Quantization as a result of matter-field interaction

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ABSTRACT. Possible stationary solutions in stochastic electrodynamics with nonrandom characteristic Fourier frequencies are studied; it is shown that the equations that describe them are just the Heisenberg equations of quantum theory. For these solutions, the response of the particle to the random field turns out to be linear. The zero point field with spectral energy density $\propto \omega^3$ is required to guarantee detailed energy balance and stability of the atomic system.

RESUMEN. Se estudian las posibles soluciones estacionarias dentro de la electrodinámica estocástica que tienen la propiedad de que sus frecuencias características de Fourier son no estocásticas. Se muestra que las ecuaciones que las describen son justamente las ecuaciones de Heisenberg de la mecánica cuántica. Para estas soluciones, la respuesta de la partícula al campo de radiación resulta lineal. Se muestra asimismo que es necesario el campo de punto cero con densidad espectral de energía $\propto \omega^3$ para garantizar la existencia de balance detallado de energía y la estabilidad atómica.

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Stochastic electrodynamics (SED) has been developed with the purpose of studying the possible effects of a real zeropoint radiation field (zpf) on matter, most particularly its possible relationship with quantum phenomena. A series of results have been obtained in SED along the years, which show good quantitative agreement with the quantum description [1,2],although the theory has also met with serious difficulties, notably when dealing with nonlinear forces [3]. More recently, a nonperturbative treatment of the action of the random field on the particle has been under development [4], which allows to treat systems subject to arbitrary nonlinear binding forces without the aforementioned difficulties. An appropriate formalism to deal with the SED system has been introduced, which leads to results that strongly suggest a close relationship with quantum mechanics.

In the present letter we report a further step along this nonperturbative approach to the stochastic problem. By studying the possible stationary solutions of the SED system that possess *nonrandom* characteristic frequencies, we are led to a set of algebraic equations which, in the radiationless approximation, coincide fully with Heisenberg's equations of motion expressed in matrix form; for such solutions the system is seen to respond linearly to the zpf and with well defined response amplitudes, whose scale is fixed by the size of the field fluctuations. Also, by considering the full equations instead of the radiationless approximation, the non-relativistic radiative corrections are obtained; in particular the Lamb shift, the lifetimes and the question of atomic stability can thus be analysed. In this letter we briefly discuss the latter possibility; the interested reader is invited to see a more detailed and complete treatment elsewhere [5,6].

Let us recall that the zeropoint field is a purely random electromagnetic field with energy $\frac{1}{2}\hbar\omega$ per normal mode; in the long-wavelength approximation the electric component can be represented in the form

$$\mathbf{E} = i\sum_{n,\sigma}\sqrt{\frac{\pi\hbar\omega_n}{V}}\,\hat{\boldsymbol{\epsilon}}_{n\sigma}a^0_{n\sigma}\exp(-i\omega_n t) + \text{c.c.}, \tag{1}$$

where V is the normalization volume and n,σ are the wavenumber and polarization indices. The $a^0_{n\sigma}$ are random variables that average to zero and have a covariance $\langle a^0_{n\sigma}a^{0*}_{n'\sigma'}\rangle = \delta_{nn'}\delta_{\sigma\sigma'}$. (For simplicity, we shall use a single greek letter instead of the double subindex: $a^0_{n\sigma} \to a^0_\lambda$.) To study the interaction of a particle with this field, one may start from the random Abraham-Lorentz equation (written in one dimension, for simplicity, and with $\tau = 2e^2/3mc^3$):

$$m\ddot{x} = m\tau\dddot{x} + F(x) + eE(t), \tag{2}$$

and look for stationary solutions of this equation. To show that these solutions, when they exist, do not depend on the initial conditions, one can write $x = x_s + x_t$ and use the mean-value theorem, $F(x_s + x_t) = F(x_s) + F'(x_s + \theta x_t)$, with $0 \leq \theta \leq 1$. If x_s is taken as a stationary solution of $m\ddot{x}_s = m\tau\dddot{x}_s + F(x_s) + eE(t)$, x_t must satisfy the equation $m\ddot{x}_t = m\tau\dddot{x}_t + F'(x_s + \theta x_t)x_t$. Since this equation describes a particle that radiates in absence of an external source to compensate for the dissipation, x_t goes to zero with time, whatever its initial value, and thus represents the transient part of x. Therefore, for long times x coincides with x_s, and is hence substantially independent of the initial conditions, since it is driven by the field. (A more detailed discussion can be seen in Ref. [5]).

