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Polarization squeezing, biphotons and new non-classical states of unpolarized light

V.P. Karassiov* and M. Cervantes
Centro de Investigación en Física, Universidad de Sonora
Apartado postal 5-088, 83190 Hermosillo, Son., México

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Abstract. The concept of squeezing is discussed for multimode quantum light with the consideration of polarization by using the polarization gauge SU(2) invariance of free electromagnetic fields, the related concept of polarization quasispin (P) and appropriate uncertainty relations. As consequence, we obtain within quantum optics new non-classical states of unpolarized light generated by specific two-photon excitations (unpolarized biphotons).

Resumen. Discutimos el concepto de compresión (squeezing) para el caso de luz cuántica multimodal con respecto a su estado de polarización mediante el uso de la invarianza de norma SU(2) de polarización de los campos electromagnéticos libres. Asimismo, los conceptos relacionados de cuasi-espín (P) y las relaciones de incertidumbre apropiadas. Como resultado obtenemos nuevos estados no clásicos de la luz no polarizada generados por excitaciones específicas de dos fotones (*i.e.*, bifotones no polarizados).

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1. Introduction

For the last several decades, polarization properties of light were widely investigated in both theoretical and applied aspects (see, *e.g.*, Refs. [1–14] and references therein). Specifically, some fundamental problems of quantum mechanics, related to "hidden" variables, Bell's inequalities and Einstein-Podolsky-Rosen (EPR) paradox, quantum chaos, different topological phases etc., are intensively examined with the help of quantum polarization optics (see, *e.g.*, Refs. [1,2,5,6,12–14] and references therein).

However, as a rule, the polarization structure of light has ben described in terms of the field correlation functions, associated Stokes parameters and the Poincaré sphere which are well adapted to classical optics experiments [3–5,13–14] but are not quite adequate to specific quantum ones (photon counting) [3]. Such a description also ignores a polarization SU(2) symmetry [14-19] of light fields though it has been widely used implicitly —thorough the Stokes parameters s_a which determine, in particular, the polarization degree $\deg P = [s_1^2 + s_2^2 + s_3^2]^{1/2}/s_0$ of monochromatic plane wave light beams [3,4,7,20]. Furthermore, the physical meaning of the Stokes parameters and their connections with

*On leave of abscence from: Lebedev Physical Institute, Leninsky prospect 53, Moscow 117924, Russia. Internet: karas@sci.fian.msk.su.

the spin properties of light fields are sufficiently studied only for plane wave light beams [3-6] although in Ref. [7] some generalized Stokes parameters were introduced for examining light beams with arbitrary wave fronts within classical statistical optics.

Recently, however, a new formalism [16–19] was proposed for a description of polarization structure of multimode quantum light fields using the polarization SU(2) symmetry and a related concept of the P–quasispin which generalized the Stokes vector notion at the quantum level and is closely related to the Stokes operators defined in Ref. [20]. This approach enabled us to gain a new insight into the polarization structure of light and quantum mechanisms of its depolarization.

At the same time, at present, so-called squeezed states of light are intensively examined within quantum optics by many investigators (see, *e.g.*, Refs. [21–25] and references therein) since these states have attractive properties of the "noise reduction" in measurements of some quantum mechanical observables that provides certain prospects of their applications, particularly, in optical communication theory, in precise and non-demolition measurements, etc. However, we note that squeezed states are sufficiently studied only for the single-mode fields [21–25] whereas for multimode fields it is not the case since even the definition of the concept of multimode squeezing is not unique that is due to a variety of the choice of measurable quantities [26,27].

The aim of this paper is to give an analysis of the concept of squeezing of the multimode light related to polarization degrees of freedom by using the above mentioned formalism of P–quasispin. Specifically, we will show (Sects. 2 and 3) that there exist new quantum states of light beams exhibiting, in a sense, an absolute squeezing in polarization degrees of freedom. Such states are generated by specific unpolarized biphoton clusters and have all characteristics of usual unpolarized light, but unlike the latter one new quantum states of unpolarized light are "polarizationally noisless" [16-19,28]. Besides we discuss briefly some generalizations and applications of new non-classical states of light (Sect. 4). Preliminary results of the work were reported by one of us (V.P.K.) at the Third International Workshop on Squeezed States and Uncertainty Relations (Baltimore, August 10–13, 1993).

2. POLARIZATION SU(2) INVARIANCE AN P–SPIN OF ELECTROMAGNETIC FIELDS; UNPOLARIZED BIPHOTONS

In quantum optics the free transverse electromagnetic field with m spatiotemporal modes described by the vector potential [1,3,19,20]

$$\begin{aligned}\mathbf{A}(\mathbf{r},t) &= c\sum_{j=1}^{m}\left(\frac{2\pi\hbar}{\omega_j V}\right)^{1/2}\sum_{\alpha=+,-,3}\left\{\mathbf{e}_\alpha(j)a_\alpha(j)\exp[i(\mathbf{k}_j\mathbf{r}-\omega_j t)]+\text{h.c.}\right\}\\ &= \mathbf{A}^{(-)}(\mathbf{r},t)+\mathbf{A}^{(+)}(\mathbf{r},t),\quad \mathbf{A}^{(+)}=\left(\mathbf{A}^{(-)}\right)^+,\end{aligned} \tag{2.1}$$

