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Nonlinear models of electroencephalographic dynamics

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ABSTRACT. Alternative nonlinear techniques to the characterization of EEG time series are proposed, which allow their modeling and the prediction of bifurcations due to changes of parameters. These techniques show clear-cut differences between normal and pathological states which can be used as diagnostic tools to the evaluation of patient's normality. We also formulate a new homeodynamical principle of health.

RESUMEN. Se proponen técnicas no lineales alternativas para la caracterización de series temporales de EEG, que permiten la modelación de las mismas, así como la predicción de la aparición de bifurcaciones al cambiar ciertos parámetros. Estas técnicas muestran diferencias marcadas entre los estados normal y patológico del individuo, las cuales pueden ser empleadas como un método diagnóstico de evaluación de la "normalidad" del paciente. También formulamos un nuevo principio homeodinámico del estado saludable del individuo.

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1. INTRODUCTION

Recent years have witnessed an increase of the interest in the application of nonlinear concepts in the analysis of brain activity, mainly through the analysis of the evolution and dynamics of the electroencephalographic (EEG) recordings (measurements of potential differences between fixed points in the scalp of the subject *vs.* time).

Most of the work has been dedicated to the use of the embedding theorem and the calculation of the fractal dimension of the EEG time series [1–8].

In this paper we propose the use of other alternative nonlinear techniques to characterize EEG signals. We will also construct some analytical models to describe EEG dynamics, which will allow us not only to reproduce some characteristics of the real time series, but also to predict the appearance of bifurcations under variations of parameters.

Finally it will be formulated a new homeodynamical principle to describe the healthy state of the individual.

2. NONLINEAR CHARACTERISTICS OF EEG DYNAMICS

Multilateral studies carried out by our group on EEG signals allow us to state beyond any doubt that these time series belong to nonlinear, generally chaotic systems.

This statement is supported by the results of phase portrait analysis, fractal dimension evaluation, study of the evolution of the distance between initially close phase trajectory points and the inverse problem of reconstruction of the nonlinear systems which modelates the series.

These methods were applied to unfiltered and filtered EEG time series corresponding to normal α rhythms an to epileptic seizures (Figs. 1–3). As can be seen in the phase portraits (Figs. 4–6) of the unfiltered data, in spite of chaos and noise, the system shows patterns which are a proof of the existence of order and a structure within itself.

Figures 7 to 9, corresponding to filtered data show that the phase portrait of the epileptic seizure has easily identifiable characteristics, plainly visible to the naked eye, which turns this techniques into one more diagnostic tool that can be used by physicians. Note specially the existence of a double loop in the epileptic's phase portrait, which is absent in normal α rhythm.

The study of these pictures show also that to seen the fully developed pattern of the attractor we have to wait some time until the transient toward the steady dynamical system dies out.

3. Reconstruction of nonlinear models

Taking only the extreme of the filtered time series we have constructed the return map (next-amplitude map) of them, selecting it from several possible Poincaré section and stroboscopic maps.

Figures 10–12 show these return maps for α normal and epileptic seizures. Note the difference between them. The set of points for the α normal map is distributed in a "curve" of roughly dimension 1, while the seizure's set of points of the map are found in several groups (generally three), of nearly zero dimension. The total number of points corresponding to maxima is much higher in the α signal than in the epileptic seizure signal for an equal number of initial time series points.

4. Analytical models

For the data set GPQ310.dat corresponding to an α normal filtered signal, we constructed an analytical model to describe the amplitude series:

$$X_{n+1} = f(X_n), \tag{1}$$

where

$$f(X_n) = C_0 + C_1 X_n + C_2 X_n^2 + C_3 X_n^3, \tag{2}$$

$$C_0 = -4.834485 \times 10^6, \qquad C_1 = 1.6794695 \times 10^5,$$

$$C_2 = -1.944587249 \times 10^3, \qquad C_3 = 7.5051.$$

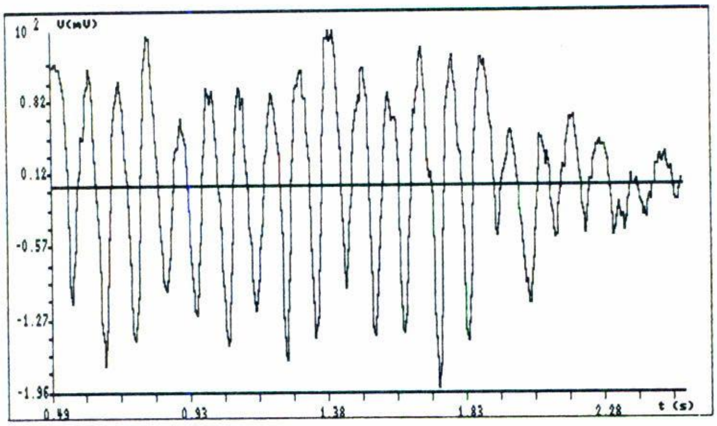

FIGURE 1. Unfiltered EEG signal from normal subject showing normal alpha rhythm.

