

The role of dissipation in quantum Hall voltage profiles

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ABSTRACT. The effects of temperature and Ohmic conduction on the field configurations in IQHE samples are investigated. The dissipative effects are identified as the cause of the Hall currents to flow through the bulk of the samples in the experiments. Two regimes for the Hall effect observation are traced out: the first one works at time intervals shorter than the relaxation time and is characterized by Hall currents circulating near the boundary. The second one is related to the stationary state in the presence of dissipation and is characterized mainly by bulk currents. An approximate analytical solution of the equations describing the steady state is found. This satisfactorily describes the oscillations of the experimental Hall voltage with the magnetic field for interior sample points and leads to the quantization of the Hall resistance to one part in 10^5 or better.

RESUMEN. Se investigan los efectos de la temperatura y la conducción óhmica en las configuraciones de campo en el efecto Hall cuántico entero. Los efectos disipativos se identifican como las causas de que las corrientes de Hall fluyan a través del volumen en las muestras experimentales. Se identifican dos regímenes para la observación del efecto Hall cuántico: el primero trabaja en intervalos de tiempo menores que el tiempo de relajación y está caracterizado por corrientes de Hall circulando cerca de la frontera; el segundo se relaciona con el estado estacionario en presencia de disipación y se caracteriza básicamente por corrientes volumétricas. Se obtiene una solución analítica aproximada de las ecuaciones que describen el estado estacionario; ella describe satisfactoriamente las oscilaciones del voltaje de Hall obtenido experimentalmente con la variación del campo magnético en puntos interiores de la muestra y produce una cuantificación de la resistencia de Hall en una parte en 10^5 o mejor.

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Although the integer quantum Hall effect (IQHE) is essentially well understood [1], certain aspects of this phenomenon are still under discussion. A particularly interesting problem is concerned with the current and voltage distributions inside a quantum Hall device. While semiclassical [2,3,4] and percolation [5] arguments lead to the propagation of currents in the bulk of the two dimensional system, the dominance of edge currents is stressed in other approaches [6,7,8].

On the other hand, the experiments [9,10,11] show currents that in the regions between plateaus are uniformly distributed across the sample, and that inside plateaus flow mainly through a zone whose spatial position is determined by the value of the applied magnetic field. This zone may be thought as the path of minimal resistivity along the sample (which should exist because, as a result of the two-dimensionality, the carrier density is

inhomogeneously distributed across the sample). Consequently, we are led to the idea that, in spite of the smallness of the Ohmic resistance in plateau regions, dissipative effects may play a major role in determining current and voltage distributions in IQHE devices. Another characteristic feature of the experiments which also underlines the role of dissipation is the existence of a relaxation time for the attainment of the steady conductive state. In the measurements reported by Zheng *et al.* [9], Ebert *et al.* [10], and Sample and Salerno [11] this time is typically a few minutes, and grows as the temperature is lowered.

Theoretical calculations of spatial field distributions have, up to now, ignored dissipation inside plateaus [12,13,14] and, consequently, can not reproduce the above mentioned experimental results. We aim at presenting in this paper a semi quantitative analysis—based on effective Maxwell equations, constitutive equations and Fermi distributions of extended electron states at finite temperatures— which explicitly models the local resistivity of the system and its effects on the Hall voltage profiles.

We start by considering an idealized Hall arrangement at approximately 4.2 K, in which the electrons are forced to move in a strip of width L of the xy plane. The current flows in the positive y direction, and the magnetic field B is oriented along the z axis.

In the steady state and neglecting diamagnetic effects, the Maxwell equations governing the electric field distribution in the strip are the following:

$$0 = (\text{curl } E)_z = \partial_x E_y, \quad (1)$$

$$\phi(x) - \phi(0) = eL \int_{-1}^1 dt \Delta n \ln |x/t - 1|. \quad (2)$$

In the first equation, we explicitly use the condition $\partial_y E_x = 0$, coming from the symmetry of the sample geometry, while in the integral Poisson equation (2), we take the difference of potentials at $y = \text{const.}$, and $\Delta n(x)$ is the electron excess at point x .

To Eqs. (1) and (2) we must add the constitutive equations

$$E_y = \rho(x)J(x), \quad (3)$$

$$E_x = \rho_H(x)J(x), \quad (4)$$

which take care of Ohmic dissipation and momentum conservation in the x direction respectively. Combining (1), (3) and (4) we may obtain another integral equation for the difference $\phi(x) - \phi(0)$, which properly stresses that this difference arises as a result of the process of conduction:

$$\phi(x) - \phi(0) = -I \frac{\int_0^x dx [\rho_H(t)/\rho(t)]}{\int_{-1}^1 dt/\rho(t)}. \quad (5)$$

Some qualitative properties follow directly from the system of Eqs. (1)–(5) (for example, the dependence $J \propto 1/\rho(x)$, which arises from Eq. (3) and the constancy of E_y), but to proceed further we must specify the properties of the 2D system, *i.e.*, the magnitudes Δn ,