where $a_\alpha(j)$ $(a_\alpha^+(j))$ are destruction (creation) operators for j–th spatiotemporal and α–th polarization modes of the field, $\mathbf{e}_\alpha(j)$ are the polarization unit vectors adapted to the helicity basis, $\mathbf{e}_3(j) = \mathbf{k}_j/\omega_j$, V is a quantization volume, etc. With the help of Eq. (2.1) one determines correlations tensors [3]

$$G^{(s,p)}_{i_1\ldots i_s j_1\ldots j_p}\big(\{\mathbf{r}_a, t_a\}; \{\mathbf{r}'_b, t'_b\}\big) = \operatorname{tr}\big[\rho E^{(-)}_{i_1}(\mathbf{r}_1, t_1)\cdots E^{(-)}_{i_s}(\mathbf{r}_s, t_s) E^{(+)}_{j_1}(\mathbf{r}'_1, t'_1)\cdots E^{(+)}_{j_p}(\mathbf{r}'_p, t'_p)\big],$$

$$\mathbf{E}^{(\pm)} = \frac{1}{c}\frac{\partial \mathbf{A}^{(\pm)}}{\partial t}, \tag{2.2}$$

which correspond to different physical quantities, measurable in optical experiments, and are expressed in terms of quantum expectations of ordered polynomials in operators $a_\alpha(j)$ and $a^+_\alpha(j)$ [12]. The most important of such measurable quantities is the field Hamiltonian H_{f} which determines the time-evolution of other field observables [3].

The starting point of our next analysis is the evident invariance of standard expressions

$$H_{\mathrm{f}} = \sum_{i=1}^{m} \omega_i \sum_{\alpha=+,-,3} a^+_\alpha(i) a_\alpha(i), \qquad \mathbf{P}_{\mathrm{f}} = \sum_{i=1}^{m} \mathbf{k}_i \sum_{\alpha=+,-,3} a^+_\alpha(i) a_\alpha(i) \tag{2.3}$$

for the Hamiltonian H_{f} and the momentum $\mathbf{P}_{\mathrm{f}}$ of the transverse electromagnetic field under the transformations [15–19]

$$\begin{aligned} a_\alpha(i) &\to \hat{a}_\alpha = \sum_{\beta=+,-} u_{\alpha,\beta}\, a_\beta, \\ a^+_\alpha(i) &\to \hat{a}^+_\alpha(i) = \big(\hat{a}_\alpha(i)\big)^+, \qquad \alpha = +,-, \quad u = \|u_{\alpha\beta}\| \in \mathrm{U}(2). \end{aligned} \tag{2.4}$$

We note the Eqs. (2.3) admit, in fact, the more vast group $\mathrm{U}(3) \supset \mathrm{U}(2)$ of polarization transformations [17], but in quantum optics it is reduced to the above mentioned U(2) group. It is due to the fact that we calculate quantum expectations of any physical quantities by averaging on the space $L_{\mathrm{phys}} = L_{\mathrm{F}}(m)$ spanned by basis vectors

$$\big|\{n^\sigma_i\}\big\rangle = N\big(\{n^\sigma_i\}\big) \prod_{i=1}^{m} \prod_{\sigma=-,+} \big[n^\sigma_i!\big]^{-1/2} \big(a^+_\sigma(i)\big)^{n^\sigma_i} |0\rangle, \tag{2.5}$$

which are generated by the creation operators $a^+_\alpha(i)$ of photons with transverse ($\alpha = +,-$) polarizations (helicities) only (that corresponds to a standard form of the gauge condition for transverse radiation fields in quantum electrodynamics [19,20]).

The transformations (2.4) correspond to the U(2) "rotations" of the polarization unit vectors $\mathbf{e}_\alpha(i)$ [15–20] in a "polarization spinor" space [20]:

$$\mathbf{e}_\alpha(i) \to \hat{\mathbf{e}}_\alpha(i) = \sum_{\beta=+,-} u_{\beta\alpha} \mathbf{e}_\beta(i), \tag{2.6}$$

and, therefore, may be interpreted as specific polarization gauge transformations. We note that this continuous polarization group SU(2) is closely related to discrete symmetries of light fields: mirror reflections and spatial inversion [17].

The generators of this polarization gauge group U(2) are of the form

$$P_0 = \frac{1}{2}\sum_{i=1}^{m}\left[a_+^+(i)a_+(i) - a_-^+(i)\right] = \sum_i P_0(i),$$
$$P_\pm = \sum_{i=1}^{m} a_\pm^+(i)a_\pm(i) = \sum_i P_\pm(i), \tag{2.7}$$
$$N = \sum_{i=1}^{m}\sum_{\alpha=+,-} a_\alpha^+(i)\,a_\alpha(i) = \sum_i N(i),$$

where N is the total photon number operator and operators P_α are generators of the SU(2) subgroup defining the polarization (P) (quasi) spin [15–19]. the operators P_β and N satisfy commutation relations

$$[N, P_\alpha] = 0, \quad [P_0, P_\pm] = \pm P_\pm, \quad [P_+, P_-] = 2P_0, \tag{2.8}$$

and in the case $m = 1$ coincide up to the factor 1/2 with Stokes operators Σ_α [20]. As is clear from Eqs. (2.7) the total P–quasispin of the electromagnetic field is obtained by adding of the appropriate quasispin quantities for single spatiotemporal modes. This allows us to consider along with the "global" polarization invariance transformations (2.4) also their "local" analogues related to P–quasispin and appropriate independent "polarization rotations" of the type (2.4) for each single spatiotemporal mode. However, from the experimental viewpoint the global polarization SU(2) invariance and the total P–quasispin of the electromagnetic field enable us to examine new interesting physical phenomena connected with correlations of different modes, in particular, with so-called "entangled states" which are widely discussed in multiparticle interoferometry [11,29].