To describe the system once it has reached such a state of stationary stochastic motion, we make a Fourier transformation of Eq. (2), with $x = \int_{-\infty}^{\infty}\tilde{z}(\omega)e^{-i\omega t}\,d\omega$ and $E = \int_{-\infty}^{\infty}\tilde{E}(\omega)a^0(\omega)e^{-i\omega t}\,d\omega$, to get

$$x = -e\int_{-\infty}^{\infty}\frac{a^0(\omega)\tilde{E}(\omega)e^{-i\omega t}}{\Delta(\omega)}\,d\omega, \tag{3}$$

where

$$\Delta(\omega) = m\omega^2 + \frac{\tilde{F}(\omega)}{\tilde{z}(\omega)} - im\tau\omega^3. \tag{4}$$

Now we introduce a fundamental assumption, namely that the dominant contributions to $x(t)$ in Eq. (3) come from the poles of the integrand, *i.e.*, from those frequencies for which $\Delta(\omega) = 0$. In the usual long-wavelength approximation the frequencies ω of interest are such that $|\tau\omega| \ll 1$, and the last term in Eq. (4) can thus be neglected. For bound systems, the equation $\Delta(\omega) = 0$ is satisfied only for certain frequencies (in general stochastic). A subindex, say α, is thus required to identify the stationary state being studied, and a second one, say β, to tag the Fourier component; thus we write more explictly $x_\alpha = \sum_\beta \tilde{z}_{\alpha\beta}(\omega)\exp(i\omega_{\alpha\beta}t) + \text{c.c.}$, and the condition $\Delta(\omega) = 0$ takes the form

$$-\frac{1}{m}\frac{\tilde{F}_{\alpha\beta}(\omega)}{\tilde{z}_{\alpha\beta}(\omega)} = \omega^2_{\alpha\beta}. \tag{5}$$

(In order that our results adjust to usual conventions of quantum mechanics we have written ω in the form $-\omega_{\alpha\beta}$, making use of the fact that $\omega_{\alpha\beta} = -\omega_{\beta\alpha}$, as is seen further on.)

Since $\tilde{F}_{\alpha\beta}$ and $\tilde{z}_{\alpha\beta}$ depend on the a^0's in the general case, the r.h.s. of Eq. (5) is in principle also a function of them. Here we introduce a second crucial requisite, namely, that the stationary motions of interest are those for which *the characteristic frequencies* $\omega_{\alpha\beta}$ *have nonrandom values.* For such motions the quotient $\tilde{F}_{\alpha\beta}/\tilde{z}_{\alpha\beta}$, and hence also Δ, must be independent of the a^0's. Then from Eq. (3) it follows that $\tilde{z}_{\alpha\beta}$ is linear in $a^0_{\alpha\beta}$, and from (5) so is $\tilde{F}_{\alpha\beta}$, so that

$$\tilde{z}_{\alpha\beta} = \tilde{x}_{\alpha\beta}a^0_{\alpha\beta}, \qquad \tilde{F}_{\alpha\beta} = \tilde{f}_{\alpha\beta}a^0_{\alpha\beta},$$

with $\tilde{x}_{\alpha\beta}$ and $\tilde{f}_{\alpha\beta}$ nonrandom coefficients which must satisfy Eq. (5), *i.e.*,

$$\tilde{f}_{\alpha\beta} = -m\omega^2_{\alpha\beta}\tilde{x}_{\alpha\beta}. \tag{6}$$

Now the problem is to investigate the conditions for such solutions to exist. Most remarkably, a general answer can be constructed; it is our purpose to sketch it here in a very concise form, inviting the interested reader to see a detailed discussion of it elsewhere [5].

Developing the external force in a Taylor series $F(x) = \sum c_n x^n$ and using Eq. (5), one can see that the Fourier coefficient $\tilde{F}_{\alpha\beta}$ of a typical term (*i.e.*, x^n) of the force contains products of n field amplitudes that must comply with the condition

$$a^0_{\alpha\lambda_1}a^0_{\lambda_1\lambda_2}\cdots a^0_{\lambda_n\beta} = a^0_{\alpha\beta}, \tag{7}$$

