Some remarks on the role of the Lorentz condition in massive Yang-Mills theory

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ABSTRACT. Slavnov-Ward identities for the massive Yang-Mills theory are considered. They are associated with infinitesimal gauge-invariance transformations of the action when the vector field satisfies the Lorentz condition. From these identities it follows that the physical amplitudes vanish after the insertion of an arbitrary number of $\partial_\mu A_\mu$ operators, when all their space-time arguments are different. In order to study, in more detail, the role of the Lorentz condition in the quantum theory of the system, the Dirac-brackets quantization is recalled. Again, it arises that the mean value of an arbitrary product of $\partial_\mu A_\mu$ operators vanishes in a physical state. It can also be argued that the evolution operator should be unaltered by adding terms dependent on $\partial_\mu A_\mu$ to the Hamiltonian. This property allows to obtain a formal derivation of renormalizable Feynman integrals for the physical amplitudes which should be further investigated for clarifying the question of unitarity.

RESUMEN. Se consideran las identidades de Slavnov-Ward para la teoría de Yang-Mills masiva. Estas identidades están asociadas a transformaciones de invariancia de norma infinitesimales de la acción, cuando el campo vectorial satisface la condición de Lorentz. De esas identidades se sigue que las amplitudes físicas se anulan después de la inserción de un número arbitrario de operadores $\partial_\mu A^a_\mu$ cuando todos los argumentos espacio temporales difieren. Con vistas a discutir en más detalle la importancia de la condición de Lorentz en la teoría cuántica del sistema, se considera la cuantización en términos de los corchetes de Dirac. De nuevo se sigue que el valor medio de un producto arbitrario de operadores $\partial_\mu A^a_\mu$ se anulan en los estados físicos. Se puede argumentar, además, que el operador de evolución debe quedar inalterado al añadir términos dependientes en $\partial_\mu A^a_\mu$ al hamiltoniano. Esta propiedad permite obtener una prueba formal de la validez de una integral de Feynman renormalizable para las amplitudes, la cual debe ser investigada posteriormente con vistas a esclarecer la cuestión de la unitariedad.

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1. INTRODUCTION

Massive Yang-Mills theory was the object of intense activity in the literature in the heyday of unified gauge theories [1–9]. After the development of the spontaneous-symmetry-breaking approach to the generation of masses, interest in this model diminished but still existed (see Ref. [10] for a review). In spite of the above, it seems that the conclusion about the absence of unitarity and renormalizability in any non-symmetry-breaking modified theory is a kind of agreement. Taking this fact into account, it is appropriate to put forward some considerations which could be useful for the analysis of this problem. The present paper aims at giving some elements which, although formal, are closely related to the above-mentioned matters.

The work starts by noticing the presence of an infinitesimal remaining gauge transformation, which leaves invariant the classical action when the field satisfies the Lorentz condition. The resulting Ward identities are written. These relations were already found by Slavnov [8], who also expressed the interest in investigating their effect on the renormalization problem. These expressions are of similar simplicity to the ones in the massless case. Here we write them in terms of the various kinds of generating functionals. Their use allow us to show that a physical amplitude in which any number of $\partial_\mu A_\mu$ operators are inserted, vanishes when evaluated on mass shell, if the arguments of the operators are different.

After that, the Dirac-brackets quantization of the system is considered as was also done by Senjanovic [11]. This is performed in order to discuss in more detail the role of the Lorentz condition in the quantum framework. We reproduce the results of Ref. [11] with a minor change in the Dirac brackets among the canonical variables. The results show that the Lorentz condition is always satisfied when applied to physical states. This fact seems to allow the modification of the evolution operator when applied to physical states by adding products of the Lorentz condition to the Hamiltonian. Then the usual reasoning in determining the functional integral permit us to arrive at a interesting conclusion. The functional integral action of the massive Yang-Mills theory can be modified, by the addition of a linear term $\rho^a \partial_\mu A_\mu^a$, when matrix elements between physical states and vanishing boundary conditions on the auxiliary field ρ^a are considered.

In Sect. 2 the infinitesimal invariance of the action is described and the Slavnov-Ward identities are used in the study of the insertions of the Lorentz condition in physical amplitudes. Section 3 presents the classical and quantum canonical procedure and the way in which the Lorentz condition is implemented. The use of these results for putting forward formal arguments concerning the modification of the functional integral is described.

