The interferometric pattern of a simple radiointerferometer for a circular source

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ABSTRACT. The pattern equation of a simple radio-interferometer is deduced. On the basis of this equation the response and the visibility for a circular source are computed. The analysis of the visibility as a function of the source diameter and the distance between the antennas is done. On the basis of the behavior of the visibility function applications of interferometers for solar observations and communications are briefly analyzed.

RESUMEN. Se deduce la ecuación que describe el patrón de un radio interferómetro simple. Con base en esta ecuación se encuentran el patrón de respuesta y la visibilidad para una fuente circular. Se hace el análisis del comportamiento de la visibilidad en función del diámetro de la fuente y de la distancia entre las antenas. A partir del comportamiento de la función de visibilidad se hace un breve análisis de posibles aplicaciones de interferómetros, tanto para observaciones del Sol como para comunicaciones.

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1. INTRODUCTION

In the last years the interferometers has been widely used in radio-astronomy. This has been due most of all to the fact that an interferometer may attain high spatial resolutions. The spatial resolution of an interferometer is determined by the distance between the antennas, while the spatial resolution of an antenna is determined by its diameter. Thus, relative small antennas distributed in an interferometric array may substitute a big antenna.

The high resolution that may be attained by an array of antennas is not the only advantage of an interferometer over an antenna. The use of interferometers may give other ones. However, the principles on which an interferometer works substantially differ from those of an antenna. So, the understanding of these principles is important in order to get real advantages in the use of an interferometer.

Some properties of the interferometric parameters, as the increasing of the resolution with the increasing of the distance between the antennas, have been widely used. However, another property such as the possibility to exclude the signal of an extended source have been less commonly used.

Here, we demonstrate that an interferometer may be used to exclude the radiation of extended sources while conserving that of relatively small ones. This possibility constitutes an important difference between an interferometer an a simple antenna. To show it we compute the interferometric pattern and the visibility for the particular case of a circular source. Finally, an application of the visibility characteristics is given for solar observations and also for communications.

2. The interferometric pattern

The power that an antenna gives at its feed for a given direction, relative to an antenna axis, is named the "antenna response". The antenna response to a radiating source is expressed by the power diagram $P_{\text{an}}(\vartheta, \varphi)$, a function of the source coordinates [1]. The power diagram is a dimensionless quantity usually normalized to the unity, it is often simply named the antenna diagram.

In the following analysis, in order to avoid confusions, we will use the term "diagram" only to refer to the antenna response, while we will use the term "pattern" to refer to the interferometer response. On the other hand, we will say that the antenna is observing a source when the maximum of the antenna diagram is directed to it.

To deduce the equation for the interferometric response let us see how to express the induced voltage, in terms of the radiation parameters (frequency and phase), induced at an antenna by a source radiating to it.

Let us assume that a parabolic antenna, whose diagram is given by P_{an} is observing to a point source on the sky. The electromagnetic field from the source is reflected by the antenna surface to the focus. We can assume that this field induces there a voltage V. This voltage will be transmitted by waveguides to a receiver (R in Fig. 1) where the signal will be amplified and recorded. Assuming monochromatic radiation of frequency $\nu = \omega/2\pi$, the induced voltage can be expressed as

$$V(t) = V_0(t) P_{\text{an}} \cos(\omega t - k\delta), \tag{1}$$

where $(k\delta)$ is the phase, k is the wave number and δ is the delay path of the incident field at the time t, $V_0(t)$ characterizes the time variation of the source intensity. The delay (δ) depends on the time that we take as reference.

When the source radiation (referred also as the signal) is received by two antennas there is, in a general case, a difference l between the beam paths of the radiation coming from the source to the antennas. For the case drawn in Fig. 1 a wave front coming from a point source arrives before to antenna 1. In this case the path to antenna 2 (A_2 in Fig. 1) is larger by a distance l, which is given by

$$l = D \sin \vartheta, \tag{2}$$

where ϑ is the zenithal distance angle on the interferometer axis direction and D is the distance between the antennas.

