [4-6]. Consequently, in trying to understand single-particle properties such as the nucleon spectral function, one must, at the very least, address simultaneously the one- and two-body problem. The solution of the one- and two-body problem, however, poses formidable challenges. Since, both, the propagation of a particle and the basic two-body interaction are modified in the medium, one must solve the problem self-consistently. Furthermore, since most of the experiments already planned for CEBAF will probe the nuclear response at high-momentum transfer, these calculations will need to use relativity.

In the traditional nonrelativistic approach based on the Schrödinger equation with static two-body potentials, the two-body problem can be solved exactly in free space. Once the dynamical equation to be solved has been selected, the only remaining task consists in constructing a NN force that will reproduce the empirical two-body data, *i.e.*, deuteron properties and NN scattering observables. This NN force then serves as input for parameter-free calculations of the many-body system.

While the goals remain unchanged in relativistic approaches to the many-body problem, the means to achieve these goals get substantially more complicated. The main difficulty lies in the fact that a simple extension of Schrödinger's equation to the relativistic two-body problem is not yet available. Strictly speaking, there is no relativistic two-body problem [7]. Due to pair creation and annihilation the number of bodies in the two-body problem is actually undefined (only the conserved baryon number has a well defined meaning). In addition, retardation effects, which introduce an extra relative-time variable into the problem, have posed very serious challenges to the search for efficient methods of solution. In effect, retardation converts the three-dimensional Schrödinger (or Lippmann-Schwinger) equation into the four-dimensional Bethe-Salpeter [8]. In fact, most methods of solution rely on three-dimensional reductions of the Bethe-Salpeter [9-11]. Although it is widely agreed that the these three-dimensional equations should satisfy fundamental physical principles such as relativistic covariance and two-body unitarity, there is still ample freedom, and hence ambiguity, on how to implement the three-dimensional reduction. Unfortunately, most, if not all, three-dimensional reductions of the Bethe-Salpeter equation contain undesirable features. These range from ignoring meson retardation and negative-energy intermediate states in the Blankenbecler-Sugar [9] and the Thompson [10] reductions, to the presence of spurious singularities in the Gross spectator [11]. In the case of Salpeter's [12], the equation employed throughout this work, the most serious drawback is the use of an instantaneous (energy independent) kernel. In Salpeter's original work on hydrogen-like atoms retardation effects were incorporated perturbatively. While a perturbative scheme may be appropriate for weak-coupling theories (*e.g.*, QED), one must be cautious whenever applying Salpeter's approach to the study of strongly interacting systems. Nevertheless, in this work we adopt the position advocated in Ref. [13], namely, if a simplification concerning the meson propagator has to be done, to simply ignore retardation is probably the best choice. In all cases, however, once a particular three-dimensional equation has been chosen, one then proceeds, as in the nonrelativistic case, to construct a phenomenological NN interaction that will reproduce the two-body data and which will serve as input for many-body calculations.

Our paper has been organized as follows. In Sect. 2 we show how Salpeter's equation can be derived from the four-point (or two-body) Green's function in the two-body limit. In this limit, the analytic structure of the two-body propagator reveals singularities located at the exact energies of a system containing, either, two baryons ($B = +2$) or two antibaryons ($B = -2$). The interacting two-time, two-body Green's function is obtained as a solution to the Bethe-Salpeter equation using an instantaneous one-boson exchange kernel. Salpeter's eigenvalue equation is then obtained by isolating the singularities at the bound-state poles. It is not our intention to provide yet another derivation of Salpeter's equation. After all, Salpeter's equation has been known for forty years [12]. What is perhaps not well known, however, is the implementation of Salpeter's approach in the calculation of relativistic two-body bound states in the *nuclear medium.* The behavior of bound pair states in the nuclear medium is a topic attracting considerable interest from theorists as well as experimentalists [14-17]. The purpose of presenting a derivation of Salpeter's equation based on Green's function theory is to illustrate that its generalization to bound states in the medium is straightforward. Indeed, a nonrelativistic study of bound pair states in nuclear matter (using Green's function theory) is based on a set of equations identical in structure to Salpeter's equation [16]. An interesting feature of Salpeter's approach is that it leads, in contrast to most three-dimensional reductions, to a non-hermitian Hamiltonian matrix having the same structure as a random phase approximation (RPA) matrix [18]. As a consequence, the solution of Salpeter's equation can lead to imaginary eigenvalues, usually, associated with the development of a pairing instability. In Sect. 3, we show how the instability develops using a simple scalar-exchange model. To further illustrate the method, we calculate the relativistic amplitude and binding energy of the deuteron in Walecka's sigma-omega model [19]. Due to the absence of isovector mesons from the Walecka model, however, the calculation should only be regarded as schematic. Finally, our conclusions and directions for future work are discussed in Sect. 4.

2. Salpeter's equation

The starting point in our derivation of Salpeter's equation is the two-time two-body propagator defined in terms of a time-ordered product of fermion-field operators [18]

$$iG_{\alpha\beta;\lambda\sigma}(x_1,x_2;y_1,y_2) \equiv \big\langle\Psi_0|T\big[\psi_\alpha(x_1)\psi_\beta(x_2)\bar{\psi}_\sigma(y_2)\bar{\psi}_\lambda(y_1)\big]|\Psi_0\big\rangle, \qquad (1)$$

where $x_1^0 = x_2^0 \equiv x^0$; $y_1^0 = y_2^0 \equiv y^0$, and Ψ_0 represents the exact vacuum wave function. The Lehmann representation of the two-body propagator is particularly useful for displaying its analytic structure. This is done by using the integral representation of the Heaviside step-function, the explicit time evolution of the fermion-field operators and, finally, by inserting a complete set of states (Ψ_n) of the exact Hamiltonian *i.e.*,

$$G_{\alpha\beta;\lambda\sigma}(x_1, x_2; y_1, y_2) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega(x^0-y^0)} G_{\alpha\beta;\lambda\sigma}(\mathbf{x}_1, \mathbf{x}_2; \mathbf{y}_1, \mathbf{y}_2; \omega),$$

