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Plasmons in three, two and one dimension

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ABSTRACT. We present a unified derivation of the plasmon dispersion relation for three, two and one dimensional systems. We obtain exact results in the long-wavelength limit with the help of a hydrodynamic model. A heuristic description is also developed to help understanding the physics behind the plasmon in the three different dimensionalities and a quantum derivation is briefly presented in order to corroborate the hydrodynamic results.

RESUMEN. Presentamos de una manera unificada la derivación de la relación de dispersión para plasmones en tres, dos y una dimensión. Obtenemos resultados exactos en la aproximación de onda larga con la ayuda del modelo hidrodinámico. También se presenta un modelo heurístico como ayuda para el entendimiento de la física detrás del plasmón en las diferentes dimensionalidades, y un tratamiento cuántico es presentado brevemente para corroborar los resultados hidrodinámicos.

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1. INTRODUCTION

In this article we would like to present a unified picture of plasmons in three (3D), two (2D) and one (1D) dimensional systems. Although there is a vast literature for plasmons in either dimension [1–10], it is very difficult to find a picture that gives the basic ingredients of these modes in a simple and unified way. We will use a hydrodynamic picture that suffices in the long wavelength approximation. Thus, the electrons will be considered like a fluid of charged particles that in 3D move without constraints, but in 2D and 1D will be restricted to move in two and one dimensions, respectively (see Appendix A for the precise meaning of a one dimensional system). We will present as well, a heuristic argument for the plasmon frequency that will give a clear and direct physical picture of the electron's motion as they participate to form the collective mode or plasmon. For completeness, we will derive the results from a purely quantum mechanical point of view, and will show that in the long-wavelength limit the results are identical with those of the hydrodynamic model.

A plasmon is a collective (normal) mode of a collection of identical charged particles. The usual model is that of a 3D system uniformly and isotropically filled with electrons and positive charges (background), so the system is neutral in equilibrium. But, plasmons

their individual equilibrium position in an orderly way. Each and every electron will leave behind a positive charge from the background. If the electrons are then "released" they will move towards their equilibrium positions (due to the Coulombic attraction of the positive charge), but due to their inertia they will go past these positions, and thus the electrons will start an oscillatory motion about these equilibrium positions. The organized or collective motion of all the electrons, that is the normal modes of the electrons as a whole, constitute a plasmon. This collective oscillations will propagate through the system and correspond to compression waves in the electron gas. The very aim of the present article is to elucidate the frequency of such collective modes when the electron gas is confined to different dimensionalities.

The article is organized as follows. In Sect. 2 we present the hydrodynamic model, in Sect. 3 we present the heuristic argument and in Sect. 4 we corroborate the results of the hydrodynamic picture using a microscopic formulation. In Sect. 5 we give the conclusions. Finally, in Appendix A we define the one dimensional system, in Appendix B we derive the Fourier transform of the Coulomb potential for the three different dimensionalities and in Appendix C we obtain its long wavelength limit in 1D.

2. HYDRODYNAMIC MODEL

We start by defining the hydrodynamic model which will give the dynamical behavior of the electrons. First, there is a linearized Navier-Stokes equation of motion [11],

$$\frac{\partial \mathbf{j}(\mathbf{r},t)}{\partial t} = \frac{n_0 e^2}{m}\mathbf{E}(\mathbf{r},t) - \beta^2 \nabla \delta\rho(\mathbf{r},t), \tag{1}$$

which attributes the acceleration of electrons to direct electrical forces and to density gradients. In Eq. (1), $\delta\rho(\mathbf{r},t)$ is the induced density, $\mathbf{j}(\mathbf{r},t)$ is the current density, m is the mass of the electron, and β is defined as the stiffness parameter, which includes the effects of compressibility and spatial dispersion. The total charge density was separated into an equilibrium value ρ_0 and a deviation $\delta\rho(\mathbf{r},t) = \rho_0 + \delta\rho(\mathbf{r},t)$. The equilibrium number density is given by $\rho_0 = en_0$, with $e < 0$ an electron's charge. Also, $\mathbf{E}$ is the total electric field, and is found (in the electrostatic limit) from $\mathbf{E} = -\nabla\phi^{\text{tot}}$, with the scalar potential given by

$$\phi^{\text{tot}}(\mathbf{r},t) = \phi^{\text{ext}}(\mathbf{r},t) + \int d\mathbf{r}' \frac{\delta\rho(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|}, \tag{2}$$

where we have separated the total ϕ^{tot} into external and induced parts. Notice that for simplicity we have not included damping processes, but they can be easily included in this formalism by adding phenomenologically a friction in the relaxation time approximation. Second, to relate the induced density $\delta\rho(\mathbf{r},t)$ and the current density $\mathbf{j}(\mathbf{r},t)$, we use the equation of continuity

$$\frac{\partial \delta\rho}{\partial t} + \nabla \cdot \mathbf{j} = 0, \tag{3}$$

