SU(6) and neutrino magnetic moment

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ABSTRACT. We use a grand unified SU(6) model to show that we can generate masses for fermions consistent with present low energy phenomenology. This model produces a magnetic moment for the electronic neutrino that solves the solar neutrino problem.

RESUMEN. Usamos el modelo de unificación SU(6) para mostrar que podemos generar masas para los fermiones consistente con la fenomenología a bajas energías. Este modelo produce un momento magnético para el neutrino electrónico que permite resolver el problema de neutrinos solares.

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1. INTRODUCTION

The solar neutrino flux detected on the Earth [1] is three times less than the theoretical value given by the solar standard model [2]. An attractive solution to this puzzle is to assume that the electronic neutrino, ν_e, has a magnetic moment of order [3]

$$\mu_{\nu_e} \simeq (0.3\text{–}1.0) \times 10^{-10} \mu_{\mathrm{B}}, \tag{1}$$

where $\mu_{\mathrm{B}} = e/2m_e$ is the Bohr magneton, which interacts with the intense magnetic fields of the sun, rotating the neutrino into a class of sterile or right handed neutrino.

Different extensions of the Standard Model (SM) have been proposed in order to obtain a larger value for the electronic neutrino magnetic moment [4]. An SU(6) Grand Unification Theory (GUT) was proposed recently [5]. In this work we are going to show how this model produces an appropriate value for the magnetic moment with a very small mass for ν_e. This model is important because SU(6) is a subgroup of E_6 [6], with a simpler gauge structure, the same predictive range at low energy, and the same fermions, as well as being the minimun extension of SU(5) [7] with an additional neutral current and neutrino mass.

The model has five neutral fermions, three are $\mathrm{SU(2)_L}$ doublets and two are singlets. Using symmetry breaking and the see-saw mechanism [8] it is possible to get a very large mass for the two doublets and one singlet neutrino 500 GeV and very small masses for the other two 10 eV and 5 eV, respectively. One of these is ν_e and the other is the sterile neutrino. The leptons e, E and the quarks d, D have the same mass matrix. Using the universal see-saw mechanism [9] and taking the electron mass to be 0.5 MeV we have obtained the second symmetry scale, 10^8 GeV, in agreement to the renormalization group equations of the SU(6) model. This is important to give masses to the neutrinos. In order to get different masses for the electron and the d quark, we can use an 84–dimensional representation, or consider radiative corrections.

2. The model

When we impose the cancellation of triangular anomalies [10] in this model the fermion content is the same as the 27–dimensional representation of E_6, distributed in two 6^* representations and one 15 representation:

$$\psi_1 = \begin{pmatrix} d_1'^c \\ d_2'^c \\ d_3'^c \\ e' \\ \nu \\ n \end{pmatrix}, \qquad \psi_2 = \begin{pmatrix} D_1'^c \\ D_2'^c \\ D_3'^c \\ E' \\ N \\ N' \end{pmatrix},$$
$$\Psi = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1' & D_1' \\ -u_3^c & 0 & u_1^c & u_2 & d_2' & D_2' \\ u_2^c & -u_1^c & 0 & u_3 & d_3' & D_3' \\ -u_1 & -u_2 & -u_3 & 0 & e'^c & E'^c \\ -d_1 & -d_2 & -d_3 & -e'^c & 0 & \tilde{N} \\ -D_1 & -D_2 & D_3 & -E^c & \tilde{N} & 0 \end{pmatrix}. \tag{2}$$

The electromagnetic charge is given by

$$Q = T_{3\mathrm{L}} + \sqrt{\frac{3}{5}}\,Y, \tag{3}$$

with

$$T_{3\mathrm{L}} = \mathrm{diag}\,(0 \quad 0 \quad 0 \quad 1/2 \quad -1/2 \quad 0)\,,$$

$$Y = \sqrt{\frac{3}{20}}\,\mathrm{diag}\,(-2/3 \quad -2/3 \quad -2/3 \quad 1 \quad 1 \quad 0)\,,$$

and the prediction for the Weinberg angle at the unification scale is

$$\sin^2\theta_{\mathrm{W}} = \frac{3}{8}. \tag{4}$$

3. The stages of symmetry breaking

Using the representation 6^*, 15, 21 and 35 for the Higgs we break the symmetry to $\mathrm{SU}(3)_c \otimes \mathrm{U}(1)_Q$ and give masses to the fermions, according to