$$G_{\alpha\beta;\lambda\sigma}(\mathbf{x}_1, \mathbf{x}_2; \mathbf{y}_1, \mathbf{y}_2; \omega) = \sum_n \left[\frac{\langle\Psi_0|\psi_\alpha(\mathbf{x}_1)\psi_\beta(\mathbf{x}_2)|\Psi_n\rangle\langle\Psi_n|\bar{\psi}_\sigma(\mathbf{y}_2)\bar{\psi}_\lambda(\mathbf{y}_1)|\Psi_0\rangle}{\omega - E_n^{(+)} + i\eta} - \frac{\langle\Psi_0|\bar{\psi}_\sigma(\mathbf{y}_2)\bar{\psi}_\lambda(\mathbf{y}_1)|\Psi_n\rangle\langle\Psi_n|\psi_\alpha(\mathbf{x}_1)\psi_\beta(\mathbf{x}_2)|\Psi_0\rangle}{\omega + E_n^{(-)} + i\eta} \right]. \quad (2)$$

The Lehmann representation reveals that the two-body propagator is an analytic function of ω except for the presence of single-poles located infinitesimally close to the real axis. The poles are located at the exact energies $E_n^{(+)}$ and $E_n^{(-)}$ of the system with two baryons, either, added or removed from the exact ground state. The residue at the pole, given by

$$\langle\Psi_0|\psi_\alpha(\mathbf{x}_1)\psi_\beta(\mathbf{x}_2)|\Psi_n\rangle\langle\Psi_n|\bar{\psi}_\sigma(\mathbf{y}_2)\bar{\psi}_\lambda(\mathbf{y}_1)|\Psi_0\rangle \equiv \chi_{\alpha\beta}(\mathbf{x}_1, \mathbf{x}_2; n)\bar{\chi}_{\sigma\lambda}(\mathbf{y}_2, \mathbf{y}_1; n), \quad (3a)$$

$$\langle\Psi_0|\bar{\psi}_\sigma(\mathbf{y}_2)\bar{\psi}_\lambda(\mathbf{y}_1)|\Psi_n\rangle\langle\Psi_n|\psi_\alpha(\mathbf{x}_1)\psi_\beta(\mathbf{x}_2)|\Psi_0\rangle \equiv \bar{\varphi}_{\sigma\lambda}(\mathbf{y}_2, \mathbf{y}_1; n)\varphi_{\alpha\beta}(\mathbf{x}_1, \mathbf{x}_2; n), \quad (3b)$$

represents, on the other hand, the relativistic two-body wave function which is related to the two-baryon (or two-antibaryon) removal amplitude.

The concept of a two-particle removal amplitude is clearly basis dependent; one is posing the question of what is the probability of removing two particles having, for example, momentum $\mathbf{k}_1$ and $\mathbf{k}_2$ from the system. To address this issue it is convenient to expand the fermion-field operator in terms of a, yet unspecified, single-particle basis,

$$\psi(\mathbf{x}) = \sum_i \left[\mathcal{U}_i(\mathbf{x})b_i + \mathcal{V}_i(\mathbf{x})d_i^\dagger\right], \quad (4)$$

where $\mathcal{U}_i$ and $\mathcal{V}_i$ are the positive- and negative-energy eigenstates of a one-body Dirac Hamiltonian and b_i and $d_i^\dagger$ are second-quantized operators that, respectively, annihilate a baryon or create an antibaryon in the corresponding single-particle state. The single-particle states are orthonormal,

$$\int d\mathbf{x}\, \mathcal{U}_i^\dagger(\mathbf{x})\mathcal{U}_j(\mathbf{x}) = \int d\mathbf{x}\, \mathcal{V}_i^\dagger(\mathbf{x})\mathcal{V}_j(\mathbf{x}) = \delta_{ij},$$

$$\int d\mathbf{x}\, \mathcal{U}_i^\dagger(\mathbf{x})\mathcal{V}_j(\mathbf{x}) = \int d\mathbf{x}\, \mathcal{V}_i^\dagger(\mathbf{x})\mathcal{U}_j(\mathbf{x}) = 0, \quad (5)$$

and, in addition, satisfy the completeness relation

$$\sum_i \left[\mathcal{U}_i(\mathbf{x})\mathcal{U}_i^\dagger(\mathbf{y}) + \mathcal{V}_i(\mathbf{x})\mathcal{V}_i^\dagger(\mathbf{y})\right] = \delta(\mathbf{x}-\mathbf{y})\mathbf{1}. \tag{6}$$

Inserting the expansion of the fermion-field operator into Eq. (3a) yields,

$$\begin{aligned}\chi_{\alpha\beta}(\mathbf{x}_1,\mathbf{x}_2;n) = \sum_{ij}\Big(&\left[\mathcal{U}_i(\mathbf{x}_1)\right]_\alpha\left[\mathcal{U}_j(\mathbf{x}_2)\right]_\beta\langle\Psi_0|b_ib_j|\Psi_n\rangle \\ &+ \left[\mathcal{V}_i(\mathbf{x}_1)\right]_\alpha\left[\mathcal{V}_j(\mathbf{x}_2)\right]_\beta\langle\Psi_0|d_i^\dagger d_j^\dagger|\Psi_n\rangle \\ &+ \left[\mathcal{U}_i(\mathbf{x}_1)\right]_\alpha\left[\mathcal{V}_j(\mathbf{x}_2)\right]_\beta\langle\Psi_0|b_i d_j^\dagger|\Psi_n\rangle \\ &+ \left[\mathcal{V}_i(\mathbf{x}_1)\right]_\alpha\left[\mathcal{U}_j(\mathbf{x}_2)\right]_\beta\langle\Psi_0|d_i^\dagger b_j|\Psi_n\rangle\Big).\end{aligned} \tag{7}$$

Since the basis functions are chosen as the eigenstates of a one-body Dirac-Hamiltonian they are, in principle, known and all dynamical information is therefore contained in the four two-baryon removal amplitudes. The first of these amplitudes, namely, $\langle\Psi_0|b_ib_j|\Psi_n\rangle$, is the two-nucleon removal amplitude familiar from nonrelativistic approaches. The other three amplitudes, containing at least one antinucleon creation operator, have a relativistic origin and, hence, have no counterpart in a nonrelativistic formalism.