Eqs. (1)-(3) and related definitions form a closed set and specify our hydrodynamic model. It is to be understood that the vector quantities as well as the number and charge densities, take the values of the particular dimensionality that is chosen.

Now, we proceed to solve Eqs. (1)-(3) for their normal modes. To this end we set ϕ^{ext} to zero, and derivate the equation of continuity, (3), with respect to t, to yield

$$\frac{\partial^2}{\partial t^2}\delta\rho + \nabla \cdot \frac{\partial \mathbf{j}}{\partial t} = 0, \tag{4}$$

and use the equation of motion to get

$$\left(-\frac{\partial^2}{\partial t^2} + \beta^2\nabla^2\right)\delta\rho(\mathbf{r},t) = \frac{n_0 e^2}{m}\nabla \cdot \mathbf{E}(\mathbf{r},t) = -\frac{n_0 e^2}{m}\nabla^2\phi(\mathbf{r},t). \tag{5}$$

To continue, we Fourier transform according to

$$\delta\rho(\mathbf{r},t) = \sum_{\mathbf{q},\omega} \delta\rho(\mathbf{q},\omega)e^{-i\omega t}e^{i\mathbf{q}\cdot\mathbf{r}}, \tag{6}$$

$$\phi(\mathbf{r},t) = \sum_{\mathbf{q},\omega} \nu(q)\delta\rho(\mathbf{q},\omega)e^{-i\omega t}e^{i\mathbf{q}\cdot\mathbf{r}}, \tag{7}$$

where $\nu(q)$ is the Fourier transform of $|\mathbf{r}-\mathbf{r}'|^{-1}$ (the Coulomb potential), ω is the frequency and $\mathbf{q}$ is the wave vector (being q its magnitude). In Eqs. (6) and (7), the sum stands for an integration over $\mathbf{q}$ and ω. From Appendix B we get that $\nu(q)$, depends on the dimensionality of the system according to

$$\nu(q) = \begin{cases} V(q) & \text{(1D)}, \\ \dfrac{2\pi}{|\mathbf{q}|} & \text{(2D)}, \\ \dfrac{4\pi}{|\mathbf{q}|^2} & \text{(3D)}, \end{cases} \tag{8}$$

where $V(q)$ is the one-dimensional Fourier transform of the Coulomb potential in the chosen "one-dimensional geometry" (see Appendix A). For instance, for a flat strip of width a (see Appendix B), $V(q) = K_0(qa)$, where K_0 is the modified Bessel function of zeroth order [12]. Taking Eqs. (6) and (7) into (5) gives

$$\left(\omega^2 - \beta^2 q^2 - \frac{n_0 e^2}{m}q^2\nu(q)\right)\delta\rho(\mathbf{q},\omega) = 0, \tag{9}$$

hence the normal modes of the system are given by

$$\omega^2 = \frac{n_0 e^2}{m} q^2 \nu(q) + \beta^2 q^2$$

$$= \frac{n_0 e^2}{m} q^2 \nu(q) \left[1 + \frac{\beta^2}{\frac{n_0 e^2}{m} \nu(q)} \right]. \tag{10}$$

Since the Fourier transform of the Coulomb potential $\nu(q)$ diverges as $q \to 0$ [see Eq. (8)], Eq. (10) reduces to

$$\omega^2 = \frac{n_0 e^2}{m} q^2 \nu(q), \tag{11}$$

in the long wavelength limit ($q \to 0$). This equation gives the plasmon frequency within the hydrodynamic model. In view of (8), this frequency will depend upon dimensionality. Then, substituting $\nu(q)$ from (8) into (11), the plasmon frequency for 3D plasmons, 2D plasmons and 1D plasmons is given by

$$\omega^2(q) = \begin{cases} \dfrac{n_0 e^2}{m} V(q) q^2 & \text{(1D)}, \\ \dfrac{2\pi n_0 e^2}{m} q & \text{(2D)}, \\ \dfrac{4\pi n_0 e^2}{m} & \text{(3D)}. \end{cases} \tag{12}$$