We note that operators P_α do not commute with components S_α of the gauge non-invariant (and hence locally non-observable) ordinary spin $\mathbf{S} = (S_1, S_2, S_3)$ of the electromagnetic field, wich defines the field transformations with respect to the SO(3) ⊂ SL(2C) group of rotations in the usual space though we have the relation

$$\langle\phi|[P_0, S_a]|\psi\rangle = 0, \quad \forall\,|\phi\rangle, |\psi\rangle \in L_{\text{phys}}. \tag{2.9}$$

Indeed, the components S_α are expressed in terms of the $\mathbf{A}(\mathbf{r}, t)$ Fourier components as follows [20,19]:

$$S_a = -i\sum_j\sum_{b,c}\epsilon_{abc}A_b^{(-)}(j)A_c^{(+)}(j),$$
$$A_a^{(+)}(j) = \sum_\alpha e_{\alpha a}(j), \quad A_a^{(-)}(j) = \left(A_a^{(+)}(j)\right)^+, \tag{2.10}$$

where ϵ_{abc} is the fully antisymmetric tensor ($\epsilon_{123} = 1$), $e_{\alpha a}(i)$ is the projection (directing cosine) of $\mathbf{e}_\alpha(i)$ on the a–th axis of a fixed spatial frame of reference. From Eqs. (2.1), (2.7) and (2.10) one gets

$$[P_0, S_a] = \tfrac{1}{2}\sum_j \big\{ e_{+a}(j)\big[a_3^+(j)a_+(j) + a_-^+(j)a_3(j)\big] + e_{-a}(j)\big[a_3^+(j)a_-(j) + a_+(j)a_3^+(j)\big]\big\}; \tag{2.11a}$$

$$[P_\pm, S_a] = \mp\sum_j \big\{ 2e_{3a}(j)a_\pm^+(j)a_\mp(j) + e_{\pm a}(j)\big[a_\pm^+(j)a_3(j) - a_3^+(j)a_\mp(j)\big]\big\}. \tag{2.11b}$$

Then Eq. (2.9) follows immediately from Eqs. (2.5) and (2.11a). We also note that in the case of plane wave beams, when all $e_{3a}(j) = \delta_{3a}$, $a = 1, 2, 3$ and $e_{\pm 3}(j) = 0$, from Eq. (2.11) one finds a relation

$$\exp(i\phi S_3)P_\alpha \exp(-i\phi S_3) = \exp(i\alpha\phi)P_\alpha, \qquad \alpha = 0, \pm, \tag{2.12}$$

defining transformations of P–spin components under rotations around the light beam axis.

Without dwelling here on a more complete analysis of other interrelationships between ordinary spin $\mathbf{S}$ and P–spin we only note that the ordinary spin $\mathbf{S}$ has some advantages as against the P–quasispin for describing of "rotation" properties of light fields and appropriate experiments [13]. Specifically, from Eqs. (2.1) and (2.10) one easily finds relations

$$[S_a, a_\pm^+(j)] = \pm a_\pm^+(j)\, e_{3a}(j) \mp a_3^+(j)\, e_{\pm a}(j), \tag{2.13a}$$

$$[S_a, a_3^+(j)] = -a_-^+(j)\, e_{+a}(j) + a_+^+(j)\, e_{-a}(j), \tag{2.13b}$$

$$[S_a, A_b^{(-)}(j)] = -i\sum \epsilon_{abc} A_c^{(-)}(j), \tag{2.13c}$$

specifying "rotation" properties of appropriate field operators. Moreover, the ordinary spin formalism allows us to expand familiar correlation tensors $G^{(n,m)}_{\dots;\dots}(\{\dots;\dots\})$ from Eq. (2.2) in sums of the SO(3) irreducible tensors (spin and higher multipole operators) which possess well-defined transformation properties with respect to the "spatial" SO(3) group (see, *e.g.*, Ref. [7], where a similar expansion was given for $G^{(1,1)}_{i;j}(\dots)$ in order to define some generalized Stokes parameters).