As an illustration of the above ideas, and also because of its importance in deriving Salpeter's equation, we now proceed to evaluate the free two-body propagator. Starting from Eq. (1), but with Ψ_0 now representing the noninteracting vacuum state, the free two-body propagator can be readily evaluated with the aid of Wick's theorem

$$G^0_{\alpha\beta;\lambda\sigma}(x_1,x_2;y_1,y_2) = i\left[G^0_{\alpha\lambda}(x_1,y_1)G^0_{\beta\sigma}(x_2,y_2) - G^0_{\alpha\sigma}(x_1,y_2)G^0_{\beta\lambda}(x_2,y_1)\right], \tag{8}$$

in terms of the free (one-body) nucleon propagator $G^0_{\alpha\lambda}(x_1,y_1)$. This expression shows that the noninteracting two-body propagator consists of direct-plus-exchange contributions representing the free propagation of two identical particles. In the two-time limit [see Eq. (2)], where only a single underlying energy variable is necessary, the free two-body propagator becomes

$$\begin{aligned}G^0_{\alpha\beta;\lambda\sigma}(\mathbf{x}_1,\mathbf{x}_2;\mathbf{y}_1,\mathbf{y}_2;\omega) = i\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\big[&G^0_{\alpha\lambda}(\mathbf{x}_1,\mathbf{y}_1;\omega-\omega')G^0_{\beta\sigma}(\mathbf{x}_2,\mathbf{y}_2;\omega') \\ &- G^0_{\alpha\sigma}(\mathbf{x}_1,\mathbf{y}_2;\omega-\omega')G^0_{\beta\lambda}(\mathbf{x}_2,\mathbf{y}_1;\omega')\big],\end{aligned} \tag{9}$$

with the free nucleon propagator

$$G^0_{\alpha\beta}(\mathbf{x},\mathbf{y};\omega) - \sum_{\mathbf{k}s}\left(\frac{[\mathcal{U}_{\mathbf{k}s}(\mathbf{x})]_\alpha[\overline{\mathcal{U}}_{\mathbf{k}s}(\mathbf{y})]_\beta}{\omega-\epsilon_{\mathbf{k}}+i\eta} + \frac{[\mathcal{V}_{\mathbf{k}s}(\mathbf{x})]_\alpha[\overline{\mathcal{V}}_{\mathbf{k}s}(\mathbf{y})]_\beta}{\omega+\epsilon_{\mathbf{k}}-i\eta}\right), \tag{10}$$

written in terms of eigenstates of the free Dirac-Hamiltonian. These free eigenstates of well-defined momentum k and energy $\epsilon_{\mathbf{k}} = +\sqrt{k^2+M^2}$ are given by

$$\mathcal{U}_{\mathbf{k}s}(\mathbf{x}) = e^{+i\mathbf{k}\cdot\mathbf{x}}\mathcal{U}(\mathbf{k},s); \quad \mathcal{V}_{\mathbf{k}s}(\mathbf{x}) = e^{-i\mathbf{k}\cdot\mathbf{x}}\mathcal{V}(\mathbf{k},s), \tag{11}$$

where ($\tilde{\chi}_s \equiv (-1)^{1/2+s}\chi_{-s}$)

$$\begin{aligned}\mathcal{U}(\mathbf{k},s) &= \left[\frac{\epsilon_{\mathbf{k}}+M}{2\epsilon_{\mathbf{k}}}\right]^{1/2}\begin{pmatrix}1\\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{k}}{\epsilon_{\mathbf{k}}+M}\end{pmatrix}\chi_s,\\ \mathcal{V}(\mathbf{k},s) &= \left[\frac{\epsilon_{\mathbf{k}}+M}{2\epsilon_{\mathbf{k}}}\right]^{1/2}\begin{pmatrix}\dfrac{\boldsymbol{\sigma}\cdot\mathbf{k}}{\epsilon_{\mathbf{k}}+M}\\ 1\end{pmatrix}\tilde{\chi}_s,\end{aligned} \tag{12}$$

are positive- and negative-energy plane-wave spinors expressed in terms of conventional two-component Pauli spinors,

$$\chi_{+1/2} = \begin{pmatrix}1\\0\end{pmatrix}, \qquad \chi_{-1/2} = \begin{pmatrix}0\\1\end{pmatrix}. \tag{13}$$

The integral in Eq. (9) can be readily evaluated by contour integration. Because of the analytic structure of the free two-time two-body propagator the only contributions to the integral arise from either two particle or two antiparticle poles. Consequently, the free two-body propagator can be written as

$$\begin{aligned}G^0_{\alpha\beta;\lambda\sigma}(\mathbf{x}_1,\mathbf{x}_2;\mathbf{y}_1,\mathbf{y}_2;\omega) = \sum_{\substack{\mathbf{k}_1 s_1\\ \mathbf{k}_2 s_2}}\Bigg[&\frac{\chi^0_{\alpha\beta}(\mathbf{x}_1,\mathbf{x}_2;n^0)\bar{\chi}^0_{\sigma\lambda}(\mathbf{y}_2,\mathbf{y}_1;n^0)}{\omega-\epsilon_{\mathbf{k}_1}-\epsilon_{\mathbf{k}_2}+i\eta}\\ &-\frac{\phi^0_{\alpha\beta}(\mathbf{x}_1,\mathbf{x}_2;n^0)\bar{\phi}^0_{\sigma\lambda}(\mathbf{y}_2,\mathbf{y}_1;n^0)}{\omega+\epsilon_{\mathbf{k}_1}+\epsilon_{\mathbf{k}_2}-i\eta}\Bigg],\end{aligned} \tag{14}$$

where $|n^0\rangle \equiv |\mathbf{k}_1 s_1;\mathbf{k}_2 s_2\rangle$ labels the free two-particle, or two-antiparticle, state and the free relativistic two-body wave functions are given by