Taking into account the difference l we will compute the induced voltages at the antennas. Let the time t, when a wave front coming from the source arrives to the point located in the middle between the antennas, be the time reference of Eq. (2) and the phase $(k\delta)$ of a beam coming to this point equal zero. Then, the delay path of a beam coming to

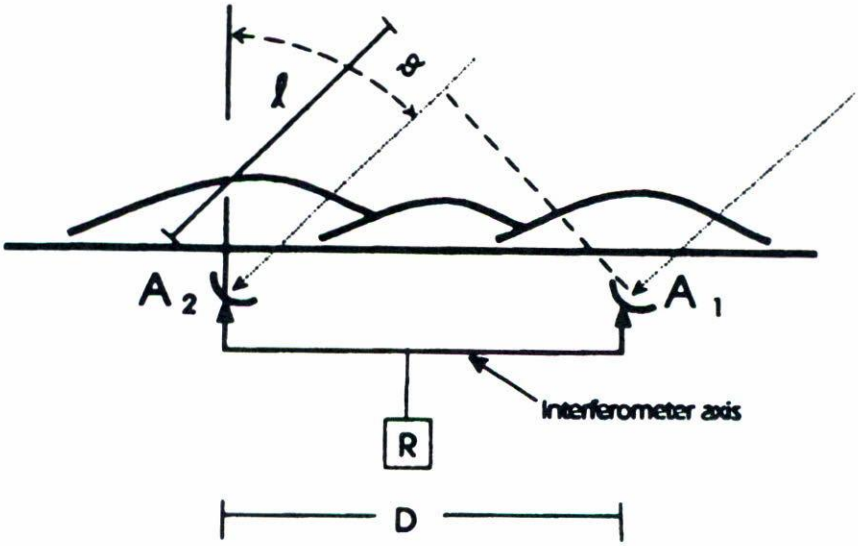

FIGURE 1. Simple interferometer of two-element interferometer. The path of the beam coming to antenna 2 (A_2) is longer, by a distance "l", than the path of the beam coming to antenna 1 (A_1).

antenna 1 will be $\delta_1 = -l/2$ and the delay of a beam coming to antenna 2 will be $\delta_2 = l/2$. The induced voltages at antenna 1 (A_1) and at antenna 2 (A_2) will be, respectively,

$$V_1 = V_0 \cdot P_{\text{an}} \cos\left[\omega t + \frac{2\pi}{2\lambda} l\right] \tag{3.a}$$

and

$$V_2 = V_0 \cdot P_{\text{an}} \cos\left[\omega t - \frac{2\pi}{2\lambda} l\right]. \tag{3.b}$$

We have assumed that the response P_{an} is the same for both antennas. On the other hand, to have a simpler notation the argument of $V_0(t)$ in Eq. (3) was omitted and will not be written in the further analysis.

For a simple interferometer (consisting of two antennas) the interferometric pattern is parallel to the direction of the join axis (Fig. 2). In the further analysis we will consider any angular distance (in particular zenithal distances) with respect to the direction of the interferometer axis. On the other hand, we will use P_{an} as a function of only one coordinate also in this direction.