We notice immediately in a very simple and unified fashion, the always referred to, but not always easily demonstrated qualitative behavior that as $q \to 0$ (see Fig. 1): the 3D plasmon is dispersionless and starts at a finite frequency at $q = 0$, the 2D plasmons starts with zero frequency and follows the $(q)^{1/2}$ behavior, and the 1D plasmon also starts at zero frequency and follows a $q(V(q)^{1/2}$ dispersion. We remark that n_0 is the equilibrium density per unit length in 1D, per unit area in 2D and per unit volume in 3D.

We should mention that according to (12) and Appendix B, in 1D $\omega \propto q(2K_0(qa))^{1/2}$. From Fig. 1, we notice that the dispersion of the mode seems to be almost acoustic, *i.e.*, linear with q, for some range of q. However, as we approach $q \to 0$, we show in Appendix C that $V(q) \to -2\log(qa)$, and thus the dominant behavior of q in the aforementioned range, no longer controls the mode's dispersion for low q. A more careful analysis on the actual numbers, reveals that the deviation of the dispersion from linear for low q is quite noticeable [13].

The stiffness parameter β of Eq. (2) can be chosen so that this hydrodynamic model will mimic a more rigorous microscopic theory [14]. Doing so, will actually allow us to set quantitatively how small q should be for the $q \to 0$ limit to be quantitatively valid. We mention though, that this choice is not unique and depends on the particular system

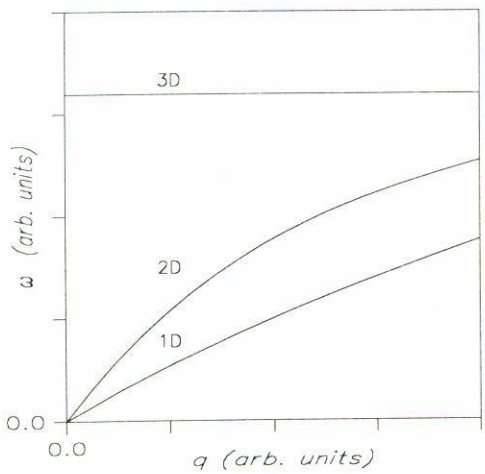

FIGURE 1. The dispersion relation is shown for the three different plasmons. Notice that both axis are given in arbitrary units since we are just interested in the qualitative functional dependence of ω with q. (See text for details).

chosen to be described by the hydrodynamic approach. Therefore, we do not put any emphasis on numerics.

3. HEURISTIC DESCRIPTION

In this section we give a heuristic approach to the functional q dependence or plasmon dispersion obtained in Sect. 2. This will give a more direct insight into the physical nature of the three differential plasmons [15]. We start from Newton's second Law by writing for an individual electron

$$m\frac{d^2}{dt^2}\mathbf{r}(t) = e\mathbf{E}(t). \tag{13}$$

Assuming the usual time dependence $\mathbf{r}(t) = \mathbf{r}(\omega)e^{-i\omega t}$, inherited from a similar behavior of $\mathbf{E}(t)$, we get that the frequency of the electron's harmonic motion, *i.e.*, the plasmon, is proportional to the force,

$$\omega^2 \propto \mathbf{F} = e\mathbf{E}. \tag{14}$$

Let's start with the 3D plasmon. In 3D, once a perturbing oscillation is introduced, the perturbed charges are in the form of oscillating sheet charges normal to $\mathbf{q}$ and separated by a distance $\ell \sim \lambda = 1/q$, with λ the associated wavelength (see Fig. 2). Since the system is 3D, and these sheets are infinite, then from Gauss's Law, it is straightforward to show that the electric field produced by them is independent of the distance away from the sheet. Thus, the restoring Coulomb force acting between these sheets being proportional to the electric field is independent of ℓ as well. Therefore, from (14) we get that ω^2 is independent

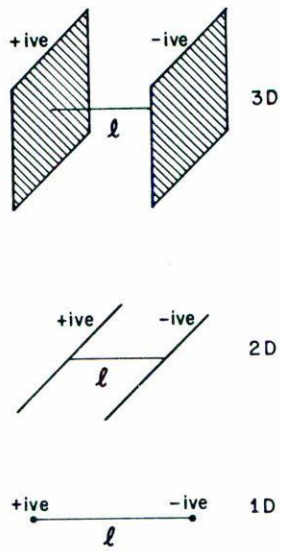

FIGURE 2. We schematically represent the perturbed charges that give rise to the plasmon in the three different dimensionalities. The positive (+ive) and negative (-ive) "charges" are shown. (See text for details).