2. Infinitesimal gauge invariance and Ward identities

The Lagrangian for the massive Yang-Mills fields can be written in the following form:

$$\mathcal{L}(x) = -\tfrac{1}{4} F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) - \frac{m^2}{2} A^a_\mu(x) A^a_\mu(x), \tag{1}$$

$$F^a_\mu(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + g f^{abc} A^b_\mu(x) A^c_\nu(x), \tag{2}$$

in which the conventions are

$$A^a_\mu(x) = (A^a_j(x), iA^a_0(x)),$$

$$x \equiv x_\mu = (x_i, ix_0) = (\vec{x}, it),$$

$$\mu = 1, 2, 3, 4.$$

The symmetry group is SU(N) with completely antisymmetric structure constants f^{abc}, and g is the gauge coupling.

The classical equation of motion of the system is given by

$$\frac{\delta S}{\delta A^a_\mu(x)} = \nabla^{ab}_\nu(x) F^b_{\nu\mu}(x) - m^2 A^a_\mu(x) = 0, \tag{3}$$

where the action and the covariant derivative are

$$\begin{aligned} S &= \int \mathcal{L}(x)\,dx, \qquad dx = i dx_0\, d^3x, \\ \nabla^{ab}_\mu(x) &= \delta^{ab}\partial_\mu + g A^{ab}_\mu(x), \\ A^{ab}_\mu(x) &= f^{acb} A^c_\mu(x). \end{aligned} \tag{4}$$

Let us now consider that $A^a_\mu(x)$ is not on the mass shell, *i.e.*, it does not obey Eq. (3), but it satisfied the Lorentz condition

$$\partial_\mu A^a_\mu(x) = 0. \tag{5}$$

Then, after performing an infinitesimal gauge transformation in the following way:

$$A^a_\mu(x) \to A^a_\mu(x) + \nabla^{ab}_\mu(x)\lambda^b(x), \tag{6}$$

with $\lambda^a(x)$ arbitrary but infinitesimal, it follows that

$$\begin{aligned} \delta S &= -m^2 \int dx \left(A^a_\mu \nabla^{ab}_\mu \lambda^b\right) = -m^2 \int dx\, A^a_\mu \partial_\mu \lambda^a \\ &= m^2 \int dx\, \lambda^a \left(\partial_\mu A^a_\mu\right) = 0. \end{aligned} \tag{7}$$

This means that the action is invariant because of the Lorentz condition. This conclusion is also valid in the Abelian case, and again in both cases (Abelian and non-Abelian) after the inclusion of gauge-invariant coupled matter fields. Also, it follows that in all these cases the condition $\partial_\mu A^a_\mu(x) = 0$ is implied by the equations of motion. Therefore, it seems interesting to investigate the role of these relationships in the quantum theory.

A speculative idea that comes to mind from Eq. (7) is related to the bad behaviour of the propagator of the model. Could such a property be a consequence of the "remaining" gauge invariance, in a similar way to that in which the singularity of the propagator in the massless case is produced by the full gauge independence? If such is the case, could there exist a way of using this fact to help in the renormalization? These questions were the motivation for this paper. Below, in this section, we shall derive the Slavnov-Ward identities arising from the above transformations.

The generating functional $\mathcal{Z}$ will be taken in the form

$$\mathcal{Z}[j] = \int \mathcal{D}(A) \exp\left(S + \int dx\, j^a_\mu A^a_\mu\right), \tag{8}$$

where $j^a_\mu(x)$ is the source of the gauge field. For $\mathcal{Z}$ the following Schwinger equation holds:

$$0 = \int \mathcal{D}(A) \left\{ \frac{\delta S}{\delta A^a_\mu(x)} + j^a_\mu(x) \right\} \exp\left(S + \int dx\, j^a_\mu A^a_\mu\right). \tag{9}$$

After applying to Eq. (9) the operator

$$\nabla^{ab}_\mu \left[\frac{\delta}{\delta j}\right] = \delta^{ab}\partial_\mu + g f^{acb} \frac{\delta}{\delta j^c_\mu(x)}. \tag{10}$$

which is equivalent to performing the transformation (6) as a change of variables in Eq. (8), the following identity arises:

$$0 = m^2 \partial_\mu \frac{\delta \mathcal{Z}}{\delta j^a_\mu(x)} + \left(\nabla^{ab}_\mu \left[\frac{\delta}{\delta j(x)}\right] \mathcal{Z}\right) j^b_\mu(x), \tag{11}$$

which is the one given by Slavnov [8]. Defining the generating functional of connected Green functions

$$W[j] = \ln \mathcal{Z}[j], \tag{12}$$

the expression (11) becomes

$$m^2 \partial_\mu \frac{\delta W}{\delta j^a_\mu(x)} + \partial_\mu j^a_\mu(x) + g f^{acb} \frac{\delta W}{\delta j^c_\mu(x)} j^b_\mu(x) = 0. \tag{13}$$

The simplicity of the mass term allow the relations (11) and (13) to become as simple as the corresponding expressions in the massless case. The invariance-breaking term is just proportional to the mean value of the Lorentz condition.