$$\begin{aligned} \mathrm{SU}(6) &\xrightarrow{M_u} \mathrm{SU}(3)_c \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1) \\ &\xrightarrow{M_I} \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \\ &\xrightarrow{M_v} \mathrm{SU}(3)_c \otimes \mathrm{U}(1)_Q, \end{aligned} \tag{5}$$

where M_u, M_I and M_v are the symmetry breaking scales. In this model ψ_1 and ψ_2 have the same quantum numbers, and the charged leptons and quarks in these representations have the same masses. Therefore, it is necessary to introduce a discrete symmetry to distingish the multiplets and get different masses for $e(d)$ and $E(D)$. By ussing an appropriate discrete symmetry we obtain the Yukawa Lagrangian

$$\begin{aligned} -\mathcal{L}_{\mathrm{Y}} = {} & \psi_2^{\mathrm{T}} C \Psi \phi^{(I)} + \psi_2^{\mathrm{T}} C \psi_2 \Phi^{(I)} + \psi_2^{\mathrm{T}} C \Psi \phi^{(v)} \\ & + \psi_1^{\mathrm{T}} C \psi_1 \Phi^{(v)}_{(a)} + \psi_2^{\mathrm{T}} C \psi_2 \Phi^{(v)}_{(b)} + \Psi^{\mathrm{T}} C \Psi \varphi^{(v)} \end{aligned} \tag{6}$$

where superindex I, v indicates that the vacuum expectation value (V.E.V.) is different from zero at M_I and M_v scales, respectively, and ϕ, Φ and φ are the 6, 21 and 15-dimensional representations, respectively.

We introduce two scales of orders $M_v \approx 264$ GeV and $M_I \approx 10^8$ GeV, in order to implement the see-saw mechanism in the sector of neutral particles. In this way we obtain high neutrino masses; and the universal see-saw mechanism to get an electron mass of order 0.5 MeV. The V.E.V.'s in $\phi^{(I)}$ and $\Phi^{(I)}$ are in the directions $\phi_6^{(I)}$ and $\Phi_{66}^{(I)}$, respectively, and $\phi^{(v)}, \Phi^{(v)}$ and $\varphi^{(v)}$ are in the directions $\phi_5^{(v)}$, $\Phi^{(v)}_{(i)56} = \Phi^{(v)}_{(i)65}$, $i = a, b$ and $\varphi_{56}^{(v)} = -\varphi_{65}^{(v)}$, respectively. The mass matrix M_ν for the neutral fermions in the basis $(\nu, N, \tilde{N}, n, N')$ is

$$M_\nu = h \begin{pmatrix} 0 & 0 & 0 & 2M_v & 0 \\ 0 & 0 & M_I & 0 & M_v \\ 0 & M_I & 0 & 0 & -M_v \\ 2M_v & 0 & 0 & M_I & 0 \\ 0 & M_v & -M_v & 0 & 0 \end{pmatrix} \tag{7}$$

with eigenvalues given by

$$
\begin{aligned}
\lambda_1 &= hM_I, \\
\lambda_2 &= h\,\frac{M_I + \sqrt{M_I^2 + 8M_v^2}}{2} \approx -h\left(M_I + \frac{2M_v^2}{M_I}\right), \\
\lambda_3 &= h\,\frac{-M_I + \sqrt{M_I^2 + 16M_v^2}}{2} \approx h\,\frac{2M_v^2}{M_I}, \\
\lambda_4 &= h\,\frac{-M_I + \sqrt{M_I^2 + 16M_v^2}}{2} \approx h\,\frac{-4M_v^2}{M_I}, \\
\lambda_5 &= -h\,\frac{M_I + \sqrt{M_I^2 + 16M_v^2}}{2} \approx h\left(M_I + \frac{4M_v^2}{M_I}\right),
\end{aligned} \tag{8}
$$

and the corresponding eigenvectors

$$
\begin{aligned}
\nu_1 &= \frac{1}{\sqrt{2}}\,(0,1,1,0,0), \\
\nu_2 &= \frac{\lambda_2 + M_I}{\sqrt{2M_v^2 + (\lambda_2 + M_I)^2}}\left(0, \frac{M_v}{\lambda_1 + M_I}, -\frac{M_v}{\lambda_1 + M_I}, 0, 1\right) \\
&\approx \frac{1}{\sqrt{2}}\left(0, 1, -1, 0, \frac{M_v}{2M_I}\right), \\
\nu_3 &= \frac{\lambda_3 + M_I}{\sqrt{2M_v^2 + (\lambda_3 + M_I)^2}}\left(0, \frac{M_v}{\lambda_3 + M_I}, -\frac{M_v}{\lambda_3 + M_I}, 0, 1\right) \\
&\approx \left(0, \frac{M_v}{M_I}, -\frac{M_v}{M_I}, 0, 1\right), \\
\nu_4 &= \frac{\lambda_4}{\sqrt{4M_v^2 + \lambda_4^2}}\left(\frac{2M_v}{\lambda_4}, 0, 0, 1, 0\right) \\
&\approx \left(1, 0, 0, -\frac{2M_v}{M_I}, 0\right), \\
\nu_5 &= \frac{\lambda_5}{\sqrt{4M_v^2 + \lambda_5^2}}\left(\frac{2M_v}{\lambda_5}, 0, 0, 1, 0\right) \\
&\approx \left(\frac{2M_v}{M_I}, 0, 0, 1, 0\right),
\end{aligned} \tag{9}
$$