At the same time, as it follows from Eqs. (2.7), (2.10) and (2.11), the P–quasispin formalism has evident advantages in comparison with the ordinary spin for describing properly polarization properties of light since its components have a clear physical meaning and are measurable in quantum optics polarization experiments related to counting photons with definite polarizations [19]. In particular, the total helicity $2P_0$ of the field

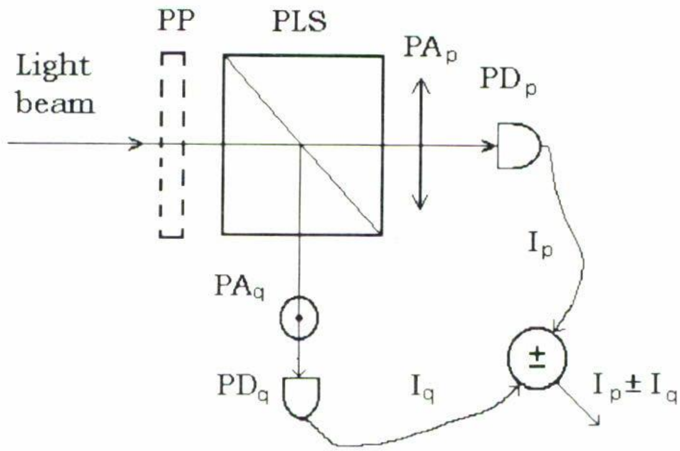

FIGURE 1. A principal scheme of the measurement of P–quasispin components.

is the difference $(N_+ - N_-)$ of the right- and left-handed photon numbers and Hermitian operators $2P_1 = (P_+ + P_-)$ and $2P_2 = i(P_+ - P_-)$ determine (cf. [4,12]) differences of photon numbers with orthogonal linear polarizations which are connected with the helicity basis by the unitary transformation

$$a_1^+(j) = \tfrac{1}{\sqrt{2}}\{a_-^+(j) - a_+^+(j)\}, \qquad a_2^+(j) = \tfrac{i}{\sqrt{2}}\{a_+^+(j) + a_-^+(j)\}, \tag{2.14a}$$

$$\hat{a}_1^+(j) = \tfrac{1}{\sqrt{2}}\{a_1^+(j) + a_2^+(j)\}, \qquad \hat{a}_2^+(j) = \tfrac{1}{\sqrt{2}}\{-a_1^+(j) + a_2^+(j)\}, \tag{2.14b}$$

implemented, for example, with the help of phase plates and polarization rotators (for Eq. (2.14b)) [4,12]. We note that from the formal viewpoint components P_1 and P_2 correspond to the choice of different subgroups SO(2) $\subset$ SU(2) unlike the helicity subgroup U(1) for P_0. Moreover, linear polarization basis functions related to Eqs. (2.14) are eigenstates of operators describing the above mentioned discrete symmetries of light fields. A typical principal scheme [28] of the measurement of components P_α of P–quasispin is presented on Fig. 1, where we use the following notations: PP denotes phases plates, PLS stands for polarization light beam splitters, PA$_a$ and PD$_a$ are, respectively, polarization analyzers and photodetectors for polarization modes a. This scheme can be realized in both single-mode ($m = 1$) and multimode ($m > 1$) regimes. However, as it will be seen later, the use of multimode regimes enables us to reveal new interesting physical phenomena, in particular, an absolutely unpolarized quantum light [16–19].

Besides, in the case of the monochromatic plane waves quantum expectations $\langle P_\alpha\rangle$ are proportional to the Stokes parameters s_α: $s_1 = 2\langle P_2\rangle$, $s_2 = -2\langle P_0\rangle$, $s_3 = -2\langle P_1\rangle$ (cf. [3,4,7,20]) which are expectation values of the Stokes operators Σ_α [20]. Therefore, one can consider that in general cases quantities $\langle P_\alpha\rangle$, $\langle N\rangle$ determine the polarization degree $\deg P$ of light beams with arbitrary wave fronts and frequencies by the relation

$$\deg P = \frac{2}{\langle N\rangle}\left[\sum_{\alpha=0,1,2}(\langle P_\alpha\rangle)^2\right]^{1/2}, \tag{2.15}$$

generalizing the definition for one-mode light beams [3]. At the same time the quantum averages $\langle|P^2|\rangle$ of the $\mathrm{SU}(2)_{\mathrm{pol}}$ Casimir operator $P^2 = \frac{1}{2}(P_+P_- + P_-P_+) + P_0^2$ are connected by the relation

$$\langle|P^2|\rangle = \sum_{\alpha=0,1,2}\left[\sigma_\alpha + (\langle|P_\alpha|\rangle)^2\right], \tag{2.16}$$

with the variances $\sigma_\alpha = \langle|P_\alpha^2|\rangle - (\langle|P_\alpha|\rangle)^2$ determining "polarization noises" [3,17,19,28] and a "radial" uncertainty relations for angular momentum operators [30-32].

Further, calculating the eigenvalue $P(P+1)$ of the operator P^2 on the subspace of one-particle states we find $P(P+1) = 3/4$, *i.e.*, the photon should be ascribed the value $P = 1/2$, as against $S = 1$ for the ordinary spin as it follows from Eqs. (2.5) and (2.10) [17,19]. This fact allows us to identify P–spin of one-photon states with the so-called effective spin (see, *e.g.*, Refs. [5,12,33]) simultaneously clarifying a physical meaning of the latter one.

Therefore one may use P–spin (P_α) as an adequate tool for studying proper polarization properties of quantum light fields in parallel to the usual apparatus of the correlation functions and Stokes vectors [3]. But unlike the latter one use of the P–spin formalism allows us to gain a more deep insight into the inner nature of the polarization structure of light beams with arbitrary wave fronts.