Let us now introduce the effective action by the usual Legendre transformation:

$$\Gamma[A] = W[j] - \int dx\, j^a_\mu(x) A^a_\mu(x),$$

$$A^a_\mu(x) = \frac{\delta W}{\delta j^a_\mu(x)}. \tag{14}$$

Then, after expressing Eq. (13) in terms of Γ, it follows that

$$m^2 \partial_\mu A^a_\mu(x) - \partial_\mu \frac{\delta \Gamma}{\delta A^a_\mu(x)} - g f^{acb} A^c_\mu(x) \frac{\delta \Gamma}{\delta A^b_\mu(x)} = 0. \tag{15}$$

That is to say that the 1P–irreducible n–point functions of consecutive order are as tightly linked as in the fully-gauge-invariant theory. It seems curious that this fact is a direct consequence of an infinitesimal "remainder" of the gauge invariance at the field values obeying the Lorentz condition.

Further, we shall consider the identities for reducible Green's functions. Taking a number n of functional derivatives of Eq. (11) with different indices (x_i, μ_i, a_i), $i = 2, \dots, n+1$, and replacing (x, μ, a) by (x_1, μ_1, a_1), the following expression can be obtained after setting $j = 0$:

$$m^2 \partial_\mu(1) \left. \frac{\delta^{n+1} \mathcal{Z}}{\prod_{r=1}^{n+1} \delta j_{\mu_r}^{a_r}(x_r)} \right|_{j=0} + \sum_{j=2}^{n+1} \partial_{\mu_1}(1) \delta(x_1 - x_j) \delta_{\mu_1 \mu_j} \delta^{a_1 a_j} \left. \frac{\delta^{n-1} \mathcal{Z}}{\prod_{i=2,\,(i \neq j)}^{n+1} \delta j_{\mu_i}^{a_i}(x_i)} \right|_{j=0} +$$

$$g \sum_{j=2}^{n+1} \delta(x_1 - x_j) f^{a_1 c a_j} \left. \frac{\delta^n \mathcal{Z}}{\delta j_{\mu_j}^{a_j}(x_1) \prod_{i=2,\,(i \neq j)}^{n+1} \delta j_{\mu_i}^{a_i}(x_i)} \right|_{j=0} = 0, \quad \partial_\mu(l) \equiv \frac{\partial}{\partial x_\mu^l}. \tag{16}$$

Equation (16) indicates that the divergence of any Green function over an arbitrary external leg vanishes whenever all the external variables have different values.

Let us now consider that the mass shell for asymptotic states determined by the one-particle propagator is defined and has a mass m. Then, after putting the $n+1, n, n-1, \dots, n+1-N$ external legs on the mass shell by applying the operators

$$\begin{gathered} \hat{\jmath}_{\mu l} = \frac{1}{\sqrt{V}} \int \exp(-i k_l x_l) \epsilon_{\mu l}(k_l) (\partial^2(x_l) - m_r^2)\, dx_l, \\ \epsilon(k_l) k_\mu^l = 0, \qquad \epsilon_\mu(k_l) u_\mu = 0, \qquad u_\mu u_\mu = -1 \end{gathered} \tag{17}$$

to Eq. (16), the following relation for the resulting function G of this transformation on $\mathcal{Z}$ arises:

$$\begin{aligned} & m^2 \partial_{\mu_1}(1) G_{\mu_1, \dots, \mu_l}^{a_1, \dots, a_l}(x_1, \dots, x_l|\mathrm{ms}) + \\ & \sum_{j=2}^{l} \partial_{\mu_j}(1) \delta(x_1 - x_j) \delta^{a_1 a_j} G_{\mu_2, \dots, \mu_i \dots \mu_l}^{a_2, \dots, a_i \dots a_l}(x_2, \dots, x_i, \dots x_l|\mathrm{ms})|_{(i \neq j)} + \\ & \qquad \sum_{j=2}^{l} g f^{a_1 c a_j} \delta(x_1 - x_j) G_{\mu_j \mu_2, \dots, \mu_i \dots \mu_l}^{c, a_2, \dots, a_i \dots a_l}(x_1, x_2, \dots, x_i, \dots, x_l|\mathrm{ms})|_{(i \neq j)} = 0, \end{aligned} \tag{18}$$

where $|\mathrm{ms})$ stands for the mass shell of N legs, $l = n + 1 - N$.