The voltages V_1 and V_2 may be added leading to a voltage

$$V = 2V_0 P_{\text{an}} \cos\omega t \cos\frac{\pi l}{\lambda}. \tag{4}$$

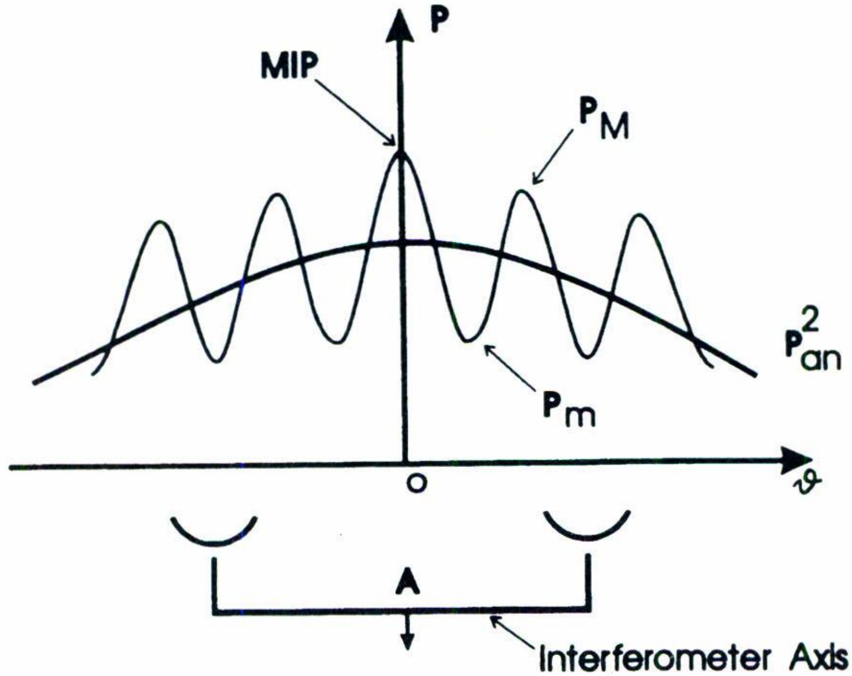

FIGURE 2. The response of a simple interferometer to a point source. The letter A denotes the point where the signals are added. The letters MIP denote the maximum of the interferometric pattern, P_M a local maximum and P_m a local minimum. P_{an} gives the power diagram of the antenna.

The adding of the voltages produces a simple interferometer. When the signals were added the power of the resulting signal (V^2) is sent to a detector which eliminates the high frequency component ($\cos \omega t$). This may be seen as the mean value in a period of $\cos^2 \omega t$ which is equal to 1/2. So, the induced power, after the detector, is equal to

$$P = \frac{V_0^2 P_{an}^2}{2} \left(1 + \cos \frac{2\pi l}{\lambda}\right). \tag{5}$$

Summarizing we may say that the P given by Eq. (5) is the power induced by a source at an adding interferometer. The curve which describes this function is constituted by several lobes (Fig. 2). Since the term P_{an} is normalized to the unity P_{an}^2 will be also normalized to the unity. On the other hand the function due to the interference between the signals $(1/2(1 + \cos \frac{2\pi l}{\lambda}))$ is also normalized to unity. So the induced power is directly related to the power of the emitting source (V_0^2).

Since the lobes produced by the interference are modulated in amplitude by the function P_{an}^2, the interferometer response to a point source has an absolute maximum (Fig. 2). We will refer to it as the maximum of the interferometer pattern (MIP). If the maximum of P_{an} for both antennas and the maximum of the function $1/2(1 + \cos \frac{2\pi l}{\lambda})$ are directed to a source then we will say that the interferometer is observing this source (the MIP will be directed to it).

For a fixed distance D between the antennas and for a given λ the interferometric pattern is a function only on the source coordinates but, as have been pointed out before, for a simple interferometer the power may be given as a function of only one coordinate (ϑ).

Since the difference between the beam paths (l) is directly related to the source position (ϑ) we can see that the interferometer pattern is also a function of l. It is clear that, if we use the first half of a period, the power (P) increaseas as l decreases and the power P will attain its maximum value at $l = 0$.

The signals may be added in a given point in the waveguides, as shown in Fig. 1. If the length of the waveguides from antenna 1 to the point A is equal to the length of the waveguides from antenna 2 to this point then, for a source located at the zenith, the path differences will be $l = 0$. If both antennas are observing to the zenith then the MIP will be also directed to this point.

3. The response to an extended source

To know the response to an extended source the MIP should cross it, *i.e.*, should be first directed to an extreme of the source and then should cross it to reach the other source extreme. The same result may be obtained directing the MIP to a point on the sky on the source trajectory and then waiting as the source crosses this point. In our analysis we will follow this variant assuming the MIP is directed to the zenith and will use ϑ as the zenithal distance on the interferometer axis direction. The coordinate ϑ will take positive values in the side from which the source is coming and will take negative values in the other side.

The power induced by an extended source depends on the brightness distribution over it. Each point on the source will induce a power P at the interferometer. This power depends on the brightness of the point which determines the voltage V, and on the difference between the paths (l) of the beams coming to the antennas from the given point.