ρ and ρ_H . A very simplified, but essentially correct, picture may be obtained by extending the analysis of Ingraham and Wilkes [15] to include the inhomogeneity in the spatial distribution of electrons. The number of extended electron states in a magnetic field, at inverse temperature β , and at point x of the sample is written as

$$n(x) = \frac{eB}{hc} \nu(x) = \frac{eB}{hc} \sum_{i=0}^{\infty} \left[1 + \exp \left\{ \beta \hbar \omega_c \left(i + \frac{1}{2} \right) - \beta \mu(x) \right\} \right]^{-1}, \quad (6)$$

where h is the Plank constant, $\omega_c = eB/(m^*c)$ is the cyclotronic frequency, and by $\mu(x)$ we understand an effective position-dependent chemical potential, which takes into account the difference between the energy of the electrons in the selfconsistent electric and magnetic fields and the electrochemical potential, as well as the variation of the density of states due to dielectric effects.

Δn , ρ and ρ_H are expressed in terms of n in the following way [15]:

$$\Delta n = \frac{eB}{hc} [\nu(x) - \nu_B], \quad (7)$$

$$\rho_H = \frac{h}{e^2} \frac{1}{\nu(x)}, \quad (8)$$

$$\rho = \frac{h}{e^2} \frac{1}{\omega_c \tau} \sum_i \frac{\nu_i (1 - \nu_i)}{\nu^2}. \quad (9)$$

Here, $\nu_B = \nu|_{\mu=\mu_0}$ is the background density, which is supposed to be independent of x , but dependent on the magnetic field in order to model the contribution of localized states. Note that in Eq. (7) the number of available electron states is not constrained. This leads to a plateau width controlled only by the temperature. On the other hand, we will make use of the hypothesis of charge neutrality, $\int_{-1}^1 dx \Delta n(x) = 0$, neglecting in this way any externally induced charge or any possible accumulation of charge as a result of the establishment of the steady conduction state. The expression for ρ is almost self-evident. The Blocking of electrons in the i -th Landau level, *i.e.*, the factors $1 - \nu_i$ prevents this state to contribute to ρ when it is entirely filled.

Once all magnitudes in (1)–(5) are well defined, we may solve the system of equations. By equating $\phi(x) - \phi(0)$ in (2) and (5) we obtain an integral equation for $\mu(x)$. An approximate analytical solution of this equation can be obtained by expanding in $\Delta\mu = \mu - \mu_0$, and solving in the linear approximation. $\beta \Delta\mu$ shall obey the equation

$$x = -\nu_B \frac{\partial \nu_B}{\partial (\beta \mu_0)} \frac{2e^4 BL}{I h^2 c} \int_{-1}^1 dt \beta \Delta\mu \ln |x/t - 1|, \quad (10)$$

i.e., we obtain the following solution:

$$\beta \mu = \beta \mu_0 + \frac{\xi x}{(1 - x^2)^{1/2}}, \quad (11)$$

with $\xi = Ih^2c[2\pi e^4 BL\nu_B\partial\nu_B/\partial(\beta\mu_0)]^{-1}$. We shall use a cutoff for the law $(1-x^2)^{-1/2}$ near the edges because the condition of validity of the linear approximation is violated in this region.

The physical picture emerging from the solution (11) is the following.

In the transition regions between plateaus the coefficient ξ is very small, the μ is essentially constant, *i.e.*, independent of x . This leads [through Eq. (9)] to a (high) resistivity that is also almost independent of x . Consequently, the Ohm law holds and the voltage drops are uniformly distributed.

On the other hand, inside plateaus the coefficient ξ grows, showing a maximum (*i.e.*, a minimum of the resistivity) at a certain value of the field, B^* . To be definite, let us consider the situation in which the first $i+1$ Landau levels are filled. Then at the value B of the magnetic field, the point of the sample in which the resistivity is minimal is determined by the equation

$$\beta\hbar\omega_c\left(i + \frac{1}{2}\right) - \beta\mu(x) = \beta\hbar\omega_c^*\left(i + \frac{1}{2}\right) - \beta\mu_0, \quad (12)$$

where ω_c^* corresponds to the field strength B^* . Actually, the current will flow through a region in the vicinity of x in which the resistivity remains small, thus guaranteeing the quantization of the Hall resistance. When $B = B^*$, the current flows mainly through the center of the strip but, as the slope of the curve $\mu(x)$ is relatively small at $x = 0$, the region of minimal resistivity in fact extends to the whole sample. When B is varied around B^* , we should observe a displacement of the current from one side of the strip to the other, as it actually takes place in the experiments [9,10,11].