of ℓ or q, in accordance with (12) for 3D. On the other hand in 2D the corresponding perturbed charges are line charges again normal to $\mathbf{q}$ and separated by $\ell \sim \lambda = 1/q$ (see again Fig. 2). Now, Gauss's Law gives that the restoring Coulomb force between this lines of charge is proportional to $1/\ell$, implying from (14) that ω^2 is proportional to $1/\ell$ or q, thereby giving the $\omega \propto (q)^{1/2}$ behavior obtained in Eq. (12) for 2D. Finally, in 1D the corresponding perturbed charges are points separated by $\ell \sim \lambda = 1/q$ (Fig. 2), and then, the restoring force is simply the usual Coulomb force which is proportional to $1/\ell^2$. From Eq. (14), we get that $\omega \propto q$ (which gives the quasi-acoustic behavior characteristic of 1D plasmons), as previously obtained in (12) for the 1D case. We should mention that the heuristic argument doesn't give the logarithmic divergent term, which is only obtained through the Fourier transform (see Appendix C).

4. MICROSCOPIC MODEL

For completeness, in this section we review the derivation of the plasmon frequency from a quantum mechanical point of view. Actually, our derivation is very similar to the ones appearing in most textbooks [16–19], however in contrast with these, we present it in a unified way for 3D, 2D and 1D systems, as we just did for the hydrodynamic model. We restrict ourselves to the so called Self-Consistent Field approximation (SCF) [20] and eventually to its long-wavelength limit, which agrees with the result obtained in Eq. (12). The SCF approximation focuses on one electron out of the N electrons and calculates its interaction with the $N-1$ remaining neighbors through the self-consistent field produced by these. The SCF reduces the N-particle hamiltonian to an effective one-

particle hamiltonian $H = H_0 + \vartheta^{\text{tot}}$, where $H_0 = p^2/2m$ is the unperturbed hamiltonian and $\vartheta^{\text{tot}}(r,t)$ is the self-consistent field which acts as a time dependent linear perturbation upon the system. To formally solve the problem, we will follow the dielectric response formalism by which we calculate the induced number density, $\delta n(\mathbf{r},t)$, which is related to the total potential by [21]

$$\delta n(\mathbf{r},t) = \chi^0 \vartheta^{\text{tot}}(\mathbf{r},t), \tag{15a}$$

where (see Eq. (2) and recall that $e\phi = \vartheta$)

$$\vartheta^{\text{tot}}(\mathbf{r},t) = \vartheta^{\text{ext}}(\mathbf{r},t) + e^2 \int d\mathbf{r}' \frac{\delta n(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|}, \tag{15b}$$

with ϑ^{ext} the external potential and χ^0 is the local non-interacting susceptibility. The self-consistent solution of these two equations as we will see, gives among other very important physical functions (like χ^0), the plasmons of the system. To proceed further, let's use also the density matrix formalism [18]. Then, let η represent the one particle density matrix of the system, which equation of motion is given by

$$i\hbar \frac{\partial \eta}{\partial t} = [H, \eta], \tag{16}$$

with [,] denoting a commutator. The density matrix is split into $\eta = \eta_0 + \delta\eta(\mathbf{r},t)$, where

$$\eta_0 |\mathbf{k}\rangle = f(\mathbf{k})|\mathbf{k}\rangle, \tag{17}$$

with f the Fermi-Dirac distribution function and $|\mathbf{k}\rangle$ an eigenstate of the unperturbed hamiltonian with $E(\mathbf{k})$ its eigenvalue,

$$H_0|k\rangle = E(k)|k\rangle. \tag{18}$$

On the other hand, $\delta\eta$ is given by

$$i\hbar \frac{\partial \delta\eta}{\partial t} = [H_0, \delta\eta] + [\vartheta^{\text{tot}}, \eta_0], \tag{19}$$

where $\delta\eta$ is linear in the perturbation and higher order terms in the perturbation have been neglected, which is consistent with the linear response being presented. Assuming the usual $e^{-i\omega t}$ harmonic dependance, and taking matrix elements between $\mathbf{k}$ and $\mathbf{k}'$, we get from Eq. (18)