Indeed, as it was shown in Refs. [16–19], one can decompose the Fock space $L_{\mathrm{F}}(m)$ spanned by the vectors (2.4) into the direct sum

$$L_{\mathrm{F}}(m) = \sum_{P,\pi} L(P\pi) \tag{2.17}$$

of infinite-dimensional subspaces $L(P\pi)$ which are specified by eigenvalues P,π of the P–spin and P_0 respectively and spanned by basis vectors $|P\pi;n,\lambda\rangle$ of the form

$$|P\pi;n,\lambda\rangle = \sum_{\substack{\sum\alpha_i=2|\pi| \\ \sum\beta_{ij}=2(P-|\pi|) \\ \sum\gamma_{ij}=n-2P}} C(\{\alpha_i,\beta_{ij},\gamma_{ij}\})\prod_i\left(a_\pm^+(i)\right)^{\alpha_i}\prod_{i,j}\left(Y_{ij}^+\right)^{\beta_{ij}}\left(X_{ij}^+\right)^{\gamma_{ij}}|0\rangle. \tag{2.18}$$

For example, in the cases $m = 1$ and $m = 2$ we have the following expressions [34]:

$$|P\pi\rangle = [(P-\pi)!(P+\pi)!]^{-1/2}$$
$$\times\left(a_+^+(1)\right)^{|\pi|+\pi}\left(a_-^+(1)\right)^{|\pi|-\pi}\left(Y_{11}^+\right)^{P-|\pi|}|0\rangle, \tag{2.19a}$$

$$\left|P\pi = \pm P; n, t\right\rangle = \left[\frac{(n+1)!(n-2P)!(P-t)!}{(2P+1)!}\right]^{-1/2} \times \left(a_\pm^+(1)\right)^{P+t}\left(a_\pm^+(2)\right)^{P-t}\left(X_{12}^+\right)^{n-2P}\left|0\right\rangle, \quad t = n(1) - n(2), \tag{2.19b}$$

for some such vectors. In general, the coefficients $C(\ldots)$ in (2.18) are determined from the defining equation

$$\begin{aligned} P^2\,|P\pi; n, \lambda\rangle &= P(P+1)\,|P\pi; n, \lambda\rangle\,; \\ P_0\,|P\pi; n, \lambda\rangle &= \pi\,|P\pi; n, \lambda\rangle\,, \\ N\,|P\pi; n, \lambda\rangle &= n\,|P\pi; n, \lambda\rangle\,, \end{aligned} \tag{2.20}$$

and some equations for fixing an extra (vector) label λ (see Refs. [17,19] and references therein).

The operators

$$Y_{ij}^+ = \tfrac{1}{2}\left(a_+^+(i)\,a_-^+(j) + a_-^+(i)\,a_+^+(j)\right) \tag{2.21a}$$

and

$$X_{ij}^+ = a_+^+(i)\,a_-^+(j) - a_-^+(i)\,a_+^+(j) \tag{2.21b}$$

in (2.18) are the solutions of the operator equations

$$\left[P_0, Y_{ij}^+\right] = 0, \quad \left[P_\alpha, X_{ij}^+\right] = 0, \qquad \alpha = 0, +, -, \tag{2.22}$$

and may be interpreted as creation operators of P_0–scalar and P–scalar biphoton kinematic clusters respectively. From Eqs. (2.21) and (2.22) one easily obtains that quantum expectations $\langle P_\alpha \rangle$, $\alpha = 0, 1, 2$, in states generated by actions on the vacuum vectors $|0\rangle$ operators $(X_{ij}^+)^a(Y_{ij}^+)^b$ only; for example, simplest states of such types are spanned by vectors (2.19) with $\pi = 0$. In general, the states (2.18) describe light beams representing a mixture of both usual photons and unpolarized P– and P_0–scalar biphotons [16–19]. We, however, note that biphotons Y_{ij}^+ exist for any number m of time-spatial modes, whereas $X_{ij}^+ \neq 0$ only for $m \geq 2$. We also emphasize that in contrast to the usual photon operators $a_\alpha^+(j)$, $a_\alpha(j)$ the operators $X_{ij} = (X_{ij}^+)^+$, X_{ij}^+, $Y_{ij} = (Y_{ij}^+)^+$, Y_{ij}^+ satisfy not the canonical commutation relations but trilinear commutation relations for quanta of parastatistical fields [17]. However it is possible to construct from them some operators obeying canonical commutation relations (see Refs. [17,19]) and representing peculiar "optical atoms" (cf. Ref. [33]). Therefore, in a sense, such construction yield a realization within quantum optics of the method of fusion by L. de Broglie for constructing composite fields from some simple ones [35].

Further, the decomposition (2.17) is invariant with respect to the Lie algebra so*$(2m)$ generated by biphoton operators X_{ij} and X_{ij}^+ [17,19]. Therefore, states $|\psi\rangle$ belonging to a subspace $L(P\pi)$ with given P,π at initial time will be in it for the time evolution governed by the interaction Hamiltonians $H_{\text{int}} = H_i'(\{X_{ij}, X_{ij}^+\})$; examples of such Hamiltonians are given by those of some parametric processes [16–19]. Extending the algebra so*$(2m)$ by adding operators Y_{ij}, Y_{ij}^+ we get the algebra $u(m,m)$ associated with Hamiltonians $H_{\text{int}} = H_{\text{int}}''(\{Y_{ij}, Y_{ij}^+; X_{ij}, X_{ij}^+\})$ (describing, for example, light propagation in Kerr media [36]) which keep invariant for time evolution subspaces $L'(\pi) = \sum_{P\geq|\pi|} L(P\pi)$ [19].