In relation (18) all those terms in Eq. (16) in which one of the variables (associated with the legs settled on the mass shell) enters into the δ–function have vanishing contributions. This is so, because it is possible to integrate by parts in at least one of the operators (17) in these terms. After that, taking the divergence over the $i = 2, \ldots, l$ variables for the insertion of n out-of-mass-shell $\partial_\mu A_\mu$ operators, we can write

$$m^2 \partial_{\mu 1}(1) \cdots \partial_{\mu l}(l) G^{a_1, \ldots, a_l}_{\mu 1 \ldots, \mu_l}(x_1, \ldots, x_l|\text{ms}) +$$

$$\sum_{j=2}^{l} \partial_{\mu_j}(1) \partial_{\mu_j}(j) \delta(x_1 - x_j) \delta^{a_1 a_j} \partial_{\mu_2} \cdots \partial_{\mu_i} \cdots \partial_{\mu_l}$$

$$\cdot\, G^{a_2, \ldots a_i \ldots a_l}_{\mu_2, \ldots \mu_i \ldots \mu_l}(x_2, \ldots, x_i, \ldots, x_l|\text{ms})|_{(i \neq j)} -$$

$$\sum_{j=2}^{l} g f^{a_1 c a_j} \partial_{\mu_1}(1) \delta(x_1 - x_j) \partial_{\mu_2}, \ldots \partial_{\mu_i} \ldots \partial_{\mu_m}$$

$$\cdot\, G^{c, a_2, \ldots a_i \ldots \mu_l}_{\mu_1, \mu_2, \ldots \mu_i \ldots \mu_l}(x_1, x_2, \ldots x_i \ldots x_l|\text{ms})|_{(i \neq j)} = 0. \quad (19)$$

Then, the physical amplitude after the insertion of m divergences of A_μ operators (taken outside mass shell in an arbitrary on-shell amplitude) vanishes whenever all its external variables are different. The recursive character of Eq. (18) may be used to further determine the spatial dependence of the first term in Eq. (19). For $m = 1$, it follows that

$$\partial_{\mu_1}(1) G^{a_1}_{\mu_1}(x_1|\text{ms}) = 0. \quad (20)$$

For $m = 2$, relation (19) reads

$$m^2 \partial_{\mu_1}(1) \partial_{\mu_2}(2) G^{a_1 a_2}_{\mu_1 \mu_2}(x_1, x_2|\text{ms}) = \left[\partial_{\mu_1}(1) \partial_{\mu_1}(1) \delta(x_1 - x_2)\right] \delta^{a_1 a_2} G(\text{ms})$$
$$+ g f^{a_1 c a_2} \partial_{\mu_1}(1) \delta(x_1 - x_2) G^{c}_{\mu_1}(x_1|\text{ms}). \quad (21)$$

In the next section we shall discuss the classical and quantum canonical versions of the model in order to study the realization of the Lorentz condition in more detail.

3. DIRAC-BRACKETS QUANTIZATION

The application of the classical canonical procedure for constrained systems to the Lagrangian (1) leads to the following total Hamiltonian [11]:

$$H_T = \int d^3x \left\{ \frac{p^a_i p^a_i}{2} + \tfrac{1}{4} F^a_{ik} F^a_{ik} + \tfrac{1}{2} m^2 A^a_\mu A^a_\mu - i A^a_4 \nabla^{ab}_i p^b_i + \lambda^a p^a_0 \right\}, \quad (22)$$

where λ^a are Lagrange multipliers and the canonical momenta are given by

$$p^a_0(x) = \frac{\delta L}{\delta \dot{A}_0(x)} = 0, \quad (23)$$

$$p^a_j(x) = \frac{\delta L}{\delta \dot{A}^a_j(x)} = i F_{4j}, \quad (24)$$

with

$$L = \int d^3x\, \mathcal{L}(x). \tag{25}$$

Imposing the consistency condition on the primary constraint (23) leads to the following set of second-class constraints:

$$F^a = p_0^a(x) \approx 0, \tag{26}$$

$$G^a = m^2 A_0^a(x) - \nabla_i^{ab} p_i^b(x) \approx 0, \tag{27}$$

with the determination of the multiplier $\lambda^a(x)$ through

$$0 = m^2 \lambda^a(x) + m^2 \partial_i A_i^a(x) - igf^{acb} G^c(x) A_4^b. \tag{28}$$