In Figs. 1 and 2 we show the Hall voltage drops calculated by inserting the approximate solution (12) into Eq. (5). The curves represent the voltage in interior points (distributed homogeneously across the sample) with one of the edges taken as ground. In Fig. 1 the voltages are calculated taking the left edge as reference, while in Fig. 2 the right edge is taken as ground (it would be equivalent to reverse the current or the magnetic field). The values of the parameters employed in the calculation are typical. That is $T = 4.2$ K, $I = 10$ μ A, $m^* = .08 m_e$, $\beta\mu_0 = 50$ (which corresponds to $\mu = 3.27 \times 10^{-14}$ ergs or to a density of 3.33×10^{11} electrons per cm^2) and $L = 2$ mm. We see that this simple calculation reproduces the main features of the experimental curves [9,10,11] *i.e.*, linear shape in the interplateaus regions, quantization of the Hall resistance (to better than one part in 10^5), and oscillations near the centers of the plateaus.

Corrections to the linear approximation should not change the qualitative behavior of the dependence $\mu(x)$ given by Eq. (12). We expect, for example, a finite μ in the whole interval, some asymmetry with respect to the center of the strip (in order to preserve charge neutrality), etc. Another correction comes from the fact that the number of available electron states is bounded. This fact puts a bound on the plateau width and leads to a sharper agglomeration of the potential curves around the value B^* .

At this point, we want to stress that the calculations by MacDonald *et al.* [12], Heinonen and Taylor [13], and Cabo *et al.* [14] are performed by setting $\rho = 0$ and leaving only the electrostatic equation for the determination of the field distribution. It means that we are working at time intervals less than the relaxation time for the attainment of the conduction state (which is taken to be infinite, *i.e.*, proportional to the inverse of the

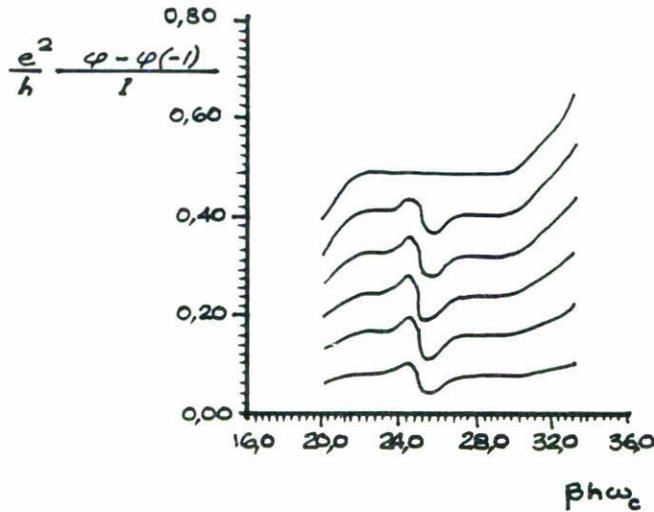


FIGURE 1. Hall voltages in equidistant interior sample points as a function of the applied magnetic field. The region corresponding to the $\nu = 2$ plateau is shown. The left edge of the sample is taken as ground.

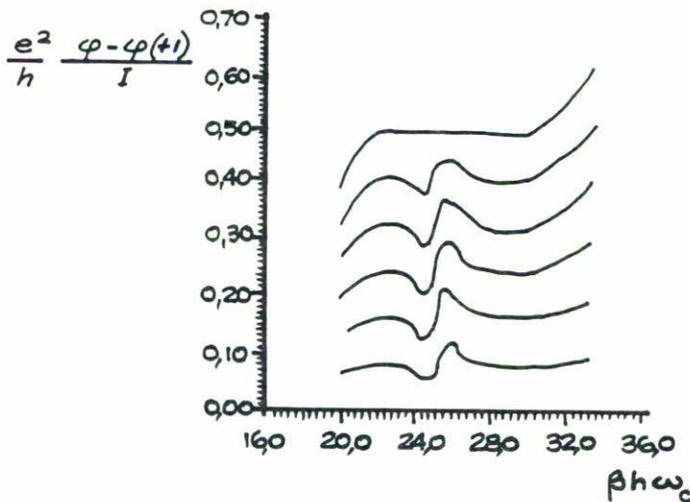


FIGURE 2. The same as in Fig. 1, but with the right edge taken as reference.

conductivity). As it follows from these papers, this regime shows currents flowing near the boundaries. Thus, we are led to a two-regime picture of the Hall effect observation: flowing near the boundary Hall and edge currents at time intervals shorter than the relaxation time, and mainly bulk (or filamentary) currents in the steady conductive state. It seems that this picture is partially confirmed by recent optical measurements of voltage distributions in heterostructures under QHE conditions [16]. The complete explanation of these experimental results and their relation with the measurements by means of electric

contacts deserves, however, a further analysis in connection with Buttiker's picture of the "zero-temperature" (edge current) regime [17].

In conclusion, we have identified the dissipative effects as the cause of the Hall current to flow through the bulk of the sample in the experiments reported by Zheng *et al.* [9], Ebert *et al.* [10], and Sample and Salerno [11]. The oscillating behavior of the Hall voltage in interior points as a function of the magnetic field is also qualitatively explained, and is found to be related to the displacement of the zone of minimal resistivity across the sample. The quantization of the Hall resistance is guaranteed to the extent in which the resistivity of this zone is near zero. The discussion seems to give a spatial description of the bulk currents defined within the model of van Son de Vries and Klapwijk [18].

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