for the quotient of Eq. (5) to be independent of the a's. A detailed analysis of the consequences of this requirement leads to the result that each amplitude must be of the form $a^0_{\alpha\lambda} = \exp(i\varphi_{\alpha\lambda})$, with $\varphi_{\alpha\lambda} = \varphi_\alpha - \varphi_\lambda$, mod 2π, and φ_μ uniformly distributed over $(0, 2\pi)$. In these expressions, the indices λ_i can have any possible value. The amplitudes have then several properties (as $a^0_{\alpha\beta} = a^{0*}_{\beta\alpha}$ or $a^0_{\lambda\lambda} = 1$,) which together with Eq. (7) show that they are no longer the original amplitudes that describe the vacuum field in the absence of matter. Similarly, the allowed frequencies must comply with

$$\omega_{\alpha\beta} = \omega_{\alpha\lambda_1} + \omega_{\lambda_1\lambda_2} + \ldots + \omega_{\lambda_n\beta}, \tag{8}$$

and in particular, $\omega_{\alpha\alpha} = 0$ for any α, and $\omega_{\alpha\beta} = -\omega_{\beta\alpha}$. From this it follows that $\omega_{\alpha\beta}$ must be written as a difference of two terms:

$$\omega_{\alpha\beta} = \Omega_\alpha - \Omega_\beta, \tag{9}$$

with Ω_μ real numbers.

The above results combine to give $(\tilde{z}^n)_{\alpha\beta} = \sum \tilde{x}_{\alpha\lambda_1}\tilde{x}_{\lambda_1\lambda_2}\cdots\tilde{x}_{\lambda_n\beta}a^0_{\alpha\beta}$, and $(\tilde{z}^n)_{\alpha\beta}/\tilde{z}_{\alpha\beta}$ becomes indeed independent of the field amplitudes (the sum is performed over all allowed values of the intermediate indices):

$$\frac{(\tilde{z}^n)_{\alpha\beta}}{\tilde{z}_{\alpha\beta}} = \frac{\sum \tilde{x}_{\alpha\lambda_1}\tilde{x}_{\lambda_1\lambda_2}\cdots\tilde{x}_{\lambda_n\beta}}{\tilde{x}_{\alpha\beta}}. \tag{10}$$

This gives for the $\tilde{x}_\alpha(t)$ that correspond to stationary solutions to Eq. (5):

$$x_\alpha(t) = \sum_\beta \tilde{x}_{\alpha\beta}a^0_{\alpha\beta}\exp i\omega_{\alpha\beta}t + \text{c.c.}, \tag{11}$$

where the powers of $\tilde{x}$ must be constructed according to the multiplication rule for matrices [see Eq. (10)]. The time factor in the components of $x(t)$ can be associated either to $\tilde{x}_{\alpha\beta}$ or to $a^0_{\alpha\beta}$, by writing $\tilde{x}_{\alpha\beta}a^0_{\alpha\beta}\exp i\omega_{\alpha\beta}t$ either as $\tilde{x}_{\alpha\beta}(t)a^0_{\alpha\beta}$ (with $\tilde{x}_{\alpha\beta}(t) = \tilde{x}_{\alpha\beta}\exp i\omega_{\alpha\beta}t$) or as $\tilde{x}_{\alpha\beta}a_{\alpha\beta}(t)$ (with $a_{\alpha\beta}(t) = a^0_{\alpha\beta}\exp i\omega_{\alpha\beta}t$). In terms of the $\tilde{x}_{\alpha\beta}(t)$, which satisfy the equation $d^2\tilde{x}_{\alpha\beta}(t)/dt^2 = -\omega^2_{\alpha\beta}\tilde{x}_{\alpha\beta}(t)$, Eq. (6) becomes

$$m\frac{d^2\tilde{x}_{\alpha\beta}(t)}{dt^2} = \tilde{f}_{\alpha\beta}(t), \tag{12}$$

where $\tilde{f}_{\alpha\beta}(t) = \tilde{f}_{\alpha\beta}\exp(i\omega_{\alpha\beta}t)$. Note that the stochastic amplitudes $a^0_{\alpha\beta}$ are entirely absent from the equations of motion (6) or (12); these equations have acquired an algebraic structure that can be expressed also in matrix form:

$$m\frac{d^2\tilde{x}}{dt^2} = \tilde{f}(\tilde{x}), \tag{13}$$

where the matrices $\tilde{x}$ and $\tilde{f}$ have elements $\tilde{x}_{\alpha\beta}$ and $f_{\alpha\beta}(\tilde{x})$, respectively [see Eq. (6]. To complete the description we use $p = m(dx/dt)$; then $\tilde{p}_{\alpha\beta} = im\omega_{\alpha\beta}\tilde{x}_{\alpha\beta}$, or in matrix notation and using Eq. (13):

$$\tilde{p} = m\frac{d\tilde{x}}{dt}, \qquad \frac{d\tilde{p}}{dt} = \tilde{f}(\tilde{x}). \tag{14}$$