The Poisson brackets among the constraints (26) and (27) turns out to be (all the time arguments of the quantities coincide)

$$\{F^a(x), F^b(y)\} = 0, \tag{29}$$

$$\{G^a(x), F^b(y)\} = m^2 \delta^{ab} \delta^{(3)}(\mathbf{x}-\mathbf{y}), \tag{30}$$

$$\{G^a(x), G^b(y)\} = gf^{acb}\big[G^c(x) - m^2 A_0^c(x)\big]\delta^{(3)}(\mathbf{x}-\mathbf{y}). \tag{31}$$

In Eq. (31) lies the small difference from the results of Eq. (11), in which the term proportional to A_0^a is absent. Let us define Dirac's brackets in the usual way:

$$\{A, B\}_{\mathrm{D}} = \{A, B\} - \{A, \varphi_i^a\}\{\varphi_i^a, \varphi_j^b\}^{-1}\{\varphi_j^b, B\}, \tag{32}$$

where φ_i^a, $i = 1, 2$ are the F^a and G^a constraints for $i = 1, 2$, respectively. Then the Dirac brackets among the canonical variables become:

$$\begin{aligned}
&\{A_i^a(x), A_j^b(y)\}_{\mathrm{D}} = 0, \quad \{p_i^a(x), p_j^b(y)\}_{\mathrm{D}} = 0,\\
&\{p_0^a(x), p_j^b(y)\}_{\mathrm{D}} = 0, \quad \{p_0^a(x), p_0^b(y)\}_{\mathrm{D}} = 0,\\
&\{p_0^a(x), A_i^b(y)\}_{\mathrm{D}} = 0, \quad \{A_0^a(x), p_0^b(y)\}_{\mathrm{D}} = 0,\\
&\{A_0^a(x), A_0^b(y)\}_{\mathrm{D}} = -\frac{g}{m^4}\, f^{acb}(G^c(x) - m^2 A_0^c(x))\delta^{(3)}(\mathbf{x}-\mathbf{y}),\\
&\{A_0^a(x), A_i^b(y)\}_{\mathrm{D}} = -\frac{1}{m^2}\nabla_i^{ab}(x)\delta^{(3)}(\mathbf{x}-\mathbf{y}),\\
&\{A_0^a(x), p_i^b(y)\}_{\mathrm{D}} = -\frac{g}{m^2} p_i^{ab}(x)\delta^{(3)}(\mathbf{x}-\mathbf{y}),\\
&\{A_i^a(x), p_j^b(y)\}_{\mathrm{D}} = \delta^{ab}\delta^{(3)}(\mathbf{x}-\mathbf{y})\delta_{ij}.
\end{aligned} \tag{33}$$

The quantization can be introduced now by replacing in the total Hamiltonian (22) the canonical variables by operators in the following way:

$$\begin{aligned} A_i^a(x) &\to \hat{A}_i^a(x), \\ A_0^a(x) &\to \hat{A}_0^a(x), \\ p_i^a(x) &\to \hat{p}_i^a(x), \\ p_0^a(x) &\to \hat{p}_0^a(x), \end{aligned} \tag{34}$$

where the operators satisfy the commutation rules obtained by assigning the commutators to the classical Dirac brackets through

$$\left[\hat{p}_i^a(x), \hat{p}_j^b(y)\right] = \left[\hat{A}_i^a(x), \hat{A}_j^b(y)\right] = 0, \tag{35}$$

$$\left[\hat{A}_i^a(x), \hat{p}_j^b(y)\right] = i\delta^{ab}\delta_{ij}\delta^{(3)}(\mathbf{x}-\mathbf{y}), \tag{36}$$

$$\left[\hat{A}_0^a(x), \hat{A}_0^b(y)\right] = -\frac{ig}{m^4} f^{acb}(\hat{G}^c(x) - m^2\hat{A}_0^c(x)) \cdot \delta^{(3)}(\mathbf{x}-\mathbf{y}), \tag{37}$$

$$\left[\hat{A}_0^a(x), \hat{A}_i^b(y)\right] = -\frac{i}{m^2}\hat{\nabla}_i^{ab}(x)\delta^{(3)}(\mathbf{x}-\mathbf{y}), \tag{38}$$

$$\left[\hat{A}_0^a(x), \hat{p}_i^b(y)\right] = -\frac{ig}{m^2}\hat{p}_i^{ab}(x)\delta^{(3)}(\mathbf{x}-\mathbf{y}), \tag{39}$$

$$\left[\hat{p}_0^a(x), \hat{p}_0^b(y)\right] = \left[\hat{p}_0^a(x), \hat{A}_0^b(y)\right] = 0, \tag{40}$$

$$\left[\hat{p}_0^a(x), \hat{p}_0^b(y)\right] = \left[\hat{p}_0^a(x), \hat{A}_i^b(y)\right] = 0. \tag{41}$$