$$\langle \mathbf{k}'|\delta\eta(\mathbf{r},\omega)|\mathbf{k}\rangle = \frac{f(\mathbf{k}) - f(\mathbf{k}')}{E(\mathbf{k}) - E(\mathbf{k}') + \hbar\omega} \langle \mathbf{k}'|\vartheta^{\text{tot}}(\mathbf{r},\omega)|\mathbf{k}\rangle, \tag{20}$$

with

$$\langle \mathbf{k}'|F(\mathbf{r})|\mathbf{k}\rangle = \int d\mathbf{r}\, \Psi^*_{\mathbf{k}'}(\mathbf{r}) F(\mathbf{r}) \Psi_{\mathbf{k}}(\mathbf{r}), \tag{21}$$

and $\Psi_{\mathbf{k}}(\mathbf{r}) = \langle \mathbf{r}|\mathbf{k}\rangle$.

The induced number density $\delta n(\mathbf{r},\omega)$ is given by

$$\delta n(\mathbf{r},\omega) = \mathrm{tr}\big(\delta(\mathbf{r}-\mathbf{r}_e)\delta\eta(\mathbf{r},\omega)\big), \tag{22}$$

where the delta function is the quantum mechanical operator for the number density, with $\mathbf{r}_e$ the position operator for the electron, and tr is the trace,

$$\delta n(\mathbf{r},\omega) = \sum_{\mathbf{k},\mathbf{k}'} \langle \mathbf{k}|\delta(\mathbf{r}-\hat{\mathbf{r}}_e)|\mathbf{k}'\rangle\langle\mathbf{k}'|\delta\eta(\mathbf{r},\omega)|\mathbf{k}\rangle. \tag{23}$$

Using (20) on (23) and $\Psi_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}$ (a plane-wave normalized to unit "volume"), we get

$$\delta n(\mathbf{r},\omega) = \sum_{\mathbf{q}} \delta n(\mathbf{q},\omega)e^{i\mathbf{q}\cdot\mathbf{r}} = \sum_{\mathbf{k},\mathbf{q}} \frac{f(\mathbf{k})-f(\mathbf{k}+\mathbf{q})}{E(\mathbf{k})-E(\mathbf{k}+\mathbf{q})+\hbar\omega}\vartheta^{\mathrm{tot}}(\mathbf{q},\omega)e^{i\mathbf{q}\cdot\mathbf{r}}, \tag{24}$$

where we used the fact that

$$\langle \mathbf{q}+\mathbf{k}|F(\mathbf{r})|\mathbf{k}\rangle = \int d\mathbf{r}\, F(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}} = F(\mathbf{q}) \tag{25}$$

is the Fourier transform of $F(\mathbf{r})$. Notice that the inverse Fourier transform of $F(\mathbf{q})$ is consistent with Eqs. (6) and (7). From Eqs. (15*b*) and (7) we have that

$$\vartheta^{\mathrm{tot}}(\mathbf{q},\omega) = \vartheta^{\mathrm{ext}}(\mathbf{q},\omega) + e^2\nu(q)\delta n(\mathbf{q},\omega), \tag{26}$$

and therefore from (24) we get

$$\left[1 - e^2\nu(q)\sum_{\mathbf{k}} \frac{f(\mathbf{k}+\mathbf{q})-f(\mathbf{k})}{E(\mathbf{k}+\mathbf{q})-E(\mathbf{k})-\hbar\omega}\right]\delta n(\mathbf{r},\omega) = \sum_{\mathbf{k}} \frac{f(\mathbf{k}+\mathbf{q})-f(\mathbf{k})}{E(\mathbf{k}+\mathbf{q})-E(\mathbf{k})-\hbar\omega}\vartheta^{\mathrm{ext}}(\mathbf{q},\omega). \tag{27}$$