Let us compute the induced power when the source center is located at a zenithal distance ϑ_0. On this situation a source region $d\vartheta$ at a distance ϑ from the source center will lead to a path difference (l) that equals to

$$l = D\sin(\vartheta_0 + \vartheta). \tag{6}$$

It is clear that for the source center the difference of the beam paths will be $l_0 = D\sin\vartheta_0$, that is the same difference given by Eq. 2. This is due to the fact that the last term was found for a point source and for the case $\vartheta = 0$.

The power induced by the element $d\vartheta$ at a distance ϑ from the source center is

$$dP = \frac{V_0^2 P_{\rm an}^2}{2}\left[1 + \cos\left(\frac{2\pi D}{\lambda}\sin(\vartheta_0 + \vartheta)\right)\right] d\vartheta. \tag{7}$$

The response to all the source may be found integrating Eq. (7) over a function $S(\vartheta, \varphi)$ which describes the source brightness distribution. As have been pointed out before, the pattern of a simple interferometer has interferometric lobes only in the direction of its axis. So, we can not compute the response, *i.e.*, the pattern, of a simple interferometer to an extended source with arbitrary brightness distribution. However, to do the analytical study of the response of a simple interferometer to an extended source we may choose a particular case where the brightness distribution over the source may be written in dependence of one coordinate. For such a case the induced power will be given by

$$P = \frac{V_0^2 P_{\rm an}^2}{2} \int_{\vartheta_s} \left[1 + \cos\left(\frac{2\pi D}{\lambda} \sin(\vartheta_0 + \vartheta) \right) \right] S(\vartheta)\, d\vartheta. \tag{8}$$

Let us compute the response to a circular source with uniform brightness distribution. In such a case the brightness of a given point does not depend on its coordinates. On the other hand the shape of the source may be written in a parametric way. So, the brightness may be simply given by the expression of a circle. A differential area of the surface may be written as

$$S(\vartheta)\, d\vartheta = 2Q \left(\frac{\vartheta_{\rm s}^2}{4} - \vartheta^2 \right)^{1/2} d\vartheta. \tag{9}$$

The term Q is a factor to normalize the integral of Eq. (9) to the unity. Since we assumed an uniform brightness distribution, the factor Q^{-1} is in fact the area of a disk which diameter is $\vartheta_{\rm s}$, and then

$$Q = \left(\frac{\pi \vartheta_{\rm s}^2}{4} \right)^{-1} \tag{10}$$

On the other hand, providing that the source is not too large, we have that $l/D \ll 1$. Then, using the approximation $\sin(\vartheta_0 + \vartheta) \simeq (\vartheta_0 + \vartheta)$ in Eq. (8) and substituting Eq. (9) on Eq. (8) we have that

$$P = \frac{V_0^2 P_{\rm an}^2}{2} Q \int_0^{\vartheta_s/2} \left[1 + \cos\left(\frac{2\pi D}{\lambda} (\vartheta_0 + \vartheta) \right) \right] \left(\frac{\vartheta_{\rm s}^2}{4} - \vartheta^2 \right)^{1/2} d\vartheta. \tag{11}$$

Expanding the term $\cos\left(\frac{2\pi D}{\lambda}(\vartheta_0 + \vartheta)\right)$ to its sin and cos-terms (the integral of the sine component is equal to zero) and introducing the notation

$$z = \frac{\pi D \vartheta_{\rm s}}{\lambda}, \tag{12}$$

we have that the power induced by all the source may be expressed as

$$P = \frac{V_0^2 P_{\rm an}^2}{2} \left[1 + \frac{J_1(z)}{z/2} \cos\left(\frac{2\pi D}{\lambda} \vartheta_0 \right) \right], \tag{13}$$

where $J_1(z)$ is the Bessel function of first order.

Equation (13) is fundamental for the understanding of the interferometer characteristics. This equation involves both interferometer parameters (D and λ), source attributes as the source shape (in this case circular) and the source diameter (ϑ_s). Summarizing, we can say that Eq. (13) gives the power induced at a simple interferometer by a circular source with uniform brightness distribution.