where the second equality is obtained with the approximation $M_I \gg M_v$.

We have introduced a finetuning parameter to get an appropriate order of magnitude for the electronic neutrino mass, $h \approx 6 \times 10^{-6}$. Here ν_1, ν_2 and ν_5 have masses of order 500 GeV; ν_4, the electronic neutrino, has mass of order 10 eV and ν_3, the sterile neutrino, has mass of order 5 eV.

We could also orient the vacuum in a particular direction in order to get the neutrino mass matrix from the Yukawa Lagrangian

$$M_\nu = h \begin{pmatrix} 0 & 0 & 0 & 2M_v & 0 \\ 0 & 0 & M_I & 0 & 2M_v \\ 0 & M_I & 0 & 0 & -M_v \\ 2M_v & 0 & 0 & M_I & 0 \\ 0 & 2M_v & -M_v & 0 & 0 \end{pmatrix}; \tag{10}$$

here there are three heavy Majorana neutrinos of 500 GeV and one light Dirac 10 eV, the electronic neutrino. But in this work we only consider the first mass matrix because the neutrino magnetic moment is of the same order for the first and second mass matrix.

The mass matrix M_l for leptons with charge -1 and quarks with charge $-1/3$ in the (e', E'), (d', D') basis is

$$M_\mathrm{l} = \begin{pmatrix} 0 & M_v \\ M_v & M_I \end{pmatrix}, \tag{11}$$

and the eigenstates for leptons are given by

$$\begin{aligned} e &= e' + \frac{M_v}{M_I} E', \\ E &= E' - \frac{M_v}{M_I} e'. \end{aligned} \tag{12}$$

4. The renormalization group equation

The renormalization group equation for the coupling constants α_c, α_L and α_Y associated to the $\mathrm{SU}(3)_c$, $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ groups, respectively, at the electroweak scale [11] are

$$\begin{aligned} \alpha_c^{-1}(2M_W) &= \alpha_u^{-1} - \frac{33-4f}{12\pi} \ln\left(\frac{M_u^2}{M_W^2}\right), \\ \alpha_L^{-1}(2M_W) &= \alpha_u^{-1} - \frac{33-4f}{12\pi} \ln\left(\frac{M_u^2}{M_I^2}\right) - \frac{22-4f}{12\pi} \ln\left(\frac{M_I^2}{M_W^2}\right), \\ \alpha_Y^{-1}(2M_W) &= \alpha_u^{-1} - \frac{\frac{1}{5}(33-4f) + \frac{4}{5}(-4f)}{12\pi} \ln\left(\frac{M_u^2}{M_I^2}\right) + \frac{4f}{12\pi} \ln\left(\frac{M_I^2}{M_W^2}\right), \end{aligned} \tag{13}$$

where $\alpha_u^{-1} = g^2/4\pi$ is the coupling constant at the unification scale, M_W is the W_μ mass and f is the number of fundamental fermion representations of the symmetry group. For M_I we obtain the relation

$$M_I = M_W \exp\left[\frac{3\pi}{11}(\alpha^{-1}(2M_W)x - \alpha_c^{-1}(2M_W)\right], \tag{14}$$

where $x = \sin^2\theta_{\rm W}(2M_W)$ and $4\pi\alpha = e^2$. Using $x \approx 0.23$, $\alpha^{-1} \approx 128$ and $\alpha_c^{-1} \approx 10$, we get

$$M_I \approx 10^8 \text{ GeV}, \tag{15}$$

in agreement with the value necesary to obtain the universal see-saw mechanism.