The constraints $\hat{F}^a$ and $\hat{G}^a$ commute with all the magnitudes as a consequence of these rules. Also in $\hat{G}^a$ the operator-ordering problem is absent due to

$$\begin{aligned} \hat{G}^a(x) &= m^2\hat{A}_0^a - \left(\delta^{ab}\partial_i + gf^{acb}\hat{A}_i^c(x)\hat{p}_i^b(x)\right) \\ &= m^2\hat{A}_0^a - \left(\delta^{ab}\partial_i\hat{p}_i + gf^{acb}\hat{p}_i^b(x)\hat{A}_i^c(x)\right), \end{aligned} \tag{42}$$

as follows from the commutation rules (36). Moreover, the absence of ordering problems is also valid for the Hamiltonian operator, which turns out to be

$$\hat{H} = \int d^3x \left\{\frac{\hat{p}_i^a\hat{p}_i^a}{2} + \tfrac{1}{4}\hat{F}_{ik}^a\hat{F}_{ik}^a + \tfrac{1}{2}m^2\hat{A}_\mu^a\hat{A}_\mu^a - i\hat{A}_0^a\hat{\nabla}_i^{ab}\hat{p}_i^b\right\}. \tag{43}$$

The following constraints can be imposed in a strong sense:

$$\hat{F}^a(x) \equiv 0, \tag{44}$$

$$\hat{G}^a(x) \equiv 0. \tag{45}$$

Now, we can study the way in which the Lorentz condition arises in the theory. For this purpose, we calculated the time derivative of the $\hat{A}_0$ coordinate operator. It results in

$$i\dot{\hat{A}}_0^a(x) = \left[\hat{A}_0^a(x), \hat{H}\right] = -i\partial_k \hat{A}_k(x). \tag{46}$$

Therefore, after applying both sides of Eq. (46) to any state $|1\rangle$, it follows that

$$\begin{gathered}\left(\partial_0 \hat{A}_0^a(x) + \partial_i \hat{A}_i^a(x)\right)|1\rangle = 0,\\ \partial_\mu \hat{A}_\mu^a(x)|1\rangle = 0,\end{gathered} \tag{47}$$

a relation firstly obtained by Salam [12].

Then the Lorentz condition should annihilate the physical states. For any matrix element of a product of $\partial_\mu \hat{A}_\mu$ factors it should also follow that

$$\left\langle 2 \left| \prod_{n=1}^{m} \left(\partial_{\mu_n} \hat{A}_{\mu_n}^{a_n}(x_n)\right) \right| 1 \right\rangle = 0. \tag{48}$$

Relation (48) shows that the possibility of non-vanishing results for the insertions considered in the previous section should be related to the time-ordering operator associated with the functional integral. This is also indicated by the fact that a non-vanishing result could exist only for coinciding arguments in Eq. (19).

Expression (48) also suggests various formal reasonings which we think have some interest. They are connected with the possibility of modifying the evolution operator. Let us consider the modified time evolution of an arbitrary state in the way

$$\begin{aligned} i\frac{\partial}{\partial t}|\Psi(t)\rangle_\rho &= \left\{\hat{H} - \int d^3x \left[\rho^a(x)\partial_k \hat{A}_k^a(x) - \hat{A}_0^a(x)\partial_0 \rho^a(x)\right]\right\}, \\ |\Psi(t)\rangle_\rho &= \left\{\hat{H} + \hat{H}_I(t)\right\}|\Psi(t)\rangle_\rho, \end{aligned} \tag{49}$$

in which space-time dependent and classical auxiliary scalar fields $\rho^a(x)$, $a = 1, 2, 3$, are introduced. The following ρ–dependent evolution operator now can be considered:

$$\hat{U}_\rho(L, -L) = T \exp\left\{-i \int_{-L}^{L} \left(\hat{H} + \hat{H}_I(t)\right) dt\right\}, \tag{50}$$

which corresponds to evolutions from an initial instant very far in the past $t = -L$ to a final one very distant at future $t = L$.