4. THE INTERFEROMETER VISIBILITY

The visibility is an interferometer function of the source size. It is defined as

$$M = \frac{P_M - P_m}{P_M + P_m}, \tag{14}$$

where P_M and P_m are respectively the maximum and the minimum induced powers.

As we can see from Eq. (13), having a fixed λ/D the values P_M and P_m are attained at those source positions where $\cos\left(\frac{2\pi D}{\lambda}\vartheta_0\right)$ takes its extreme values. As a result the visibility (M) does not depend on ϑ_0.

Substituting in Eq. (14) the values of P_M and P_m computed from Eq. (13) we obtain that the visibility for a disk with uniform brightness distribution is given as

$$M = \frac{J_1(z)}{z/2}. \tag{15}$$

As we can see from Eq. (15) the visibility depends on z only. If we have a fixed λ/D the visibility will depend only on the source size.

It should be noted that the visibility, as well as the power pattern, depends on the brightness distribution over the source. However, they represent quite different interferometer characteristics; the power pattern is a function of the source position while the visibility is a function of the source size.

The visibility maximum is attained at $z = 0$ (Fig. 4). This implies, as we can see from Eq. (12), that $M = 1$ for $\vartheta_s = 0$, *i.e.*, the visibility is maximum for point sources. For extended sources the visibility decreases. This is due to the decreasing of the difference between P_M and P_m, since for extended sources the interferometric lobes are smoothed (Fig. 3).

From Eq. (15) we can see that the visibility has a zero at $z = 3.83$ (the first zero of J_1). When the visibility is equal to zero the interferometric lobes disappear, *i.e.*, they are smoothed to the level of the thick line of Fig. 2. In other words, the interferometric pattern disappears. We can see that this situation takes place [see Eq. (15)] when

$$\frac{\pi D \vartheta_s}{\lambda} = 3.83. \tag{16}$$

The fact that the visibility may be equal to zero allows us to compute D/λ in order to do the visibility zero for a selected ϑ_s.

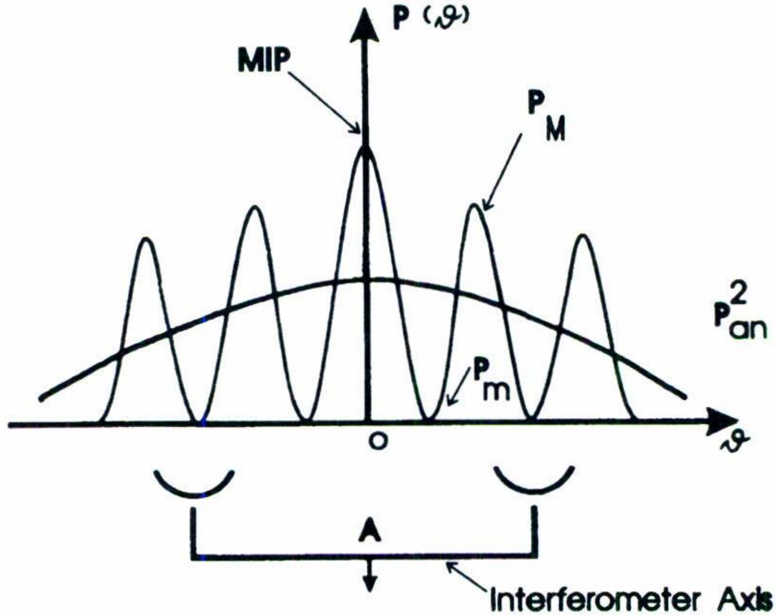

FIGURE 3. The response of a simple interferometer to an extended source. Note that the interference lobes have been smoothed.