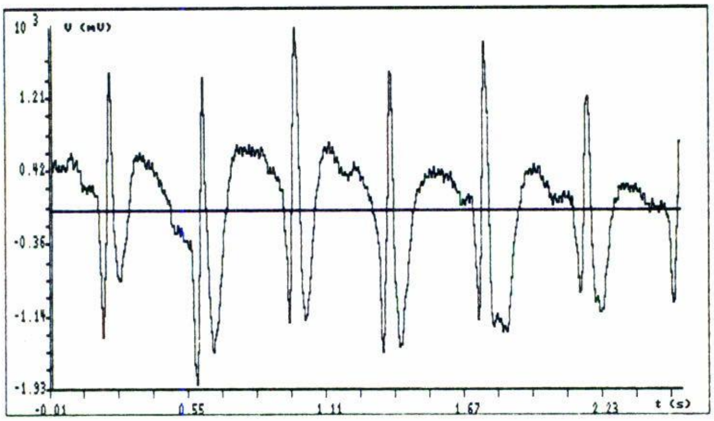

FIGURE 2. Unfiltered EEG signal of subject during an epileptic seizure. Example 1.

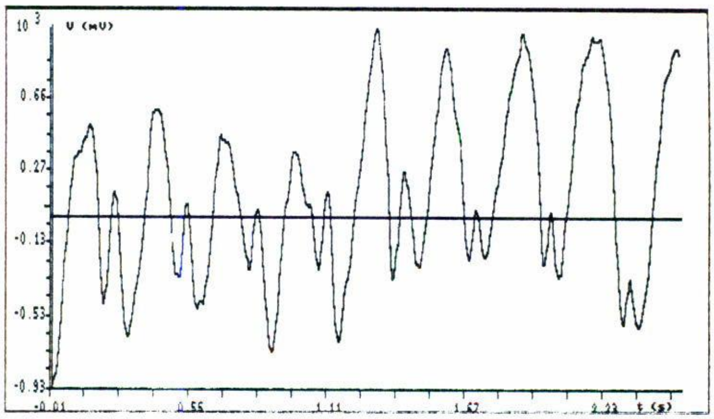

FIGURE 3. Unfiltered EEG signal of subject during an epileptic seizure. Example 2.

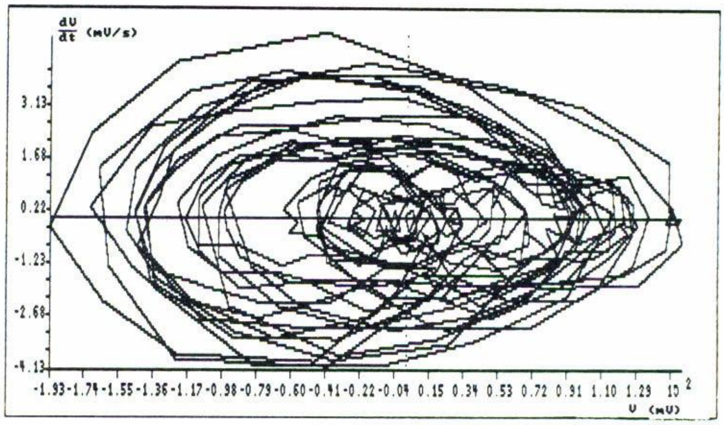

FIGURE 4. Phase portrait of EEG signal corresponding to normal subject with normal alpha rhythm. (unfiltered).