For the normal modes, we set $\vartheta^{\mathrm{ext}} = 0$, and thus require

$$\epsilon(\mathbf{q},\omega) \equiv 1 - e^2\nu(q)\sum_{\mathbf{k}} \frac{f(\mathbf{k}+\mathbf{q})-f(\mathbf{k})}{E(\mathbf{k}+\mathbf{q})-E(\mathbf{k})-\hbar\omega} = 0, \tag{28}$$

which is the SCF wavevector and frequency dependent dielectric function, also known as the Linhard function [22], explicitly written for any dimensionality. To get the plasmon frequency in the long-wavelength limit, we need then to take $q \to 0$, for fixed ω, in the sum of Eq. (28). We rewrite the sum appearing in Eq. (28) as

$$S(\mathbf{q}) \equiv \sum_{\mathbf{k}} 2f(\mathbf{k})\frac{E(\mathbf{k}+\mathbf{q})-E(\mathbf{k})}{[\hbar\omega]^2 - [E(\mathbf{k}+\mathbf{q})-E(\mathbf{k})]^2}, \tag{29}$$

where, using $E(\mathbf{k}) = \hbar^2 k^2/2m$, gives (for fixed ω)

$$\lim_{q\to 0} S(\mathbf{q}) = \frac{1}{m\omega^2} \sum_{\mathbf{k}} f(\mathbf{k})(2\mathbf{k}\cdot\mathbf{q} + q^2). \tag{30}$$

The first term is odd in $\mathbf{k}$ and thus vanishes, whereas the second term involves $\sum_{\mathbf{k}} f(\mathbf{k}) = n_0$, the electronic number density for the chosen dimensionality, that in virtue of Eqs. (30) and (28) gives

$$\epsilon(|\mathbf{q}| \to 0, \omega) = 1 - \frac{n_0 e^2}{m\omega^2} q^2 \nu(q). \tag{31}$$

Requiring $\epsilon(\mathbf{q}, \omega) = 0$ would give the same answer as the one obtained in Eq. (11) through the hydrodynamic model. This gives a more rigorous (microscopic) proof for the plasmon frequency, although the physics is not as transparent as before. In passing we remark that (31) is the usual long-wavelength limit of the dielectric function, now valid in 3D, 2D or 1D.

5. Conclusion

In this article we have derived the plasmon frequency for 3D, 2D and 1D systems in a unified way. We have done so by using a clear and simple hydrodynamic model which determines the dynamical behavior of the electrons. By the use of a more rigorous quantum approach we have checked that the hydrodynamic model gives the correct results in the long-wavelength limit. Also, we have given a heuristic picture which gives a simple physical picture for the different behavior (dispersion) of the plasmons.

Appendix A

In this appendix we would like to give a more precise definition of what we mean by a one dimensional system. We consider a very long system with a cross section being much smaller than the length. For instance, consider a parallepiped of length ℓ, width a and height b. Then $\ell \gg a, b$ and a and b could be of the same order of magnitude. Furthermore ℓ is also such that the energy quantization due to its large spatial confinement is continuous. However a and b are such that the energy quantization due to their confinement is experimentally discernible, or in other words the energy spacing is large. The electrons will occupy these energy states up to some maximum level, and we set $b < a$ in such a way that the electrons will always be in the ground energy level associated with b. This is possible if the excited states associated with the spatial confinement imposed by b are so far apart that they are inaccessible for the energies being used. In this way we are left with a system that looks like a flat strip of width a with energy levels well separated and experimentally accessible, and length ℓ with continuous energy. as we will see in the next appendix, a will have a minimum value that stems from physical grounds. It is in the

aforementioned fashion that we conceive our *one dimensional* system. We finally mention that this kind of systems are actually achieve by the new nano-fabrication techniques (see Refs. [1], [4] and [7]).

APPENDIX B

Here, we derive the Fourier transform of the Coulomb potential $\nu(q)$ for 3D, 2D and 1D. However, we start with the scalar potential in order to justify (7) first, and then we get $\nu(\mathbf{q})$. $\phi(\mathbf{r})$ is given by

$$\begin{aligned}\phi(\mathbf{r}) &= \int d\mathbf{r}' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \\ &= \sum_{\mathbf{q}} \rho(\mathbf{q}) \int d\mathbf{r}' \frac{e^{i\mathbf{q}\cdot\mathbf{r}'}}{|\mathbf{r}-\mathbf{r}'|},\end{aligned} \tag{B1}$$

(since we are only interested int he spatial variation, we use the time independent potential $\phi(\mathbf{r})$ and density $\rho(\mathbf{r})$. Taking $\mathbf{r}-\mathbf{r}' = \mathbf{R}$, gives

$$\phi(\mathbf{r}) = \sum_{\mathbf{q}} \rho(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{r}} \int d\mathbf{R} \frac{e^{-i\mathbf{q}\cdot\mathbf{R}}}{R}, \tag{B2}$$

from where we justify the spatial dependence of (7) and we identify

$$\nu(\mathbf{q}) = \int d\mathbf{R} \frac{e^{-i\mathbf{q}\cdot\mathbf{R}}}{R}, \tag{B3}$$

as the Fourier transform of the Coulomb potential, which of course imply

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \sum_{\mathbf{q}} \nu(\mathbf{q}) e^{i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')}. \tag{B4}$$