$$\chi^0_{\alpha\beta}(\mathbf{x}_1,\mathbf{x}_2;n^0) = \frac{1}{\sqrt{2}}\Big([\mathcal{U}_{\mathbf{k}_1 s_1}(\mathbf{x}_1)]_\alpha[\mathcal{U}_{\mathbf{k}_2 s_2}(\mathbf{x}_2)]_\beta - [\mathcal{U}_{\mathbf{k}_2 s_2}(\mathbf{x}_1)]_\alpha[\mathcal{U}_{\mathbf{k}_1 s_1}(\mathbf{x}_2)]_\beta\Big), \tag{15}$$

$$\phi^0_{\alpha\beta}(\mathbf{x}_1,\mathbf{x}_2;n^0) = \frac{1}{\sqrt{2}}\Big([\mathcal{V}_{\mathbf{k}_1 s_1}(\mathbf{x}_1)]_\alpha[\mathcal{V}_{\mathbf{k}_2 s_2}(\mathbf{x}_2)]_\beta - [\mathcal{V}_{\mathbf{k}_2 s_2}(\mathbf{x}_1)]_\alpha[\mathcal{V}_{\mathbf{k}_1 s_1}(\mathbf{x}_2)]_\beta\Big). \tag{16}$$

So far, we have been referring to Salpeter's equation as an eigenvalue equation. Notice, however, that the relative sign difference in the energy E between Eq. (22a) and Eq. (22b), precludes us from writing Salpeter's equation as a *Hermitian* eigenvalue equation. Instead, Salpeter's eigenvalue equation has the same algebraic structure as an RPA equation familiar from the study of nuclear collective excitations [18]. In contrast to most relativistic approaches that calculate the two-body propagator by only allowing two positive-energy particles to propagate in the intermediate state, Salpeter's approach enables, in addition, the propagation of negative-energy particles. In particular, this gives rise to the so-called Z-graphs, known to be present in the one-body Dirac equation. Ignoring these Z-graphs, by retaining only two-nucleon intermediate states, is tantamount to the Tamm-Dancoff approximation (TDA) in the study of nuclear collective excitations. As in the simpler TDA case, the resulting equation is indeed a Hermitian eigenvalue equation and thus guaranteed to yield real eigenvalues. In contrast, the RPA equation can be transformed into a Hermitian eigenvalue equation but for the *square* of the energy [20]. In principle, then, although guaranteed to be real, E^2 can become negative and give rise to imaginary eigenvalues. In the case of nuclear collective excitations the presence of imaginary eigenvalues signals the instability of the mean-field ground state against the formation of particle-hole pairs. In analogy, imaginary eigenvalues of Salpeter's equation gives an indication of a pairing instability against the formation of strongly bound particle and antiparticle pairs. Also notice, that because of the structure of Salpeter's equation, the binding energy is independent of the sign of the coupling potential $V^{+-} = V^{-+}$. In particular, the coupling potential, which arises from the inclusion of Z-graphs, always leads to additional binding as compared to the Breit (non Z-graph) equation.

Salpeter's equation was solved in the center of momentum frame after a partial wave decomposition was performed (details of this procedure are given in the appendix). Salpeter's integral equation was formally evaluated using a Gauss quadrature scheme. After transforming Salpeter's equation into a Hermitian eigenvalue equation for the square of the energy, it was, then, solved by direct matrix diagonalization [20]. The outcome of the diagonalization procedure was a set of eigenvalues (bound-state energies) and eigenvectors (two-baryon removal amplitudes) with the latter ones given by

$$B_{LSJ}(k) = \sum_{\substack{s_1 s_2 \\ M_L M_S}} \langle s_1 s_2 | S M_S \rangle \langle L M_L ; S M_S | J M \rangle \int d\hat{\mathbf{k}}\, Y^*_{L M_L}(\hat{\mathbf{k}}) B_{s_1 s_2}(\mathbf{k}), \tag{25a}$$

$$D_{LSJ}(k) = \sum_{\substack{s_1 s_2 \\ M_L M_S}} \langle s_1 s_2 | S M_S \rangle \langle L M_L ; S M_S | J M \rangle \int d\hat{\mathbf{k}}\, Y^*_{L M_L}(\hat{\mathbf{k}}) D_{s_1 s_2}(\mathbf{k}), \tag{25b}$$

and where the following definition have been used ($\tilde{d}^{\dagger}_{\mathbf{k}s} \equiv (-1)^{1/2-s} d^{\dagger}_{\mathbf{k}-s}$):

$$B_{s_1 s_2}(\mathbf{k}) \equiv \langle \Psi_0 | b_{\mathbf{k}s_1} b_{-\mathbf{k}s_2} | \Psi_E \rangle, \tag{26a}$$

$$D_{s_1 s_2}(\mathbf{k}) \equiv \langle \Psi_0 | \tilde{d}^{\dagger}_{\mathbf{k}s_1} \tilde{d}^{\dagger}_{-\mathbf{k}s_2} | \Psi_E \rangle. \tag{26b}$$

In addition, the two-baryon removal amplitudes satisfy (for $E > 0$) the RPA normalization condition [18, 20]

$$\int_0^\infty \frac{k^2\,dk}{(2\pi)^3} \sum_{LS} \left[B^2_{LSJ}(k) - D^2_{LSJ}(k)\right] = 1. \tag{27}$$