3. Squeezing in polarization quantum optics. Unpolarized quantum light

The decomposition (2.17) implies a new classification of the polarization states of quantum light fields from the physical viewpoint [17,19]. This classification is closely related to a specific sort of squeezing of multimode light beams with consideration of polarization.

In fact, a definition of squeezing in quantum mechanics is based on an analysis of different uncertainty relations of a set $\{A_i, i = 1, \ldots, r > 1\}$ of non-commuting Hermitian operators A_i representing some quantum observables [21–27,30–32,37–39]. Then, unlike the case of classical mechanics, there exist some restrictions on a possible accuracy of results of joint measurements of all quantities A_i in a given quantum state $|\,\rangle$ that is expressed in the form of different "uncertainty relations" for expectations $\langle|(A_i)^s|\rangle$ [32]. These relations represent specific measure of admissible quantum fluctuations ("noises") for observables A_i in the state $|\,\rangle$.

Specifically, the most widespread uncertainty relation (of the Heisenberg type) has the form [1,5,30–32]

$$\Delta A_i\, \Delta A_j \geq \tfrac{1}{2}\, |\langle|[A_i, A_j]|\rangle|\,, \tag{3.1}$$

where $(\Delta A)^2 \equiv \sigma_A = \langle|(A)^2|\rangle - (\langle|A|\rangle)^2$. Then the problem of squeezing consists in finding quantum states minimizing both the product $\Delta A_i\, \Delta A_j$ of two uncertainty measures and one of them. If the right hand side of inequality (3.1) is a c-number this problem is easily solved and lead to definition of the usual concept of squeezing related to generalized coherent states of the group SU$(1,1)$ [21–25]. For example, it is the case for single-mode electromagnetic field when we use as observables A_i two quadrature components of the field expressed by linear combination of the field operators of creation and destruction [21–25].

However, for multimode fields the situation becomes more complicated since in this case we have a more vast set of observables which obey non-trivial commutation relations [26,27,17,18]. Therefore, we have many possibilities of definition of squeezing related to a choice (from physical considerations) some subsets of observables for which a solution of this problem is comparatively simple. For example, as we established above, in polarization quantum optics as such subsets it is natural to take components of the P–quasispin obeying the commutation relations (2.8) of the su(2) algebra as well as subsets of unpolarized biphoton operators of X– and Y–types. That enables us to define a specific polarization squeezing which is closely related to a new physical phenomenon of quantum unpolarized light [19].

Specifically, for the states $|P_0; n, \lambda\rangle \in L(P0) \subset L'(\pi = 0)$ and $|00; n, \lambda\rangle \in L(00)$ from (2.7), (2.10) and (2.18) we find $\langle|P_\alpha|\rangle = 0$, $\langle|S_\alpha|\rangle = 0$ for all α that is a characteristic property of unpolarized light (cf. Refs. [3,4]). Besides the calculations [17] showed that correlation tensors $G_{ij}^{(1,1)}(\mathbf{r}, \mathbf{t}; \mathbf{r}, \mathbf{t})$ have for these states a form corresponding to unpolarized light beams with, in general, arbitrary wave fronts [7].

But unlike classical (chaotic) unpolarized light, for the states $|00; n, \lambda\rangle$ and $|P0; n, \lambda\rangle$ we have additional characteristics of light depolarization which follow from Eqs. (2.17)–(2.21) and are expressed in terms of higher moments for P_α; for example, we have

$$\langle|(P_0)^s|\rangle - (\langle|(P_0)|\rangle)^s = 0 \quad \text{for} \quad |\,\rangle = |P0; n, \lambda\rangle, \tag{3.2a}$$

$$\langle|(P_\alpha)^2|\rangle - (\langle|(P_\alpha)|\rangle)^2 = 0, \quad \alpha = 0, +, - \quad \text{for} \quad |\,\rangle = |00; n, \lambda\rangle, \tag{3.2b}$$

showing the absence of appropriate polarization "noises" of any order measured by appropriate noises of difference photocurrents in schemes of Fig. 1; herewith, as it follows from Eq. (2.12), for axial light beams results of measurements do not depend on rotations of analyzers around beam axis.

Thus, for states $|\,\rangle \in L(00)$ all proper polarization properties are identical with those for vacuum state $|0\rangle$, but unlike the latter the light intensity in these states (measured as the quantum expectation of the Hamiltonian (2.3)) is not equal to zero. Consequently, they may be recognized as states describing absolutely unpolarized light, while the states $|\,\rangle \in L'(0)$ have a hidden polarization structure revealed in measurements of linear polarization noises. Moreover, the states $|00; n, \lambda\rangle$ minimize both the aforementioned "radial" uncertainty measure (2.16) as well as uncertainty relation of the (3.1) type for angular momentum operators [30–32]; besides these states form the infinite-dimensional space on which three non-commuting operators P_α behave themselves as c-numbers exhibiting an "absolute squeezing" in polarization degrees of freedom (that it is of interest for designing different experiments related to the EPR-paradox and "hidden variable" theories [1,5,11]). However, we have not analogous relations for components S_α of the ordinary spin as one can see it from Eqs. (2.10) and (2.13).