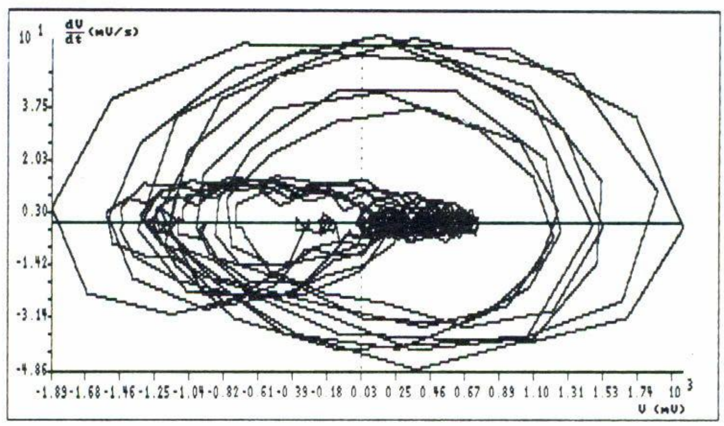

FIGURE 5. Phase portrait of EEG signal corresponding to epileptic subject during a seizure. (unfiltered). Example 1.

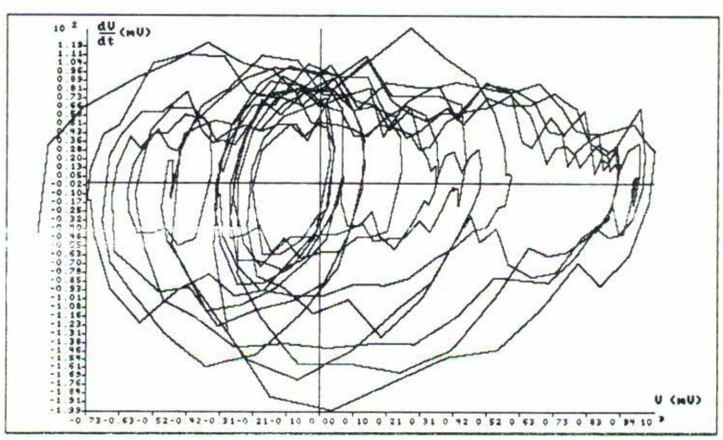

FIGURE 6. Phase portrait of EEG signal corresponding to epileptic subject during a seizure. (unfiltered). Example 2.

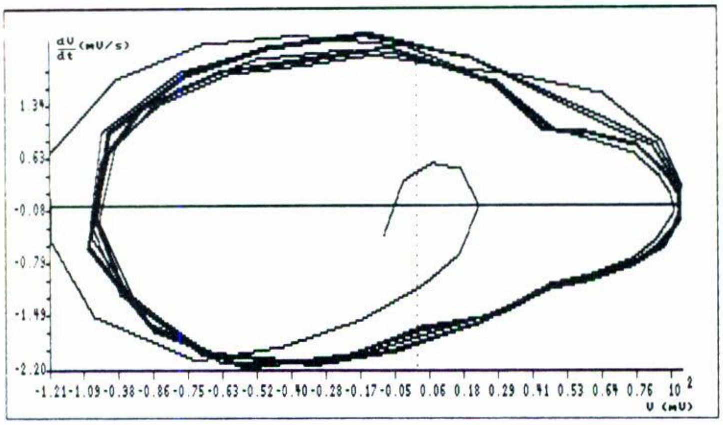

FIGURE 7. Phase portrait of EEG signal corresponding to normal subject with normal alpha rhythm. (filtered).

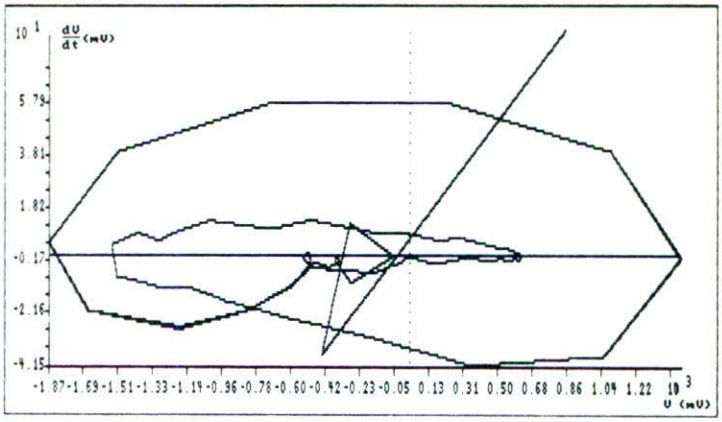

FIGURE 8. Phase portrait of EEG signal corresponding to epileptic subject during a seizure. (filtered). Example 1.

