Soliton solutions in optical fiber devices possessing periodical high gain profiles

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Abstract. We study the behavior of optical pulses propagating in optical fibers possessing periodical gain profiles which do not satisfy the adiabatic amplification conditions. We demonstrate that it is possible to obtain first and higher order soliton solutions, and we predict their final asymptotic amplitudes and widths as well as some of their transient characteristics. These predictions agree with results of numerical simulations and allow to describe the global behavior of optical fiber devices that use periodical gain profiles, such as high gain doped fiber amplifiers.

Resumen. Estudiamos el comportamiento seguido por pulsos luminosos que se propagan en fibras ópticas que poseen perfiles de ganacia peíodicos que no satisfacen las condiciones de adiabaticidad. Demostramos que es posible obtener soluciones del tipo solitón, tanto de primer como de órdenes más altos, de los cuales predecimos sus amplitudes y anchuras asintóticas, así como algunas de sus características transitorias. Estas predicciones concuerdan con resultados numéricos y permiten caracterizar de una manera general a los dispositivos de fibra óptica sujetos a procesos períodicos de ganancia, como es el caso de los amplificadores de fibra óptica activada con impurezas resonantes.

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1. Introduction

There are many physical and practical situations where an optical pulse is subjected to repetitive cycles of gain and loss in optical fibers. We can classify them into two main and broad groups according to the fundamental problem they dealt with. In the first group we consider those where the amplification offset the energy loss caused by the pulse propagation through long enough lengths in an optical fiber, and into a second group those where absorption controls the optical gain given to the pulse by an external source.

The first group is at the core of long distance optical fiber communications, and it includes the special case of soliton-based systems. There, the solitons are subject to such a periodical loss-gain cycle not only to recover its original energy but also to maintain its characteristics shape [1]. The stability of the solitons under these kind of models; that is, its perfect recovering after each cycle, has been successfully accomplished in laboratories

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when Raman Stokes conversion is used as the source of gain [2]. Physically, the stability of the solitons occurs because both the absorption and the amplification process are carried out in an adiabatic way, and the amplitude and the duration of the pulse are smoothly adjusted through each cycle, leaving its hyperbolic secant profile basically unaffected [3]. From the mathematical point of view, the stability of the soliton through a periodical adiabatic absorption-amplification process has also been proved [4], and it occurs provide the following two conditions are satisfied: i) the absorption (or amplification) coefficient times the soliton period will be much less than unity, and ii) that the amplifiers separation will be less than the soliton period [5].

On the other hand, we can label the second group as optical fiber lasers as those that require of losses to control the pulse amplification caused by an active medium. There, an optical fiber doped with active atoms acts as the gain source and the output mirror represents the loss mechanism. A soliton laser, the special case in which the output pulses are optical solitons, is of remarkable practical interest but, in order to be usefull, it needs also to reach the stability criterion. Experimental works on erbium doped fiber laser gave evidence of stability [6], and a physical argument for supporting such a stable results was given later [7]. The key idea for obtaining a uniform soliton train at the output of a doped-fiber soliton laser is that the average of the pulse intensity given to the pulse within the active fiber has to be identical to that of the soliton.

In this paper we study, both analytically and numerically, the behavior of a soliton subjected to a periodic gain profile in conditions of sudden amplification. We show that, after a transient regime, an initial pulse should evolve into a soliton. Our analysis predicts that the fundamental characteristic of the resulting soliton, its order and its width, just depend on the spatial average of the energy transferred to the pulse by the gain profile. We also show that it is possible to get physical information on the transient process. In Sect. 2 we detail the physical model we are concerned, and in Sect. 3 we reduce, using perturbation theory, the equation of motion to an exact NLSE. The asymptotic soliton solutions are given and the influence of a specific gain profile is discussed. In Sect 4 we will test the predicted soliton parameter with the results of numerically solving the original equation of motion. Finally, Sect. 5 contains the conclusions of our study.

2. THE PHYSICAL MODEL

We consider an optical pulse propagating in the anomalous dispersion region of an optical fiber consisting of M segments of length L. In each segment a given gain profile $g(Z)$, containing both amplification and absorption, acts on the pulse. Figure 1 shows the two specific gain profiles which we will considered through this paper. In (a) a simple step-like distribution, representing constant amplification followed by constant absorption, is presented. In (b), an harmonic distribution is assumed. The first one becomes of practical interest in the limit $L_a \to L$, where it can describe the conditions found in rare-earth doped fiber lasers. In such a limiting case, the resulting abrupt loss should represent the output laser mirror. On the other hand, the gain profile of Fig. 1b becomes of fundamental importance for describing the general case of an arbitrary spatial gain profile which is expanded in fourier series. Let us assume that the adequate physical conditions are