After considering that the fields $\rho^a(x)$ satisfy the following boundary conditions at the initial and final times $t = \pm L$:

$$\rho^a(-L, \mathbf{x}) = \rho^a(L, \mathbf{x}) = 0, \tag{51}$$

it follows that

$$\hat{U}_\rho(L,-L) = T\exp\left\{-i\int_{-L}^{L}\left[\hat{H} - \int_V d^3x\,(\rho^a\partial_\mu A_\mu)\right]dt - i\int_{-L}^{L}\int_V \partial_0\left(\rho^a\hat{A}_0^a\right)dt\,d^3x\right\}$$
$$= T\exp\left\{-i\int_{-L}^{L}\hat{H}\,dt_0\right\}$$
$$= \hat{U}(L,-L), \tag{52}$$

where the Lorentz condition (46) and boundary conditions (51) have ben used after integrating by parts in the time integral appearing in (50) for H_I. The operator $\hat{U}$ is the original evolution operator of the theory, thus, (52) says that $\hat{U}_\rho$ is independent of ρ^a.

Therefore integrating over the fields ρ^a (52) with a gaussian weight defined by the differential operator $\hat{Q}$, the following relation for the matrix elements of $\hat{U}$ can be written:

$$\langle 2|\hat{U}|1\rangle = \frac{1}{N_\alpha}\int \mathcal{D}[\rho^a]\left\langle 2\left|T\exp\left[-i\int_{-L}^{L}dt\,\hat{H}_{\text{eff}}\right]\right|1\right\rangle\exp\left(\int dx\,\frac{1}{2\alpha}\rho^a\hat{O}\rho^a\right), \tag{53}$$

with

$$N_\alpha = \int dx\,\mathcal{D}[\rho^a(x)]\exp\left[\int dx\,\frac{1}{2\alpha}\rho^a(x)\hat{O}\rho^a(x)\right],$$
$$\hat{H}_{\text{eff}} = \int d^3x\left\{\frac{\hat{p}_i^a\hat{p}_i^a}{2} + \frac{\hat{F}_{ik}^a\hat{F}_{ik}^a}{4} + \tfrac{1}{2}m^2\hat{A}_k^a\hat{A}_k^a\right. \tag{54}$$
$$\left. + \frac{1}{2m^2}\hat{\nabla}_i^{ab}\hat{p}_i^b\hat{\nabla}_i^{ac}\hat{p}_i^c + \rho^a\partial_k\hat{A}_k^a - \hat{A}_0^a\partial_0\rho^a\right\}.$$

The functional integral version of Eq. (53) can be obtained as usual, by partitioning the interval (t_1,t_2) into N small pieces and decomposing the exponential into a product of infinitesimal evolutions. Then, denoting by t_n any of the time values in the partition, the unit operator is introduced for each value of t_n in the usual way

$$\hat{I} = \int D[A_i^a(\mathbf{x},t_n)]D[p_i^a(\mathbf{x},t_n)]\,\frac{1}{\sqrt{M}}\exp\left\{-i\int d^3x\,A_i^a(x)p_i^a(x)\right\}$$
$$\times\left|p_i^a(\mathbf{x},t_n)\right\rangle\left\langle A_i^a(\mathbf{x},t_n)\right|, \tag{55}$$

where D indicates integration over space dependent functions at a fixed time slice, M is a normalization constant and $|A_i^a(x)\rangle$ and $|p_i^a(x)\rangle$ are the eigenfunctions of the spatial components of the fields and their respective momenta. After going to the limit $N \to \infty$, Eq. (53) transforms into

$$\langle 2|\hat{U}|1\rangle = \frac{1}{N_{\mathrm{T}}} \int D[A_i^a(\mathbf{x}_2,t_2)]\, D[A_j^b(\mathbf{x}_1,t_1)] \langle 2|A_i^a(\mathbf{x}_2,t_2)\rangle \left[\int \mathcal{D}[A_i^a(x)]\, \mathcal{D}[p_i^a(x)]\, \mathcal{D}[\rho^a(x)] \right.$$
$$\times \exp\left\{ i \int_{-L}^{L} dt\, d^3x \left[p_i^a(x)\dot{A}_i^a(x) - \frac{p_i^a(x)p_i^a(x)}{2} - \tfrac{1}{4}F_{ik}^a F_{ik}^a - \frac{m^2}{2}A_k^a A_k^a \right.\right.$$
$$\left.\left.\left. - \frac{1}{2m^2}\nabla_i^{ab}p_i^b \nabla_i^{ac}p_i^c - \rho^a\partial_k A_k^a + \frac{1}{m^2}\nabla_i^{ab}p_i^b\partial_0\rho^a + \frac{1}{2\alpha}\rho^a\hat{O}\rho^a \right]\right\}\right] \langle A_j^b(\mathbf{x}_1,t_1)|1\rangle, \tag{56}$$