Therefore, states $|\phi\rangle \in L'(0)$ generated by biphotons Y_{ij}^+, X_{ij}^+ and $|\psi\rangle \in L(00) \subset L'(0)$ generated only by biphotons X_{ij}^+ describe new types of unpolarized light due to strong quantum phase correlations rather than random mixing light beams as it is the case for the classical unpolarized light [3,4]. Examples of such states are yielded by generalized coherent states related with interaction Hamiltonians $H_{\text{int}} = H_{int}^1 + H_{\text{int}}^2$, where

$$H_{\text{int}}^1 = \sum_{i<j} (g_{ij} X_{ij} + g_{ij}^* X_{ij}^+) \tag{3.3a}$$

and

$$H_{\text{int}}^2 = \sum_{i,j} (f_{ij} Y_{ij} + f_{ij}^* Y_{ij}^+) \tag{3.3b}$$

describe some specific parametric processes [17]. In particular, generalized coherent states of the SU(1, 1) group orbit type

$$|\alpha\rangle_P = \exp\left[\alpha X_{12} - \alpha^* X_{12}^+\right] |0\rangle \tag{3.4}$$

discussed together with some related models in Refs. [16–18] are generated by $H_{\rm int}^1$, whereas $H_{\rm int}^2$ produces generalized coherent states of the group Sp$(2m, R)$,

$$|\beta_{ij}\rangle_{P_0} = \exp\left[\sum(\beta_{ij} Y_{ij}^+ - \beta_{ij}^* Y_{ij})\right] |0\rangle, \tag{3.5}$$

coinciding in the case $m = 1$ with two-mode squeezed states introduced in Ref. [23]. We also note that acting by the group displacement from Eqs. (3.4) and (3.5) on usual multimode Glauber coherent states $|\{\alpha_i^+, \alpha_j^-\}\rangle = \prod_i \exp(\alpha_i^+ a_+^+(i) + \alpha_i^- a_-^+(i) - \alpha_i^{+*} a_+(i) - \alpha_i^{-*} a_-(i))\,|0\rangle$, $\alpha_i^\pm \neq 0$, we get in general cases states of partially polarized light which contains (for special values of parameters $\alpha_i^\pm$) a subclass of states corresponding to unpolarized light. In particular, all states related in such a manner to $|\{\alpha_i^+, \alpha_i^-\}\rangle$ display properties of usual unpolarized light when $|\alpha_i^+| = |\alpha_i^-|$ [8,28].

Thus, our analysis displays inner mechanisms of the light depolarization at the quantum level (cf. Ref. [40], where a conjecture was uttered about a quantum nature of unpolarized light) by contrast to the generally accepted viewpoint [4] that randomization is the only way of obtaining unpolarized light. Besides the P–spin formalism yields some new natural measurable quantitative characteristics of light depolarization, namely, degrees $\mathrm{dep}_P = (1 - 2\bar{P}/\bar{N})$ and $\mathrm{dep}_{P_0} = (1 - |2\bar{\pi}|/\bar{N})$ of the content of P–scalar and of P_0–scalar biphotons, where $\bar{P}$, $\bar{\pi}$, $\bar{N}$ denote expectation values of appropriate operators; herewith $\bar{P} = -\frac{1}{2} + [\frac{1}{4} + \langle |P^2| \rangle]^{1/2}$ is determined from Eqs. (2.15) and (2.16) as a function of $\deg P$, $\bar{N}$ and variances σ_α. Evidently, dep_{P_0} is connected with the well-known degree of circular polarization $|\langle N_+\rangle - \langle N_-\rangle|/\langle N\rangle$, whereas dep_P provides a new quantitative characteristic of polarization structure of light related to measurements of polarization noises.

We also note that analysis above can be extended by considering modifications of the decomposition (2.17), where instead of P_0 any other Hermitian operator $P_\alpha, \alpha = 1, 2$ corresponding to a linear polarization basis is diagonalized [19]. Such extensions lead to new states of quantum unpolarized light generated by P_1– or P_2–scalar biphotons of the (2.21a) type and having characteristics similar to those described by Eqs. (3.2) but with some peculiarities concerning their "rotation" properties determined by Eqs. (2.1)), (2.12), (2.15) and (2.16).

4. Generalizations, applications and conclusion

In the previous sections we have shown that in the Fock space $L_{\rm F}(m)$ of multimode light with consideration of polarization one can select with the help of Eq. (2.17) subspaces $(L(P = 0\pi = 0)$, $L'(\pi = 0)$ and someones related to them) of quantum sates describing different new types of unpolarized light and, simultaneously, manifesting specific forms of

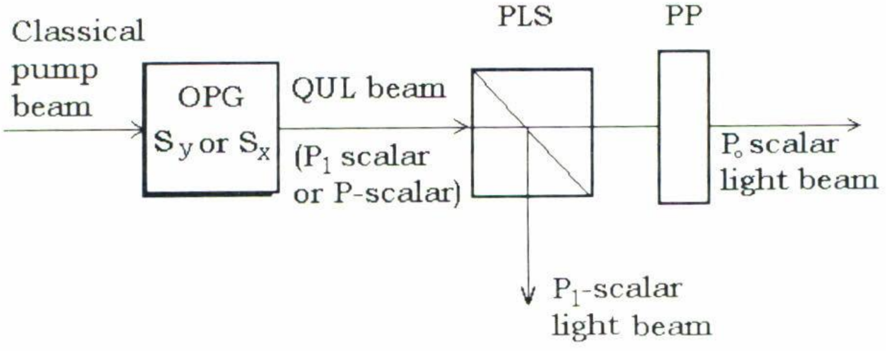

FIGURE 2. Schemes of production of quantum unpolarized light (QUL).