The two-baryon removal amplitudes, B_{LSJ} and D_{LSJ}, represent the probability amplitude of removing two particles from the system, or adding two antiparticles to the system, with relative momentum k. Within the framework of Salpeter's approach, they contain all dynamical information about the nature of the two-body bound state. Furthermore, given the total angular momentum (J), the parity (π) and the isospin (T) of the bound state, the orbital angular momentum (L) and the total spin (S) of these amplitudes are constrained, by parity and the Pauli principle, to satisfy

$$(-1)^L = \pi; \quad \text{and} \quad L + S + T = \text{odd}.$$

In contrast, the relativistic two-body amplitude $\chi_{\alpha\beta}(k)$, is not constrained by any of the above two relations. The amplitude, which is readily recovered from the two-baryon removal amplitudes, can be written as

$$[\chi_{\alpha\beta}(k)]_{\mathcal{L}SJ} = \sum_{LS} \left[\mathcal{F}^{\alpha\beta}_{\mathcal{L}S;LSJ}(k) B_{LSJ}(k) + (-1)^{\mathcal{L}} \mathcal{F}^{\bar{\alpha}\bar{\beta}}_{\mathcal{L}S;LSJ}(k) D_{LSJ}(k)\right], \tag{28}$$

where $\mathcal{F}^{\alpha\beta}_{\mathcal{L}S;LSJ}(k)$ has been defined in the appendix [Eq. ($A6$)] and $\bar{\alpha} \equiv 1 - \alpha$. Due to the presence of lower components in the relativistic one-body wave functions, the two-body amplitude contains, in addition to the conventional parity allowed amplitudes, parity "forbidden" amplitudes. The only constraint imposed on the quantum numbers is for them to satisfy

$$(-1)^{\mathcal{L}+\alpha+\beta} = \pi.$$

In the particular case of the deuteron, where only two nonrelativistic amplitudes exist ($^3S_1, {}^3D_1$), the relativistic two-body wave function contains eight allowed amplitudes [21]; these are given by, $^3S_1^{++}$, $^3D_1^{++}$, $^3P_1^{+-}$, $^1P_1^{+-}$, $^3P_1^{-+}$, $^1P_1^{-+}$, $^3S_1^{--}$, $^3D_1^{--}$, where we have used the spectroscopic notation

$$[\chi_{\alpha\beta}]_{\mathcal{L}SJ} = {}^{2S+1}\mathcal{L}_J^{\alpha\beta}$$

with the $+(-)$ sign corresponding to $\alpha = 0(1)$.

3. Results

In this section we apply Salpeter's formalism to the calculation of relativistic bound states. We illustrate the method developed in the previous section by assuming that the nucleon-nucleon interaction consists exclusively of (isoscalar) scalar plus vector exchange. In the

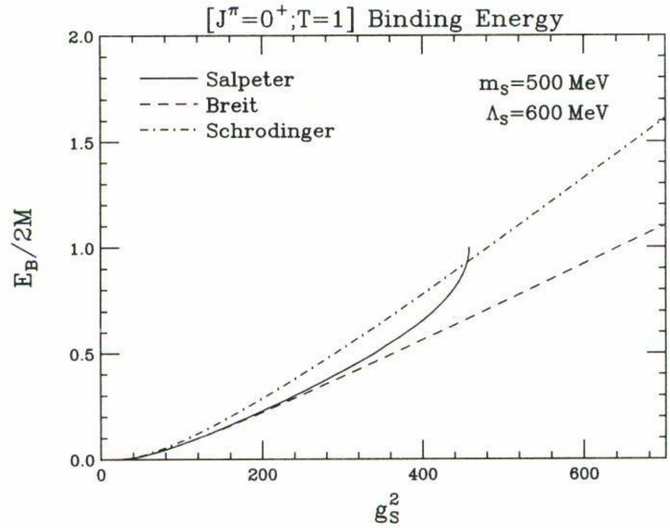

FIGURE 1. Binding energy as a function of the strength of the scalar exchange in the case of Salpeter (solid line), Breit (dashed line) and Schrödinger (dashed-dot line) approximation to the $J^{\pi}=0^{+}$; $T=1$ two-body bound state.

We have also solved Salpeter's equation in the deuteron $J^{\pi}=1^{+}$; $T=0$ channel. The calculations were done using Walecka-model parameters as obtained from a relativistic mean-field calculation that saturates nuclear matter at the correct energy and density [19]. Since the nuclear matter calculation is insensitive to the presence of formfactors, we have used monopole formfactors with the scalar cutoff parameter adjusted to reproduce the binding energy of the deuteron $E_{\rm B}=2.225$ MeV. The vector cutoff parameter was (arbitrarily) fixed at $\Lambda_v=1100$ MeV. In Table II we have listed the scalar cutoff parameter together with channel probabilities. Since the nonrelativistic limit of a scalar plus vector exchange does not generate a tensor force, there is no source of ${}^3S_1$–${}^3D_1$ mixing in Schrödinger's equation. By enabling the presence of lower components in Breit's equation, one can generate a small D-state probability through the space part of the vector potential. Due to the absence of isovector (π,ρ) mesons from the Walecka model, however, the calculated D-state probability is two orders of magnitude smaller than the accepted value. Allowing for the presence of Z-graphs in Salpeter's approach increases the D-state probability but only by a factor of three. More importantly perhaps, due to the RPA normalization, *i.e.*, $[(P_S^{+}+P_D^{+})-(P_S^{-}+P_D^{-})]=1$, one obtain a ${}^3S_1$ probability (P_S^{+}) that is (slightly) bigger than one. It has recently been claimed that this normalization effect might be responsible for solving the longstanding difficulty concerning static deuteron moments in nonrelativistic potential model [21].

We conclude this section by showing results for the two-baryon removal amplitude. In Fig. 2 we have plotted the large ${}^3S_1$ component as a function of the relative momenta for a Salpeter (solid line), Breit (dashed line) and Schrödinger (dashed-dot line) approximation to the two-body equation. In the case of the Schrödinger equation, the two-nucleon removal amplitude is simply the momentum-space wave function. Due to the small binding

TABLE II. Scalar cutoff parameter and channel probabilities for a $J^{\pi} = 1^{+}$; $T = 0$ state in the Walecka model. The calculations were performed using a monopole formfactor with the vector cutoff fixed at $\Lambda_v = 1100$ MeV. The scalar cutoff was adjusted to reproduce the binding energy of the deuteron $E_B = 2.225$ MeV.