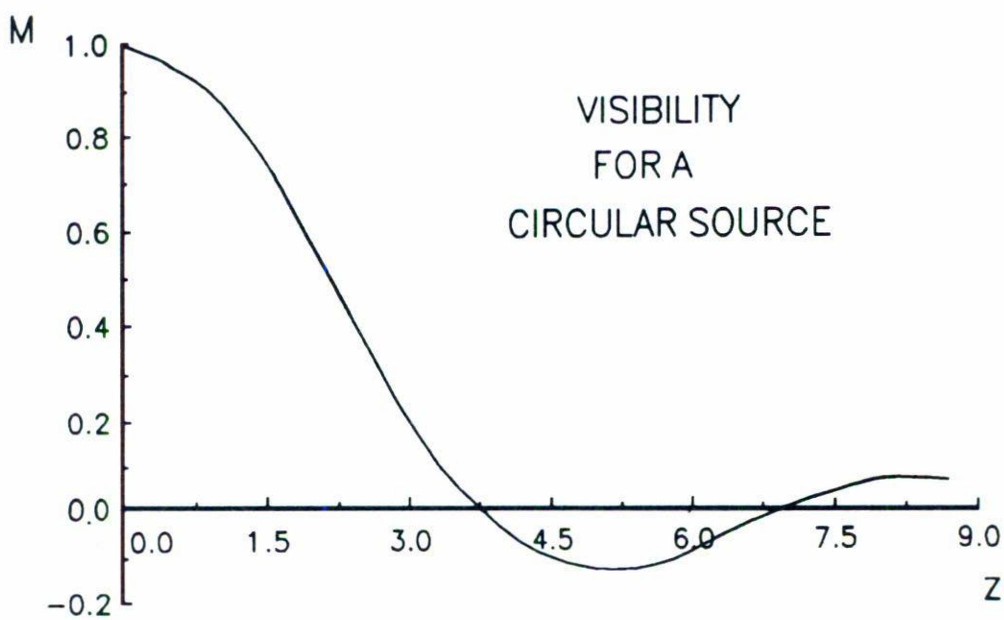

FIGURE 4. The visibility (M) as a function of $Z = \frac{\pi D \vartheta_s}{\lambda}$ for a circular source.

5. APPLICATIONS

The use of a distance equal to 131 wavelengths ($D/\lambda = 131$) between the antennas has been studied in Ref. [2] with the aim to be applied for the observation of solar active regions.

The Sun diameter is $\vartheta_s = 32'$. As may be seen from Eq. (16) the visibility for the solar disk will be equal to zero if

$$\frac{D}{\lambda} = 131 \text{ (radians)}. \tag{17}$$

The solar disk radiation, under the condition $D/\lambda = 131$, will not be observed by a simple interferometer while the maximum visibility of the interferometer will be attained for point sources.

In microwaves the active regions (AR) and the burst sources (BS) are small compared with the solar disk. So, using a distance $D = 131\,\lambda$ between the antennas a simple interferometer is able to receive radiation from these sources without solar disk emission.

In communications, and specially for satellite reception, centimeter and decimeter wavelength bands are wide used. Up to now millimeter wavelengths almost are not applied in satellite receivers since the terrestrial atmosphere strong affects the signal.

The millimeter radiation is extinguished by clouds of water vapor. On the other hand, density inhomogeneities in the atmosphere lead to signal fluctuations. So, even without clouds these fluctuations may to hide the information given in millimeter wavelengths.

The atmospheric inhomogeneities, seen from the earth, are comparatively larger than the emitting antenna of a satellite. On the other hand, the mean shape of the inhomogeneities may be taken as circular. So, the fluctuations may be considerable diminished with the use of a receiving interferometer. The question is, what is the range of sizes of the atmospheric inhomogeneities seen from the earth?. If the range is not too large it is possible to compute the distance between the antennas at which the visibility is equal or near zero for the inhomogeneities. In such a way it seems possible to get a more clean signal from satellites in some frequencies of the millimetric band.

We should note that in general the finite size of a source (ϑ_s), different of that of a mathematical point, in the classical experiment of interference, will lead (for a given λ) to the disappearing of the interference lobes when the distance between the holes will be equal to

$$D = \frac{3.83\,\lambda}{\pi \vartheta_s}. \tag{18}$$

Alternatively, for a given distance D the lobes will disappear for a wavelength given by Eq. (18). These results were found for electromagnetic waves, however they may be extended to waves of other nature.

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