These are evidently the Heisenberg equations of motion, and $\tilde{x}_{\alpha\beta}(t)$ the elementary oscillators of matrix mechanics. To recapitulate, we list the assumptions that must hold for these solutions to exist: a) the stationary part of the solution can be approximated by a resonant-type response to certain field modes, that is dominated by the poles at the characteristic frequencies; b) such characteristic frequencies are nonrandom and real (*i.e.*, the motions are radiationless), and c) the rest of the random field has no appreciable effect on the motion. From this it follows that the field that sustains the oscillations is no more the free vacuum field, but a modified field whose active modes (*i.e.*, those with which the particle resonates) are correlated [see Eq. (7)]. Under these assumptions we conclude that the matrix algebra of quantum mechanics follows as the algebra that guarantees nonrandom values for the characteristic frequencies of the stationary SED system; we say then that the system has reached the *quantum regime.* Note that the stationary solutions are independent of the initial data of any given particle, up to an "initial" reference time (*i.e.*, up to the phase with which the trajectory is percorsed); thus, each trajectory describes the set of all those particles that enter into the given state of motion, irrespectively of their initial conditions, a situation most naturally amenable to a statistical description. The possible identification of these stationary solutions with limit cycles is considered elsewhere [6].

An astonishing feature of this solution is its dependence on the field amplitudes: it shows that the mechanical system, *whatever the nonlinearities of the external forces*, responds linearly to the field, without mingling the frequencies of the different field modes. Such response is effective once the system has reached the quantum regime; before this, the dynamics is surely much more complicated and the present description does not apply. Within the quantum regime, the mechanical system behaves as a set of independent harmonic oscillators, something which was already well known since Heisenberg's times; the difference is that we are now disclosing the nature of these oscillators: they are mechanical modes resonantly driven by the modes of the field. As to the properties of the amplitudes of the active field modes, the unexpected relations given by Eq. (7) seem to indicate that in the process leading to the quantum regime, also the field has been modified, so that correlations appear among the amplitudes whereas initially randomness was dominant.

The solution is still incomplete, since the equations of motion (14) do not fully define the problem: the scale of the $\tilde{x}_{\alpha\beta}$ is not yet fixed. This is due to the fact that up to now only the poles of Eq. (3) were investigated; the rest of the information contained in it —and in particular, the strength of the random field, Eq. (1)— has been left aside. There are several possible ways to fix the scale of the response. The simplest one is of course to demand that the average energy of a harmonic oscillator be given by $\hbar\omega/2$, which is the value obtained from solving directly Eq. (2) for this specific case, as is well known [1–4]. A much more general result can be obtained by resorting to the *poissonian formalism* derived earlier [4]. As a result of this formalism, in the quantum regime the poissonian of x and p, given by

$$\langle x; p\rangle_\alpha \equiv \sum_\beta \left[\frac{\partial x}{\partial a_\beta}\frac{\partial p}{\partial a_\beta^*} - \frac{\partial p}{\partial a_\beta}\frac{\partial x}{\partial a_\beta^*}\right], \tag{15}$$

has a universal value for any stationary state α:

$$\langle x; p\rangle = i\hbar. \tag{16}$$

Inserting here the linear solution Eq. (11) and $p_\alpha = m(dx_\alpha/dt)$, this equation leads to

$$(\tilde{x}\tilde{p} - \tilde{p}\tilde{x})_{\alpha\alpha} = i\hbar, \tag{17}$$

and hence to the Thomas-Reiche-Kuhn sum rule

$$-\sum_\beta \omega_{\alpha\beta}|\tilde{x}_{\alpha\beta}|^2 = \frac{\hbar}{2m}. \tag{18}$$

This expression, which is equivalent to the quantization rule $[\hat{x},\hat{p}]=i\hbar$, fixes the scale of the solutions as a consequence of the energy $\hbar\omega/2$ per normal mode of the vacuum field. Also, from the poissonian counterpart of the dynamical Eqs. (14): $i\hbar\dot{x}=\langle x;H\rangle$, $i\hbar\dot{p}=\langle p;H\rangle$ with $H=p^2/2m+V$, and using Eqs. (9) and (11), it follows that $\hbar\omega_{\alpha\beta}=\mathcal{E}_\alpha-\mathcal{E}_\beta$, where $H_{\mu\nu}=\mathcal{E}_\mu\delta_{\mu\nu}=\hbar\Omega_\mu\delta_{\mu\nu}$. The characteristic frequencies of the present theory are thus related to the energy eigenvalues of the quantum stationary states through Bohr's formula.