Model	Λ_s (MeV)	P_S^+	P_D^+	P_S^-	P_D^-	P[a]
Schrödinger	1552.98	1.0000	0.000	0.000	0.000	1.000
Breit	1941.15	0.9994	6.335×10^{-4}	0.000	0.000	1.000
Salpeter	1052.25	1.0002	1.719×10^{-3}	1.905×10^{-3}	2.637×10^{-5}	1.000

[a] $P \equiv [(P_S^+ + P_D^+) - (P_S^- + P_D^-)]$.

energy of the deuteron, most of the removal strength is concentrated around a region of small relative momenta where all three approaches give essentially the same result. Some differences, however, are clearly visible in the high-momentum tail which, although small, will play an important role in the high-momentum transfer experiments already scheduled at CEBAF. In addition to the large $^3S_1$ component, the tensor force, driven by the space part of the vector exchange, is responsible for generating a small D-state admixture in, both, Breit's and Salpeter's two-nucleon removal amplitudes. From these, one can proceed to construct the full two-body amplitude as given in Eq. (28). Fig. 3 shows the four two-baryon removal amplitudes as a function of the relative momenta with the large $^3S_1^+$ component reduced by a factor of 20. Elucidating the effect of the smaller components of the amplitude on deuteron properties using more realistic models of the NN interaction is currently under investigation.

4. CONCLUSIONS

We have calculated relativistic two-body bound states using Salpeter's instantaneous approximation to the Bethe-Salpeter equation. In addition to the intermediate propagation of two positive-energy particles, Salpeter's approach allows for the propagation of two antiparticles (Z-graphs). As a consequence, the underlying symmetry between two-particle and two-antiparticle states present in the full Bethe-Salpeter equation is preserved in Salpeter's approach. Although Salpeter's equation can be formulated as an eigenvalue problem, the coupling to negative-energy states, *i.e.*, Z-graphs, precludes one from writing it as a Hermitian eigenvalue equation. Instead, Salpeter's equation becomes identical in structure to an RPA equation and can, thus, lead to imaginary eigenvalues. The presence of imaginary eigenvalues is associated with a pairing instability against the formation of strongly bound particle and antiparticle pairs.

We have investigated the pairing instability in a simple model of nucleons interacting via a scalar meson exchange. By increasing the strength of the scalar coupling we showed that a pairing instability developed once the binding energy of the two-particle bound state became equal to twice the mass of the individual constituents. Because of the symmetry of Salpeter's equation, this condition was equivalent to the disappearance of the energy gap between two-particle and two-antiparticle states.

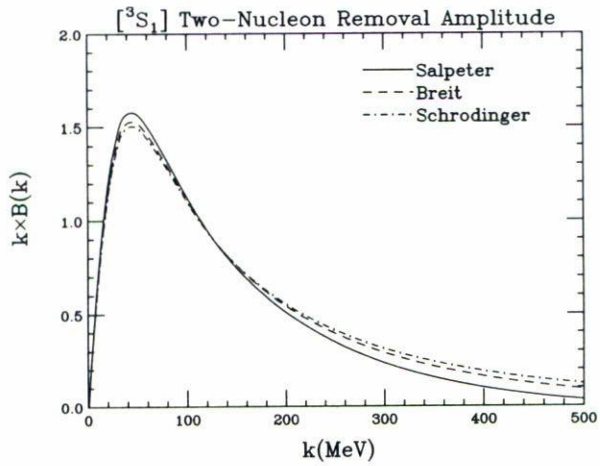

FIGURE 2. The deuteron $^3S_1$ two-nucleon removal amplitude as a function of the relative momentum in the Walecka model using Salpeter (solid line), Breit (dashed line) and Schrödinger (dashed-dot line) approximation to the two-body equation.

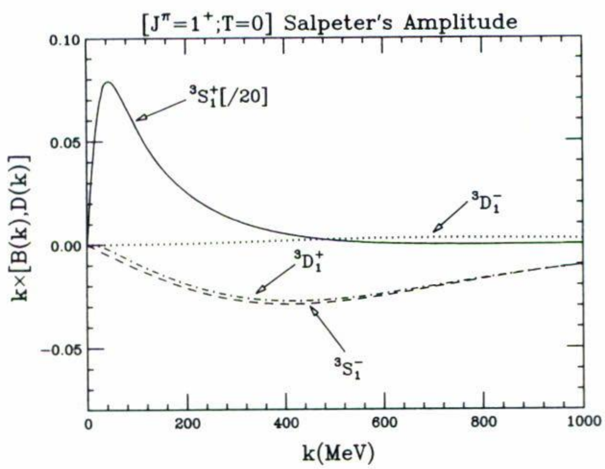

FIGURE 3. The two-nucleon removal amplitudes as a function of the relative momentum in the Walecka model. The $^3S_1^+$ amplitude has been reduced by a factor of 20.

A very simple, hence unrealistic, model of the deuteron was studied withing the framework of Walecka's scalar-vector model. Since a mean-field calculation of ground-state properties of nuclear matter is insensitive to the presence of formfactors, we have adjusted the scalar and vector cutoff parameters to reproduce the correct binding energy of the deuteron. We have also compared Salpeter's results with those obtained from a calculation using the Breit (*i.e.*, non Z-graphs) equation and a nonrelativistic Schrödinger equation. Since the presence of Z-graphs invariably leads to additional attraction, a much softer scalar formfactor was needed in Salpeter's equation as compared with the one used in the Breit equation. We have also noticed that because of the (RPA) normalization condition of the Salpeter amplitude, some channel probabilities were greater, albeit only slightly, than one. In fact, it has recently been suggested, that this normalization effect might be responsible for solving the long-standing difficulty concerning static deuteron moments in nonrelativistic potential models [21].