squeezing in polarization optics. Physical realizations of such states are connected with actions of P_0– and P–scalar biphoton squeezed operators,

$$S_Y(Z) = \exp(z_{ij}Y_{ij}^+ - z_{ij}^* Y_{ij}) \tag{4.1a}$$

and

$$S_X(Z) = \exp(z_{ij}X_{ij}^+ - z_{ij}^* X_{ij}), \tag{4.1b}$$

on the vacuum vectors $|0\rangle$ that is represented schematically on Fig. 2, where OPG stands for parametric oscillator generators corresponding to the operators (4.1) and other notations are the same as on Fig. 1. We note that, in practice, it is easier to realize such schemes corresponding to Eq. (4.1a) rather than Eq. (4.1b) because the latter require parametric oscillator crystals with highly anisotropic properties. Therefore, for production of P–scalar light it is preferable to combine more simple schemes of production of P_0-scalar light together with some interferometric schemes [28].

All other subspaces $L(P\pi)$, $L'(\pi)$, $\pi > 0$, in the decomposition (2.17) describe, generally speaking, states of partially depolarized quantum light (see Refs. [16,17], where we also examined various types of polarization generalized coherent states of light, including those which are eigenfunctions of the operators P^2, P_0, X_{ij}, Y_{ij} and generalize the Agarwal's pair coherent states [39]). However, in real physical experimental situations states of light beams do not belong to a single subspace $L(P\pi)$ but are superpositions of states from different subspaces $L(P\pi)$. Therefore, it is of interest to study polarization squeezing properties (with using measurement devices of schemes on Fig. 1) of partially polarized light beams obtained by actions of the biphoton squeezed operators (4.1) together with the "proper" (related to generalized coherent states of the polarization invariance group SU(2) [17,30,31]) polarization squeezed operators $S_P(\zeta) = \exp(\zeta P_+ - \zeta^* P_-)$ on states of some input light beams that is presented schematically on Fig. 3. As a result we can obtain new classes of non-classical states of partially polarized light.

The P–spin formalism and nonclassical states of light described above have also several potential applications, one, for example, is in optical communication theory [41–43].

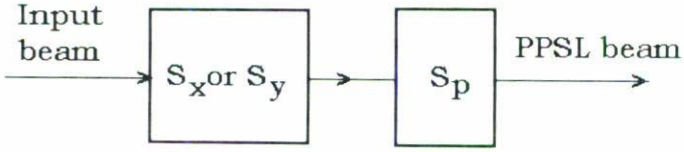

FIGURE 3. Principal schemes of production of partially polarized squeezed light (PPSL).

Specifically, the decomposition (2.17) and the properties (3.2) of states $|\phi\rangle \in L(00)$ and $|\psi\rangle \in L'(0)$ appear to be promising for designing the quantum channels of communication systems [41,42]. Such communication channels are realized by light beams using mainly both the amplitude and phase modulations for encoding transmitted information. But polarization methods of its encoding appear to be more preferable because of certain (mainly energetic) reasons [42]. We sketch a scheme of using quantum unpolarized light within such an approach following [41,19].

For discrete channels their efficiency is usually estimated with the aid of the conditional error probability $P_e[\hat{m} \neq m]$ (where m is an input message and $\hat{m}$ is the appropriate output one) [41]. Then, using states $\rho_1 \in L'(\pi \neq 0)$ for transmitting the logical "1" in binary discrete channels, we can use the above results for optimizing $P_e[\ldots]$ (cf. Ref. [41]). For this end it is also of interest to estimate the information capacity [43] of the states $|\psi\rangle \in L'(0)$ as compared with that of other quantum states.

From other lines of possible applications of the results above, we point out precise measurements in spectroscopy of anisotropic media [28] and studies of interaction of light in different new polarization states with optically active biological macromolecules [44].

In conclusion, we emphasize that the above results give a more deep insight into polarization structure of light enabling to determine new nonusual states in quantum optics. In a sense, the results of Sects. 2,3 and those of papers [16–19] yield all necessary prerequisites for developing a quantum description of unpolarized light waves whose existence has not yet an adequate solution within the classical optics [4,45]. Besides, we established some interrelations between proper polarization (related to the P-spin) and rotation (connected with the ordinary spin $\mathbf{S}$) characteristics of light fields [Eqs. (2.11) and (2.12)] that enables us to examine a behaviour of polarization characteristics of light beams in dependence on rotation of measurement devices (on schemes of Fig. 1) with respect to light beam directions.

All this opens some possibilities in setting new optical experiments related, in particular, to "hidden" variables, "entangled states" and EPR paradox [1,2,5,11,29], polarization chaos, spontaneous symmetry breaking and bistability [6,8,40,42], "optical atoms" and polarization solitons [9,33], reduction of quantum noises [10,12,29], etc. We are planning to discuss these topics as well as some practical schemes and mechanisms of producing new quantum polarization states of light (including those obtained with devices of both Fig. 1 and Fig. 2) in forthcoming papers.

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