Another more interesting matrix form of the equations of motion can be constructed by following a different path: Consider a set of square matrices $\hat{a}^{\alpha\beta}$ with elements

$$\left(\hat{a}^{\alpha\beta}\right)_{\mu\nu}=a_{\alpha\beta}(t)\delta_{\alpha\mu}\delta_{\beta\nu}. \tag{19}$$

Each of these matrices has only a single element different from zero, given by

$$\left(\hat{a}^{\alpha\beta}\right)_{\alpha\beta}=a_{\alpha\beta}(t)=\exp[i(\varphi_\alpha-\varphi_\beta)+i\omega_{\alpha\beta}t]. \tag{20}$$

For $\alpha\neq\beta$ this element is random, whereas for $\alpha=\beta$ it is equal to 1. It follows from their definition that a product of such matrices gives another one:

$$\left(\hat{a}^{\alpha\beta}\hat{a}^{\gamma\delta}\right)_{\mu\nu}=a_{\alpha\delta}\delta_{\alpha\mu}\delta_{\beta\gamma}\delta_{\delta\nu}=\delta_{\beta\gamma}\left(\hat{a}^{\alpha\delta}\right)_{\mu\nu}$$

Let us use these matrices to represent the dynamical variables once the system has reached the quantum regime [4], *i.e.*, the quantum observables. This is achieved by writing for example (this is the $\hat{a}$–representation),

$$\hat{x}=\sum_{\alpha,\beta}\tilde{x}_{\alpha\beta}\hat{a}^{\alpha\beta}=\sum_\alpha\hat{x}_\alpha, \tag{21}$$

where

$$\hat{x}_\alpha=\sum_\beta\tilde{x}_{\alpha\beta}\hat{a}^{\alpha\beta}. \tag{22}$$

Now it is straightforward to verify that the observables so constructed satisfy the rules of quantum mechanics. As the simplest example, consider the square of $\hat{x}$:

$$\left(\hat{x}^2\right)_{\mu\nu}=\sum_\lambda\tilde{x}_{\mu\lambda}\tilde{x}_{\lambda\nu}a_{\mu\lambda}a_{\lambda\nu}=\left(\tilde{x}^2\right)_{\mu\nu}a_{\mu\nu},$$

where $(\tilde{x}^2)_{\mu\nu}=\sum_\lambda\tilde{x}_{\mu\lambda}\tilde{x}_{\lambda\nu}$.

Further, note that the matrix $\hat{a}^{\alpha\beta}$ can be written as the product of two vectors $|\alpha\rangle$ and $\langle\beta|$, where

$$|\alpha\rangle = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ a_\alpha \\ \vdots \end{pmatrix} \tag{23}$$

and $\langle\beta| = |\beta\rangle^\dagger$. Each vector has a single element different from zero, $(|\alpha\rangle)_\lambda = a_\alpha \delta_{\alpha\lambda}$, where

$$a_\alpha = \exp i(\varphi_\alpha + \Omega_\alpha t), \tag{24}$$

which means that

$$a_\alpha a_\beta^* = a_{\alpha\beta}.$$

Therefore, from Eqs. (19) and (23) we have indeed

$$\hat{a}^{\alpha\beta} = |\alpha\rangle\langle\beta|. \tag{25}$$

As seen from Eq. (24), the vectors $|\alpha\rangle$ (which form a complete basis in the Hilbert space of the stationary states, as is easily verified [5]) involve the eigenvalues $\mathcal{E}_\alpha = \hbar\Omega_\alpha$ rather than the $\omega_{\alpha\beta}$; hence, in the transition from the $\hat{a}$–representation to the Hilbert space formulation the accent is shifted from the characteristic frequencies to the energy eigenvalues.