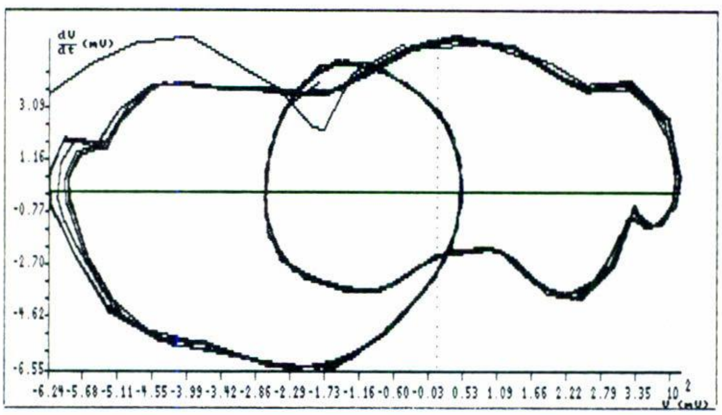

FIGURE 9. Phase portrait of EEG signal corresponding to epileptic subject during a seizure. (filtered). Example 2.

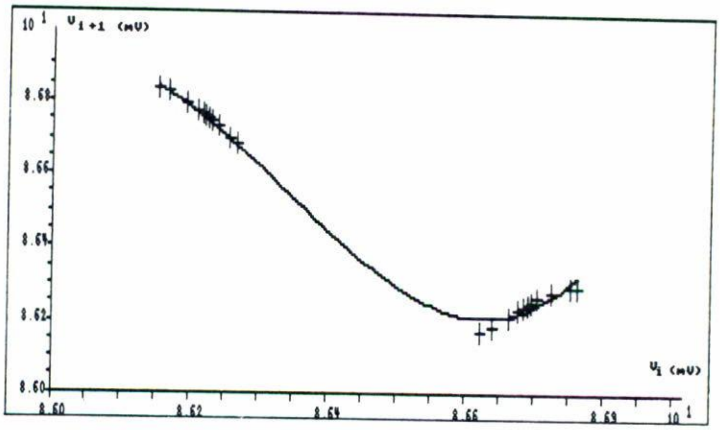

FIGURE 10. Return map of filtered alpha normal EEG signal and interpolation with polynomial fitting of the data.

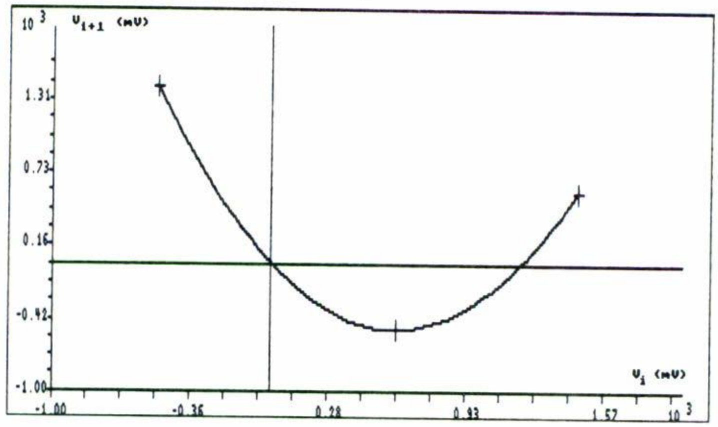

FIGURE 11. Return map of filtered alpha normal EEG signal and interpolation with polynomial fitting of the data. (Example 1.)

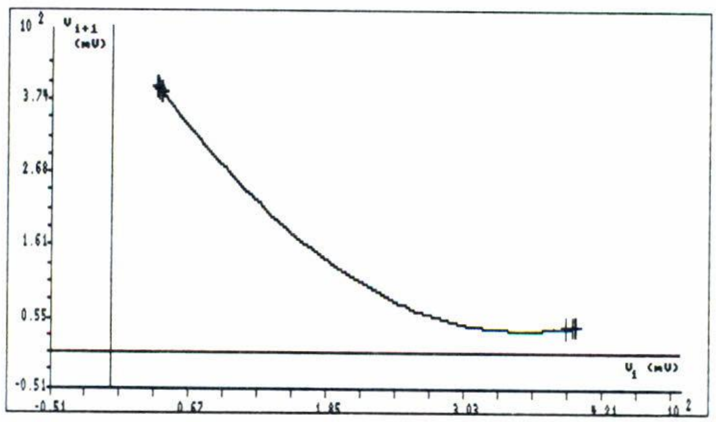

FIGURE 12. Return map of filtered alpha normal EEG signal and interpolation with polynomial fitting of the data. (Example 2.)

The results of this modeling can be seen in Figs. 10-12 on that data and also in epileptic EEGs.