5. Magnetic moment

We will suppose that the largest contribution to the magnetic moment of the neutrino is given by the electroweak part, where the lagrangian for the charge current is

$$\mathcal{L}_{\rm CC} = \frac{ig}{\sqrt{2}}\left[\bar{e}'\gamma_\mu P_L \nu + \bar{E}'\gamma_\mu P_L N + \bar{E}'\gamma_\mu P_R n\right] W^\mu, \tag{16}$$

which, written in terms of the mass eigenstates, is

$$\mathcal{L}_{\rm CC} = \frac{ig}{\sqrt{2}}\left[\bar{E}\gamma_\mu(v_{E4} - a_{E4}\gamma_5)\nu_4 + \bar{E}\gamma_\mu(v_{E3} - a_{E3}\gamma_5)\nu_3\right] W^\mu. \tag{17}$$

Here we have considered only the terms in the lagrangian which produce the largest contributions to the magnetic moment and have taken

$$v_{Ei} = U_{eE}U_{\nu i} + U_{EE}U_{Ni} + U_{EE}U_{\tilde{N}i},$$

$$a_{Ei} = U_{eE}U_{\nu i} + U_{EE}U_{Ni} - U_{EE}U_{\tilde{N}i}.$$

For $i = 3, 4$ the invariant matrix for the transition $\nu_4 \to \nu_3 + \gamma$ is [12]

$$\sigma_{\mu\nu}q^\nu(a + b\gamma_5), \tag{18}$$

where $q = p_3 - p_4$ is the momentum transfer, and a, b are

$$a = -b = -\frac{G_{\rm F}}{4\sqrt{2}\,\pi^2}\mu_{\rm B} m_e M_E f(r_E)\left[v_{E4}v_{E3} - a_{E4}a_{E3}\right].$$

The function $f(r_E) = f(M_E/M_W)$ is of order one for large values of the E mass, M_E. In the approximation $M_W \ll M_I$ we get

$$v_{E4}v_{E3} - a_{E4}a_{E3} = -2\frac{m_e}{M_E}, \tag{19}$$

so

$$a = -b \approx \frac{G_{\mathrm{F}}}{4\sqrt{2}\,\pi^2} m_e^2 \mu_{\mathrm{B}} \approx 10^{-13}\ \mu_{\mathrm{B}}. \tag{20}$$

The lagrangian for the neutral currents is of the form

$$\mathcal{L}_{\mathrm{NC}} = \frac{ig}{2\cos\theta_{\mathrm{W}}} \left[\bar{\nu}\gamma_\mu P_L \nu + \bar{N}\gamma_\mu P_L N + \bar{n}\gamma_\mu P_L n\right] Z^\mu, \tag{21}$$

which in terms of the mass eigenstate is

$$\mathcal{L}_{\mathrm{NC}} = \frac{ig}{4\cos\theta_{\mathrm{W}}} \sum_{i,j=3,4} \bar{\nu}_i \gamma_\mu (v_{ij} - a_{ij}\gamma_5)\nu_j, \tag{22}$$

where

$$v_{ij} = U^*_{\nu i} U_{\nu j} + U^*_{N i} U_{N j} + U^*_{n i} U_{n j},$$

$$a_{ij} = U^*_{\nu i} U_{\nu j} + U^*_{N i} U_{N j} - U^*_{n i} U_{n j}.$$

The decay of the Z into neutrinos is given by

$$\sum_{i,j=3,4} \Gamma(Z \to \nu_i \nu_j) \approx \Gamma_{\mathrm{SM}}(Z \to \nu_e \nu_e) \left(1 + \left(\frac{M_v}{M_I}\right)^4\right), \tag{23}$$

where Γ_{SM} is the value predicted by the SM.

The model doesn't have problem with the LEP phenomenology [13] because the decay of the Z into steriles neutrino is suppresed by a $(M_v/M_I)^4$ factor.

6. Conclusion

We have shown that choosing the vacuum expectation values of order 10^2 GeV and 10^8 GeV we can obtain the appropriate electron mass, a mass matrix that generates masses for the neutrinos consistent with the Z phenomenology, and renormalization group equations, as well as generating a magnetic moment large enough to get an appropriate number of sterile neutrinos.

The magnetic moment tht we have obtained is two orders of magnitude lower than the necessary value to solve the solar puzzle appropriately. Notice that this value is consisted with the analysis of the observed neutrino flux from the supernova SN1987A [14].

To get the appropriate relation of neutrino-electron mass we need to finetuning parameters which makes the model less natural. However, the magnetic moment does not depend on this finetuning.

Presently, we are working on this model without finetuning parameters and using the Grand Unification scale, 10^{15} GeV, to generate the fermion masses. Another solution without finetuning parameters is to leave the neutrinos massless in the symmetry breaking, and get these masses by radiative corrections and thus solve the solar neutrino problem.

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