Now any observable $\hat{g}$ can be written as a linear combination of the $\hat{a}^{\alpha\beta}$ [see Eq. (21)]:

$$\hat{g} = \sum_\alpha \hat{g}_\alpha = \sum_{\alpha\beta} \tilde{g}_{\alpha\beta}\hat{a}^{\alpha\beta} = \sum_{\alpha\beta} \tilde{g}_{\alpha\beta}|\alpha\rangle\langle\beta|, \tag{26}$$

whence $\tilde{g}_{\alpha\beta} = \langle\alpha|\hat{f}|\beta\rangle$. In this new representation the basic Eqs. (14) and (17) take the form

$$[\hat{x}, \hat{p}] = i\hbar\hat{1},$$

$$\frac{d\hat{x}}{dt} = \frac{\hat{p}}{m} = \frac{1}{i\hbar}[\hat{x}, H], \qquad \frac{d\hat{p}}{dt} = \hat{f} = \frac{1}{i\hbar}[\hat{p}, H].$$

Observe that all quantities involved in this system of equations depend linearly on the random (c- or q-numbers) $a_{\alpha\beta}$, in contrast to the usual quantum equations where these quantities do not appear; they can therefore be considered hidden variables from the point of view of quantum mechanics.

Now since the stationary solutions of quantum mechanics have been obtained after neglecting the radiative terms in the equations of motion (see Eqs. (5) and the following), it is important to reintroduce these terms and analyze their possible effect on the stability of the quantum solutions. For this purpose, consider a bound particle (*e.g.* an atomic electron) subject to the combined action of the random field and its own radiation (in addition to the Coulomb force). The net statistical effect of these forces on the energy balance is obtained by multiplying Eq. (2) by $\dot{\mathbf{x}}$ and averaging over the realizations of the field. Since in equilibrium $\langle \dot{H} \rangle = 0$, one gets to second order in e:

$$m\tau\left\langle \dot{\mathbf{x}}^{(0)} \cdot \dddot{\mathbf{x}}^{(0)} \right\rangle + e\left\langle \dot{\mathbf{x}}^{(1)} \cdot \mathbf{E} \right\rangle = 0, \tag{27}$$

where the superinidices indicate the order in e of the various terms. For the calculation of $\dot{\mathbf{x}}^{(1)}$ one can go back to Eq. (2) and solve it by perturbation around the unperturbed solutions of the stationary states; one thus obtains (the index i is a cartesian index) [6]

$$\dot{x}^{(1)}_{\alpha i} = -\frac{2e}{\hbar} \sum_{\beta} \left| \tilde{x}^{i}_{\alpha\beta} \right|^{2} \int_{0}^{t} E_i(t-t') \sin \omega_{\alpha\beta} t'. \tag{28}$$

This gives for Eq. (27):

$$\sum_{\beta} \left(m\tau\omega^{4}_{\alpha\beta} - \frac{4\pi^2 e^2}{3\hbar} \omega_{\alpha\beta} \rho(\omega_{\alpha\beta}) \right) \left| \tilde{x}^{i}_{\alpha\beta} \right|^{2} = 0, \tag{29}$$

where $\rho(\omega)$ is the spectral energy density of the field $\mathbf{E}$. For the vacuum field of SED, $\rho = \hbar\omega^3/2\pi^2 c^3$ and the expression within parentheses vanishes identically for every $\omega_{\alpha\beta} > 0$, independently of the dynamics of the system, *i.e.*, of the values of the $\tilde{x}_{\alpha\beta}$. Hence, at each frequency interval there is a balance between the average power absorbed by the particle from the zeropoint field and the average power lost by radiation reaction; in other words, the system in its ground state is in detailed equilibrium with the radiation field. Assuming that the average taken over the realizations of the field is equal to the time average for an individual system, we conclude that every individual system (an atom, for example) is stable against the combined effect of radiation reaction and of the vacuum field force. It is, therefore, the average effect of the zeropoint field what prevents the atomic electron from falling towards the nucleus. This is a characteristic behaviour of quantum systems that clearly distinguishes them from classical systems, which attain equilibrium only with the Rayleigh-Jeans distribution, as is well known.

This tight relationship that is seen to exist between the average energy rate delivered by the zeropoint field and the average rate of energy radiated by the particle, can be expressed in more general terms as a relationship between a diffusion coefficient and a friction coefficient, *i.e.*, as a fluctuation-dissipation relation [7] specific of SED (see Ref. [6]).

Finally, we would like to mention that by considering the radiative terms as perturbations to the system described previously in the radiationless approximation, it is also possible to obtain the radiative corrections to the observables of the stationary states, *e.g.* to the energy. The calculations involved are rather lengthy, and are therefore reported elsewhere [6]. This work was supported in part by CONACyT through project 068–E9109.

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