To proceed with the evaluation of $B3$, we take cartesian coordinates since as we mentioned in Appendix A, the one dimensional system we have in mind is a "flat strip". We mention, though, that one must proceed with care in order to avoid inconsistencies that could arise in other coordinate systems. We take $\mathbf{R} = (x, y, z)$ and $\mathbf{q} = (q_x, q_y, q_z)$, then in 3D

$$\nu(\mathbf{q}) = \int_{-\infty}^{\infty} dz\, e^{-iq_z z} \int_{-\infty}^{\infty} dy\, e^{-iq_y y} \int_{-\infty}^{\infty} dx \frac{e^{-iq_x x}}{\sqrt{x^2+y^2+z^2}}. \tag{P5}$$

From Ref. [23] (3.754.2)

$$\int_{-\infty}^{\infty} dx \frac{e^{-iq_x x}}{\sqrt{x^2+y^2+z^2}} = 2K_0\left(|q_x|\sqrt{y^2+z^2}\right). \tag{B6}$$

Integrating over y (see Ref. [23] (6.677.5))

$$2\int_{-\infty}^{\infty} dy\, e^{-iq_y y} K_0\left(|q_x|\sqrt{y^2+z^2}\right) = \frac{2\pi}{\sqrt{q_x^2+q_y^2}} \exp\left[-|z|\sqrt{q_x^2+q_y^2}\right]. \tag{B7}$$

Finally, integrating over z (see Ref. [23] (2.663.3))

$$\nu(\mathbf{q}) = \frac{2\pi}{\sqrt{q_x^2+q_y^2}} \int_{-\infty}^{\infty} dz\, e^{-iq_z z} \exp\left[-|z|\sqrt{q_x^2+q_y^2}\right] = \frac{4\pi}{q^2} \tag{B8}$$

or

$$\nu(\mathbf{q}) = \nu(q) = \frac{4\pi}{q^2} \quad \text{with} \quad q = \sqrt{q_x^2+q_y^2+q_z^2}, \tag{B9}$$

the usual result for 3D. For 2D we focus in ($B7$)'s integral realizing that $z = 0$, and also that no further integration over z is required, then (see Ref. [23] (6.671.14))

$$2\int_{-\infty}^{\infty} dy\, e^{-iq_y y} K_0(|yq_x|) = \frac{2\pi}{q}$$

and then

$$\nu(q) = \frac{2\pi}{q} \quad \text{with} \quad q = \sqrt{q_x^2+q_y^2}, \tag{B10}$$

the result for 2D. Finally for 1D we proceed as follows. We cannot allow strictly one dimension (*i.e.* $y = z = 0$) because the Coulomb integral ($B6$) will diverge, to circumvent this, we appeal to physical grounds. The electron gas must coexist with a positive background, thus, there should be a minimum distance, a_0, between the electrons and the positive charge. This minimum distance would be a limit to the "size" of the cross section of our 1D system. In a sense, this a_0 would be like a "Bohr radius" similar to that of the hydrogen atom. Therefore we need a system which extends a macroscopic distance, *e.g.*, along x, and has a microscopic (finite) cross section, which should be at least the a_0. Then, from ($B6$) we could identify $a \equiv \sqrt{y^2+z^2}$ as the cross section or width of the 1D system and write (in here the integration over y and z is no longer required)

$$\nu(q) = 2K_0(qa) \equiv V(q), \quad \text{with} \quad q = |q_x|. \tag{B11}$$

Notice that this choice of integration limits, correspond to a parallepiped, and thus ($B11$) is best suited for a flat strip of width a, in accordance with Appendix A. In general

other geometry would have a slightly different functional dependence with q, but would be calculated along the same lines that lead to $(B6)$.

From $(B9)$–$(B11)$ it's clear that q in 3D is the magnitude of the three dimensional wavevector, in 2D is the magnitude of the two dimensional wavevector and in 1D is the magnitude of the one dimensional wavevector.