Using Eq. (1) we can reproduce the sequence of amplitudes of the original series.

These reproduced sequences are shown in Figs. 13–15. It can be seen that the sequences for epileptic signal correspond to limit cycles of period 1 and 2, while it can be shown that the sequence for the α signal is chaotic and forms a strange attractor.

The fixed points for system (1) are

$$X_1 = 85.964809, \qquad X_2 = 86.068942, \qquad X_3 = 87.068617.$$

The points X_1 and X_3 are unstable. Any x_0 outside the interval $X_1 < x_0 < X_3$ lead to trajectories that space away from that interval. If the initial value lies inside the interval, then the systems falls into an attractor.

If we take as x_0 a value which is not in the vicinity of the set of experimental values of the return map series of the epileptic signals, then it can also lead to sequences with a high degree of chaoticity, as can be seen in Fig. 16, for the same map of Fig. 14.

In principle it is now possible to take the parameters C_i as variables and study the different bifurcations of the system, by varying one parameter with the others held constant.

The map (1) presents period doubling bifurcations like the well known cubic map, until transition to chaos is stablished.

The chaotic states are identifiable by the value of the Lyapunov's exponent:

$$\sigma = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \ln \left| \frac{df(n)}{dn} \right| . \tag{3}$$

We have found that slight variation of C_3 can produce Thom's elementary-catastrophe-type bifurcations. IF C_3 goes above, or below certain limits, two of the fixed points (either X_1 and X_2 of X_2 and X_3 can disappear. The surviving critical point is unstable and the system must jump to another regime.

5. HOMEODYNAMICAL PARADIGM OF HEALTH

Several authors [8] have noted shortcomings in the so called principle of homeostasis, which states that any physiological variable must return to its normal stationary state (in dynamical systems concepts that state would be a stable critical point) after being perturbed.

These have been also some discussions [8] about certain homeodynamical principle which postulates the existence of multiple metastable states for every physiological variable.

The erratic behavior of healthy physiological systems must be associated with a new paradigm of health. From our point of view, the "normal" state of a physiological variable, more than to a stable critical point, or a stable limit cycle, must be associated to a strange attractor. (The attractor is stable as a whole, but with an intrinsic variability due to instabilities in the phase trajectories.)

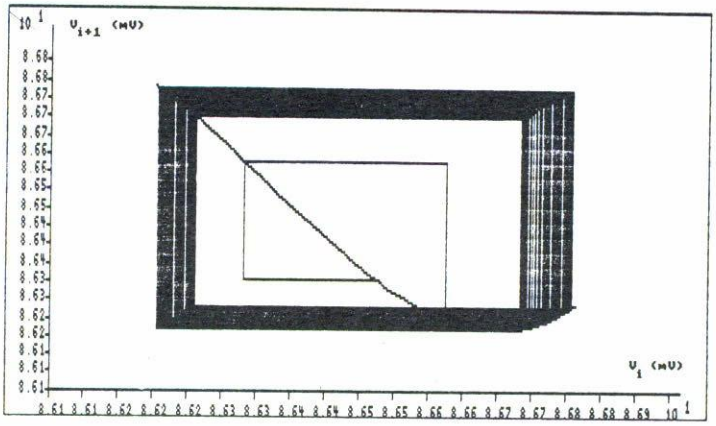

FIGURE 13. Reconstruction of the sequence of points in the return map of alpha normal EEG signal with the use of the polynomial fitting.

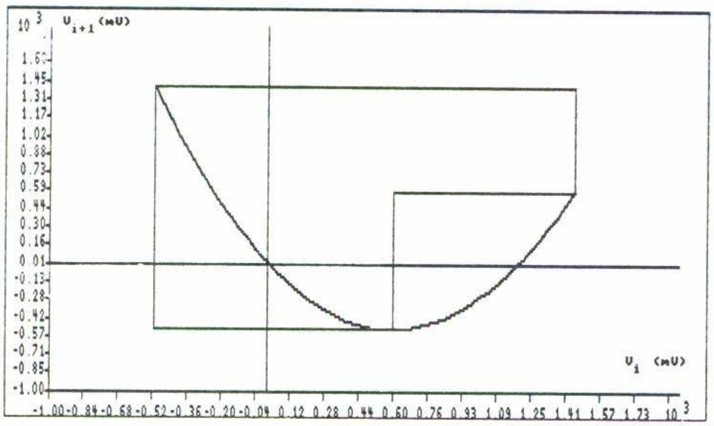

FIGURE 14. Reconstruction of the sequence of points in the return map of the epileptic subject's EEG signal with the use of the polynomial fitting. Example 1.