There are several directions in which we would like to proceed. For example, we would like to use a more realistic model of the NN interaction, that will necessarily include isovector mesons, to study deuteron properties. With a relativistic model for the deuteron, and a suitable constructed electromagnetic current, one could then proceed to explore the consequences of Salpeter's approach on static deuteron moments and high Q^2 elastic formfactors. In addition, one could use Salpeter's approach to study the behavior of two-body bound states in the nuclear medium. The pairing instability develops once the binding energy of the pair becomes equal to twice the mass of the individual constituents. For a deuteron in free space one is clearly very far from the region of the instability. The deuteron binding energy, however, arises from a sensitive cancellation between strongly attractive and repulsive contributions. Since this sensitive cancellation is known to be upset in the nuclear medium and, at least within the Walecka model, the nucleon mass becomes substantially reduced relative to its free-space value, the question of pairing instabilities might become interesting at finite nuclear density. In addition, a study of bound pair states in the medium might reveal the nature of the ground state of nuclear matter. Although, in principle, one should attempt to solve the one- and two-body problem self-consistently, one might start with a simple one-body propagator. In fact, using these ideas, it has been recently suggested, on the basis of a nonrelativistic study of bound states in nuclear matter, that the ground state of nuclear matter corresponds to a superfluid with pairing occurring with deuteron quantum numbers [16]. Ultimately, of course, the self-consistent solution of the one- and two-body problem might also prove essential for the analysis of single-nucleon $(e, e'p)$ and two-nucleon $(e, e'\mathrm{NN})$ knockout experiments.

Finally, it might also be interesting to study Salpeter's equation for the quark-antiquark $(q\bar{q})$ problem. For example, one might calibrate the flavor-independent interaction using heavy quarkonia (*e.g.*, charmonium and bottomonium) and then apply it in the study of light mesons. Indeed, a preliminary study of light mesons using Salpeter's approach indicates that the large vector-to-pseudoscalar mass difference arises from the strong coupling generated by the space part $(\boldsymbol{\gamma}(1) \cdot \boldsymbol{\gamma}(2))$ of the vector interaction [22] (the importance of this term has already been shown in Table I). In addition, one would like to understand how, if at all, a pairing instability develops in these light systems. Although there has already been some work done along these lines, some of these studies have used, either, a simplified form of Salpeter's equation where the notion of instabilities is ambiguous at best [23], or have failed to identify the instabilities by discarding those solutions of Salpeter's equation having imaginary eigenvalues [22].

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APPENDIX: PARTIAL WAVE EXPANSION

We now proceed to perform the partial wave decomposition of the two-body amplitude. We illustrate the procedure with the direct term of V^{++} [Eq. (23a)]. Including spin indices and working in the CM frame this term reduces to

$$\langle \mathbf{k}s_1; -\mathbf{k}s_2|V_D^{++}|\mathbf{k}'s_2'; -\mathbf{k}'s_2'\rangle = \overline{\mathcal{U}}(\mathbf{k}, s_1)\overline{\mathcal{U}}(-\mathbf{k}, s_2)V(\mathbf{k}, \mathbf{k}')\mathcal{U}(\mathbf{k}', s_1')\mathcal{U}(-\mathbf{k}', s_2'), \qquad (A1)$$

where the Fourier-transform of the potential is given by

$$V(\mathbf{k}, \mathbf{k}') = \int d\mathbf{x}\, e^{-i(\mathbf{k}-\mathbf{k}')\cdot x} V(\mathbf{x}) \equiv \sum_{\mathcal{L}M_{\mathcal{L}}} Y_{\mathcal{L}M_{\mathcal{L}}}(\hat{\mathbf{k}})V_{\mathcal{L}}(\mathbf{k}, \mathbf{k}')Y^*_{\mathcal{L}M_{\mathcal{L}}}(\hat{\mathbf{k}}). \qquad (A2)$$

In order to construct states of good total angular momentum we write the free two-body state as a direct product of two Pauli spinors, *i.e.*,

$$[\mathcal{U}(\mathbf{k}, s_1)]_\alpha[\mathcal{U}(-\mathbf{k}, s_2)]_\beta = \left(\frac{\epsilon_{\mathbf{k}} + M}{2\epsilon_{\mathbf{k}}}\right) \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{k}}{\epsilon_{\mathbf{k}} + M} \end{pmatrix}_\alpha \begin{pmatrix} 1 \\ \dfrac{-\boldsymbol{\sigma}\cdot\mathbf{k}}{\epsilon_{\mathbf{k}} + M} \end{pmatrix}_\beta |s_1 s_2\rangle$$

$$= C_{\alpha\beta}(\mathbf{k}) \sum_\lambda \langle \alpha 0; \beta 0|\lambda 0\rangle \left[Y_\lambda(\hat{\mathbf{k}})(\sigma_\alpha\sigma_\beta)_\lambda\right]_{0,0}|s_1 s_2\rangle, \qquad (A3)$$

where $C_{\alpha\beta}(\mathbf{k})$ has been defined by

$$C_{\alpha\beta}(k) = \sqrt{4\pi}\,(-1)^\alpha \left(\frac{\epsilon_{\mathbf{k}} + M}{2\epsilon_{\mathbf{k}}}\right) \xi_\alpha(k)\xi_\beta(k); \quad \xi_\alpha(k) = \begin{cases} 1 & \text{if } \alpha = 0; \\ \dfrac{k}{\epsilon_{\mathbf{k}} + M} & \text{if } \alpha = 1. \end{cases} \qquad (A4)$$

Equation ($A3$) can now be combined with the partial wave expansion of the two-baryon removal amplitudes [Eqs. (25a)] to give

$$\sum_{s_1 s_2} [\mathcal{U}(\mathbf{k}, s_1)]_\alpha[\mathcal{U}(-\mathbf{k}, s_2)]_\beta B_{s_1 s_2}(\mathbf{k}) = \sum_{\substack{LS\mathcal{L}\mathcal{S} \\ JM}} \mathcal{F}^{\alpha\beta}_{\mathcal{L}\mathcal{S};LSJ}(k) B_{LSJ}(k)\langle \hat{k}|\mathcal{L}\mathcal{S}JM\rangle, \qquad (A5)$$

where we have defined

$$\mathcal{F}^{\alpha\beta}_{\mathcal{L}\mathcal{S};LSJ}(k) = C_{\alpha\beta}(k) \sum_\lambda \langle \alpha 0; \beta 0|\lambda 0\rangle \langle \mathcal{L}\mathcal{S}J\|[Y_\lambda(\sigma_\alpha\sigma_\beta)_\lambda]_0\|LSJ\rangle. \qquad (A6)$$