where N_{T} represents all the normalization constants. In order to circumvent the presence of a $\delta^{(3)}(0)$ in Eq. (56) arising from the term $\nabla_i^{ab}p_i^b\nabla_i^{ac}p_i^c$, a small "point splitting" in the three space coordinates of the $\nabla_i^{ab}p_i^b$ operators was assumed. Expression (56) coincides with the known canonical functional integral in the physical phase space when the ρ field is not introduced [11]. The form of Eq. (56) corresponding to the canonical theory with constraints follows after introducing unity in the following two ways:

$$\int \mathcal{D}[A_0^a(x)]\, \delta\left(A_0^a(x) - \frac{1}{m^2}\nabla_i^{ab}p_i^b(x)\right) = 1, \tag{57}$$

$$\int \mathcal{D}[p_0^a(x)]\, \delta(p_0^a(x)) = 1. \tag{58}$$

They permit Eq. (56) to be written as

$$\langle 2|\hat{U}|1\rangle = \frac{1}{N_{\mathrm{T}}} \int D[A_i^a(\mathbf{x}_2,L)]\, D[A_j^b(\mathbf{x}_1,-L)]$$
$$\times \langle 2|A_i^a(\mathbf{x}_2,L)\rangle \left[\int \mathcal{D}[A_i^a(x)]\, \mathcal{D}[p_i^a(x)]\, \mathcal{D}[\rho^a(x)]\, D[A_0(x)] D[p_0^a(x)] \right.$$
$$\times \exp\left\{ i \int_{-L}^{L} dt\, d^3x \left[p_i^a\dot{A}_i^a - p_0^a\dot{A}_0^a - \frac{p_i^a p_i^a}{2} - \tfrac{1}{4}F_{ik}^a F_{ik}^a \right.\right.$$
$$- \frac{m^2}{2}(A_k^a A_k^a - A_0^a A_0^a) - A_0^a\nabla_i^{ab}p_i^b - \rho^a\partial_k A_k^a + A_0^a\partial_0\rho^a$$
$$\left.\left.\left. + \frac{1}{2\alpha}\rho^a\hat{O}\rho^a \right]\right\} \delta(p_0^a)\delta\left(A_0^a - \frac{1}{m^2}\nabla_i^{ab}p_i^b\right)\right] \langle A_j^b(\mathbf{x}_1,-L)|1\rangle. \tag{59}$$

In Eq. (59) it is possible to represent the δ–function, which depends on A_0^a, as a Fourier transform over an auxiliary variable λ^a. Then by performing a change of variables $A_0^a \to A_0^a - \lambda^a$ as well as the integration over the momenta and λ^a, relation (59) takes the form

$$\langle 2|\hat{U}|1\rangle = \frac{1}{N_{\mathrm{T}}^{*}} \int D[A_i^a(\mathbf{x}_2,t_2)]\, D[A_j^b(\mathbf{x}_1,t_1)]$$

$$\times \langle 2|A_i^a(\vec{x}_2,L)\rangle \Bigg[\int \mathcal{D}[A_\mu^a(x)]\, \mathcal{D}[\rho^a(x)]$$

$$\times \exp\Bigg\{ i\int_{-L}^{L} dt \int d^3x \left(-\tfrac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a - \frac{m^2}{2}A_\mu^a A_\mu^a + \tfrac{1}{2}\rho^a \left[\frac{1}{\alpha}\hat{O} - \frac{1}{m^2}\partial_0^2\right]\rho^a \right.$$

$$\left. - \rho^a \partial_\mu A_\mu^a \right) \Bigg\} \Bigg] \langle A_j^b(\mathbf{x}_1,-L)|1\rangle, \tag{60}$$

where it was again necessary to integrate by parts using $\rho^a = 0$ at t_1 and t_2.

Then, the above formal reasoning leads to Eq. (60), which indicates that the matrix elements of the evolution operator between physical states should not be affected by the introduction of the ρ–dependent terms. In arriving at Eq. (60) it was important to take into consideration the vanishing boundary conditions for the ρ field at the initial and final times. This fact should play a role, but we do not yet have a clear idea of it.

To conclude, it can be expressed that the formal analysis presented here could have some useful implications for the discussion of the renormalizability versus unitary problem associated to massive "Yang-Mills" theory. In order to clarify this possibility, further work is needed to explore the connections of the arguments with perturbation theory. Results of the investigation of this problem will be presented elsewhere.

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