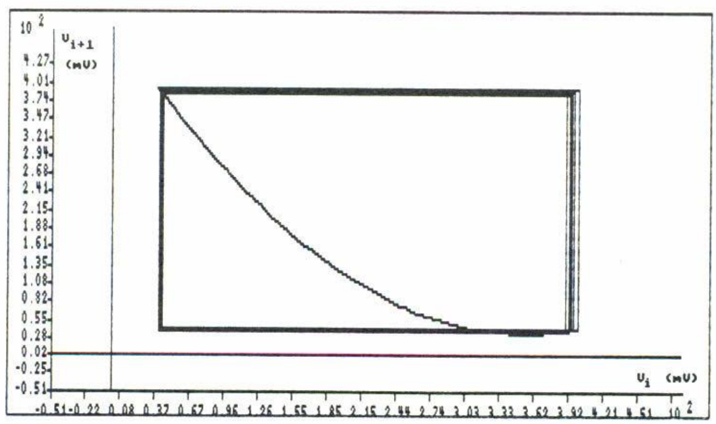

FIGURE 15. Reconstruction of the sequence of points in the return map of the epileptic subject's EEG signal with the use of the polynomial fitting. Example 2.

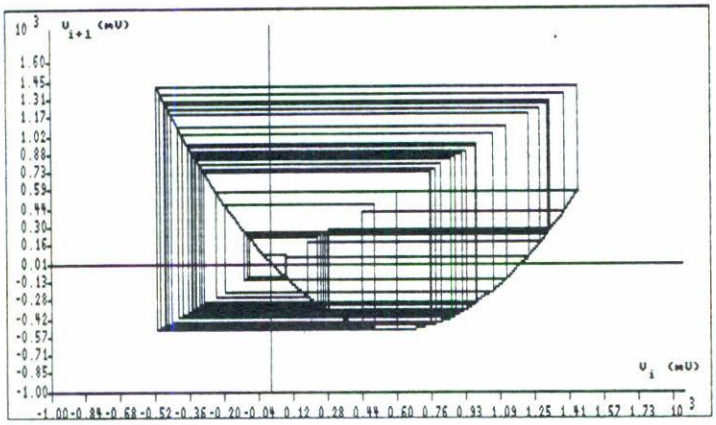

FIGURE 16. Reconstruction of the sequence of points in the return map of the epileptic subject's EEG signal with the use of the polynomial fitting using a different starting point for the sequence.

The physiological variable, after a perturbation must return to a normal state, but not necessarily to a stable fixed value, and instead will oscillate chaotically around it.

There seem to be an interval of values of the fractal dimension which is optimum for the paradigm of health. Any bifurcation which takes the system out of that interval leads to sickness of aging of the physiological system.

Specially a transition to more orderly states can indicate the presence of some diseases, even if generally authors relate chaos with specific pathologies [8]. A great increase in chaoticity, too, is a sign of abnormality. There are some data related to this in cognition studies, schizophrenia and several other pathologies of high fractality [13].

6. CONCLUSIONS

The possibility of writing equations of dynamical system which include attractors that can be used to reproduce characteristics of EEG time series is another proof of the existence of nonlinear coherent structures which describe different brain states.

We have also found that EEG time series have a somewhat long transient before entering into the attractor which describes them.

There are several unstable points in the vicinity of the attractor. An initial condition outside the normal interval of values can lead the system to unstablities that can drive it to another stable regime outside the previous interval of values.

Therefore there is probably multistability in the dynamical system which describes the EEG signal. This multistability reflects in the existence of multiple stable states and the coexistence of several types of attractors.

In many cases, small variations of parameters can produce Thom's catastrophe-type bifurcations which produce the appearance and disappearance of pairs of stable-unstable fixed points. In the case the system passes to another stable regime with another attractor.

The existence of multistability and nonlinear spatio-temporal interaction leads to the possibility of the existence of spatio temporal coherent structures which can be solitonlike [9,10] (not necessarily of the time-independent shape, travelling wave type, but pulsating time dependent solitons or even chaotic solitons [11,12]). These structures can be responsible for the transmission of different perturbations from a focus to the whole brain.

Our research shows the possibility of the existence of an optimal interval of chaoticity (fractal dimension) in the healthy individual. This could solve the controversy existing in literature about the significance of chaos in physiology and the characterization of normality.

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