Given the total angular momentum (J), parity ($\pi = (-1)^L$) and isospin (T) (the latter one necessary to determine the total spin (S)) one can now isolate the channel of interest in a straightforward manner. This procedure leads directly to Salpeter's equation in partial wave form

$$(+E - 2\epsilon_{\mathbf{k}})B_{LSJ}(k) = \sum_{L'S'} \int \frac{dk'\,k'^2}{(2\pi)^3} \Big[\langle k; LSJ|V^{++}|k'; L'S'J\rangle B_{L'S'J}(k') + \langle k; LSJ|V^{+-}|k'; L'S'J\rangle D_{L'S'J}(k') \Big], \quad (A7a)$$

$$(-E - 2\epsilon_{\mathbf{k}})D_{LSJ}(k) = \sum_{L'S'} \int \frac{dk'\,k'^2}{(2\pi)^3} \Big[\langle k; LSJ|V^{-+}|k'; L'S'J\rangle B_{L'S'J}(k') + \langle k; LSJ|V^{--}|k'; L'S'J\rangle D_{L'S'J}(k') \Big], \quad (A7b)$$

where, for local interactions, *i.e.*, non-derivative coupling, the matrix elements of the potential are given by (a sum over greek indices is implicitly assumed and $\bar{\alpha} \equiv 1 - \alpha$)

$$\langle k; LSJ|V^{++}|k'; L'S'J\rangle = \langle k; LSJ|V^{--}|k'; L'S'J\rangle = \sum_{\mathcal{LS}} (-1)^{\alpha+\beta} \mathcal{F}^{\alpha\beta}_{\mathcal{LS};LSJ}(k) \langle \mathcal{S}\|[V_{\mathcal{L}}(k,k')]_{\alpha\beta;\alpha'\beta'}\|\mathcal{S}\rangle \mathcal{F}^{\alpha'\beta'}_{\mathcal{LS};L'S'J}(k'), \quad (A8a)$$

$$\langle k; LSJ|V^{+-}|k'; L'S'J\rangle = \langle k; LSJ|V^{-+}|k'; L'S'J\rangle = \sum_{\mathcal{LS}} (-1)^{\alpha+\beta+\mathcal{L}} \mathcal{F}^{\alpha\beta}_{\mathcal{LS};LSJ}(k) \langle \mathcal{S}\|[V_{\mathcal{L}}(k,k')]_{\alpha\beta;\alpha'\beta'}\|\mathcal{S}\rangle \mathcal{F}^{\bar{\alpha}'\bar{\beta}'}_{\mathcal{LS};L'S'J}(k'). \quad (A8b)$$

We conclude the appendix by evaluating the matrix elements of the potential. Before displaying our results for an arbitrary Lorentz structure, however, we illustrate the procedure in the case of an isoscalar vector-meson exchange, *i.e.*,

$$\begin{aligned}\langle \mathcal{S}\|[V_{\mathcal{L}}(k,k')]_{\alpha\beta;\alpha'\beta'}\|\mathcal{S}\rangle &= V_{\mathcal{L}}(k,k') \langle \mathcal{S}\| \big[\gamma^0_{\alpha\alpha'}\gamma^0_{\beta\beta'} - \boldsymbol{\gamma}_{\alpha\alpha'} \cdot \boldsymbol{\gamma}_{\beta\beta'} \big] \|\mathcal{S}\rangle \\ &= V_{\mathcal{L}}(k,k')(-1)^{\alpha+\beta} \big[\delta_{\alpha\alpha'}\delta_{\beta\beta'} - \delta_{\bar{\alpha}\alpha'}\delta_{\bar{\beta}\beta'}\zeta_{\mathcal{S}} \big], \end{aligned} \quad (A9)$$

where we have defined

$$\zeta_{\mathcal{S}} \equiv \langle \mathcal{S}\|\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\|\mathcal{S}\rangle = [2\mathcal{S}(\mathcal{S}+1) - 3]. \quad (A10)$$

Results for an arbitrary Lorentz structure of the potential can be obtained in a similar fashion and are given by

$$\langle \mathcal{S}\|[V_{\mathcal{L}}(k,k')]_{\alpha\beta;\alpha'\beta'}\|\mathcal{S}\rangle \equiv V_{\mathcal{L}}(k,k') \langle \mathcal{S}\|\Gamma_{\alpha\beta;\alpha'\beta'}\|\mathcal{S}\rangle, \quad (A11)$$

where

$$\langle \mathcal{S} \| \Gamma_{\alpha\beta;\alpha'\beta'} \| \mathcal{S} \rangle = \begin{cases} \delta_{\alpha\alpha'}\delta_{\beta\beta'} & \text{for scalar;} \\ (-1)^{\alpha+\beta}(\delta_{\alpha\alpha'}\delta_{\beta\beta'} - \delta_{\bar{\alpha}\alpha'}\delta_{\bar{\beta}\beta'}\zeta_S & \text{for vector;} \\ 2(\delta_{\alpha\alpha'}\delta_{\beta\beta'} + \delta_{\bar{\alpha}\alpha'}\delta_{\bar{\beta}\beta'}\zeta_S & \text{for tensor;} \\ \delta_{\bar{\alpha}\alpha'}\delta_{\bar{\beta}\beta'} & \text{for pscalar;} \\ (-1)^{\alpha+\beta}(\delta_{\bar{\alpha}\alpha'}\delta_{\bar{\beta}\beta'} - \delta_{\alpha\alpha'}\delta_{\beta\beta'}\zeta_S) & \text{for avector.} \end{cases} \qquad (A12)$$

Finally, if the exchanged meson is of isovector nature (*e.g.*, π or ρ exchange) an additional factor of

$$\zeta_T \equiv \langle T \| \tau_1 \cdot \tau_2 \| T \rangle = [2T(T+1) - 3], \qquad (A13)$$

multiplies the above expressions.

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