APPENDIX C

In this appendix we derive the $q \to 0$ limit of $V(aq)$ given by $(B11)$. We start from $(B6)$, but written in a different way, to get

$$V(aq) = \lim_{L\to\infty} \int_{-L}^{L} dx \frac{e^{-iqx}}{\sqrt{x^2+y^2+z^2}}. \tag{C1}$$

Following the same argument that led to $(B11)$, we take $y^2 + z^2 = a^2$, and let $q \to 0$, to get

$$\begin{aligned} V(aq \to 0) &= \lim_{L\to\infty} \int_{-L}^{L} dx \frac{1}{\sqrt{x^2+a^2}} \\ &= \lim_{L\to\infty} \left\{ 2\log\left[L + \sqrt{L^2+a^2}\right] - 2\log[a] \right\} \\ &= 2\log\left[\frac{L}{a}\right]. \end{aligned} \tag{C2}$$

Now, we take $L \to 1/q$, which implies

$$V(aq \to 0) = -2\log[qa], \tag{C3}$$

result that gives the $q \to 0$ limit of the Fourier transform of the Coulomb potential in Q1D.

REFERENCES

1. We list several conference proceedings where both experimental and theoretical examples and many further references can be found: *Interfaces, Quantum Wells and Superlattices*, edited by C.R. Leavens and R. Taylor, Plenum, New York (1988); *Nanostructure Physics and Fabrication*, edited by M.A. Reed and W.P. Kirk, Academic, New York (1989); *Surf. Sci.* **228** (1990) and **229** (1990).
2. F. Stern, *Phys. Rev. Lett.* **18** (1967) 546.
3. J. Ruvalds, *Advances in Physics* **30** (1981) 677-695.
4. T. Aruga, H. Tochihara and Y. Murata , *Phys. Rev. Lett.* **53** (1984) 372.
5. T.N. Theis, *Surf. Sci.* **98** (1980), 515.
6. S.J. Allen, Jr, D.C. Tsui and R.A. Logan, , *Phys. Rev. Lett.* **38** (1977) 980.

7. A.R. Goñi, A. Pinczuk, J.S. Weiner, J.M. Calleja, B.S. Dennis, L.N. Pfeiller and K.W. West , *Phys. Rev. Lett.* **67** (1991) 3298.
8. B.S. Mendoza and Y.C. Lee, *Phys. Rev.* **B40** (1989) 12063.
9. B.S. Mendoza, and W.L. Schaich, *Phys. Rev.* **B43** (1991) 9275.
10. Y.C. Lee, S.E. Ulloa and P.S. Lee, *Jour. Phys. C: Solid State Phys.* **16** (1983) L995.
11. I.G. Currie, *Fundamental Mechanics of Fluids*, McGraw-Hill, New York (1974); M. del Castillo, W.L. Mochán y B.S. Mendoza, *Journal of Physics: Condensed Matter* (in press).
12. Like the 3D and 2D Coulomb potentials, the 1D counterpart also diverges as $q \to 0$, however this divergence being logarithmic is slower than in 3D and 2D. (See Appendix C).
13. To tell this mode apart from the 3D and 2D, we may want to say that in Q1D, the plasmon has a quasi-acoustic dispersion in the long wavelength limit, and that for $q \to 0, d\omega/dq$ has a logarithmic divergence.
14. Bernardo S. Mendoza and W.L. Schaich, *Phys. Rev.* **B43** (1991) 6590.
15. Y.C. Lee, B.S. Mendoza and S. Ulloa, *Supercond. Sci. Technol.* **1** (1989) 352.
16. N.W. Ashcroft and N.D. Mermin, *Solid State Physics*, Saunders College, Philadelphia (1976) p. 337.
17. D. Madelung, *Introduction to Solid State Physics*, Springer-Verlag, New York (1978) p. 114.
18. D. Pines, *Elementary Excitations in Solids*, W.A. Benjamin, New York (1964) p. 117.
19. C. Kittel, *Quantum Theory of Solids*, Wiley, New York (1963) p. 99.
20. H. Ehrenrich and M.H. Cohen, *Phys. Rev.* **15** (1959) 786.
21. R.F. Wallis and M. Balkanski, *Many-Body Aspects of Solid State Spectroscopy*, North-Holland, New York (1986).
22. This dielectric function is formally identical to the one obtained within the so called Random Phase Approximation (RPA) (See for instance Ref. [17]).
23. I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products*, Academic Press, New York (1980) [Eqs. (3.754.2), (6.677.5) and (2.663.3)].