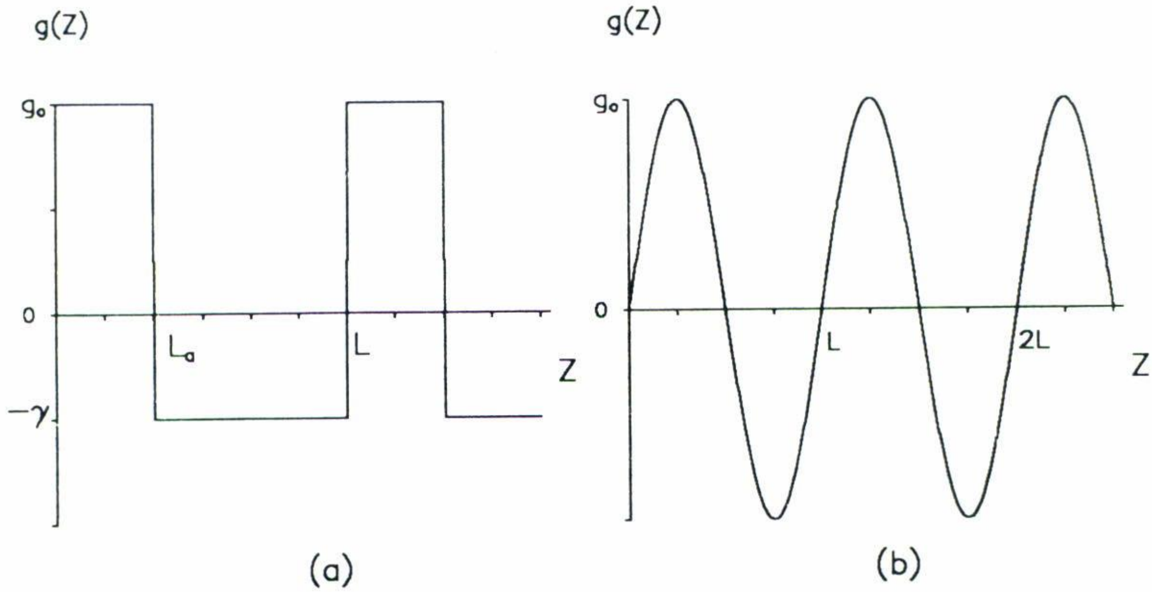

FIGURE 1. Periodical gain profiles considered through the paper. In (a) a step-like profile with constant amplification and absorption regions and in (b) a sinusoidal gain profile.

satisfied [8] to make unnecessary the further consideration of high-order effects. Then, the evolution of the complex pulse envelope $q(Z,T)$ is accurately described by

$$i\frac{\partial q}{\partial Z} = \frac{1}{2}\frac{\partial^2 q}{\partial T^2} + |q|^2 q + i\tfrac{1}{2}g(Z)q, \tag{1}$$

where $Z = z/L_\mathrm{d}$ and $T = (t - z/V_\mathrm{g})/t_0$ are the normalized propagation distance and local time, respectively. $L_\mathrm{d} = t_0^2/|\beta_2|$ is the dispersion length, t_0 the initial pulse duration, V_g the group velocity and β_2 the group velocity dispersion at the wavelength λ.

We do not consider any physical or mathematical restriction on the shape of $g(Z)$, other that being periodic with period L. However, Eq. (1) implies an exponential grow for the pulse energy, $W(z) = \int |q|^2\, dT$, given by

$$W(Z) = W_0 \exp\left[\int_0^z g(s)\, ds\right], \tag{2}$$

where W_0 is the initial pulse energy. Therefore, in order be able to achieve soliton propagation we require $W(L) = W_0$ as a stationary condition for the pulse energy. From Eq. (2), such a condition imposes the additional restriction

$$\int_0^L g(s)\, ds = 0 \tag{3}$$

to the gain profile $g(Z)$. For the step-like gain profile (Fig. 1a) Eq. (3) means that the amplification and absorption coefficients are related by $g_0 L_a = \gamma(L - L_a)$. For the sinusoidal gain profile of Fig. 1b Eq. (3) is automatically satisfied regardless its gain amplitude g_0 or its period L.

In addition, we are not restricted to adiabatic amplification or absorption process, which requires $|g_0| \ll 1$ in Eq. (1). Indeed, we are namely interested in cases of high gain at short propagation distances with respect to the dispersion length L_{d}, because its direct applications to doped optical fiber lasers. Under conditions of high gain, $|g_0| > 1$, the parameters of an initial pulse are considerably affected during one propagation period, In Fig. 2a we show the temporal evolution of an initial pulse of the form $q(0,T) = \operatorname{sech} T$ as it propagates through a fiber of length $L = \pi/2$ ($= z_0$, the soliton period), which posses the harmonic gain profile of Fig. 1b with $\Omega = 2\pi/L = 4$ and $g_0 = 2$. In order to quantify the pulse evolution we plot in Fig. 2b the spatial behavior of (a) the pulse energy $W(Z)$, of (b) the pulse peak intensity $I_{\mathrm{p}}(Z)$ and of (c) the normalized pulse width (FWHM) $\sigma(Z)$.

As it can be seen, the pulse amplitude increases faster than the pulse width reduces, a typical behavior of a sudden amplification process. However, notice that the pulse energy reaches its initial value at the end of the period according to Eq. (3). It is clear that the pulse at $Z = \pi/2$ does not reproduce the initial pulse and a question arises whether subsequent propagation through similar segments of fiber will lead to a stationary solution.

Recent numerical work on high-gain erbium-doped fiber lasers [7] has revealed that stationary single pulse solution should exist for the periodic gain profile distribution of Fig. 1a taking in the limit $L_a \to L$. Such a solution is possible if the following two conditions are satisfied: i) that $L \ll L_{\mathrm{d}}$ and ii) that the average (peak) power of the pulse within one period equals the fundamental soliton power. In the next section we analytically demonstrate that, after a transient regime, soliton solutions to Eq. (1) indeed exist. As we can see later, these theoretical soliton solutions require a generalization of the physical conditions (i) and (ii) given above.

3. Soliton solutions

From Fig. 2 it can be inferred that stationary solutions to Eq. (1), if they exist, will appear after an initial transient and oscillatory behavior. Let us now look for such an asymptotic stationary solution. We start by defining a new pulse envelope variable $Q(Z,T)$ through

$$q(Z,T) = Q(Z,T)\exp\left[\tfrac{1}{2}\int_0^z g(s)\,ds\right], \tag{4}$$

which transform Eq. (1) to

$$i\frac{\partial Q}{\partial Z} = \frac{1}{2}\frac{\partial^2 Q}{\partial T^2} + \exp\left[\int_0^Z g(s)\,ds\right]|Q|^2\,Q. \tag{5}$$

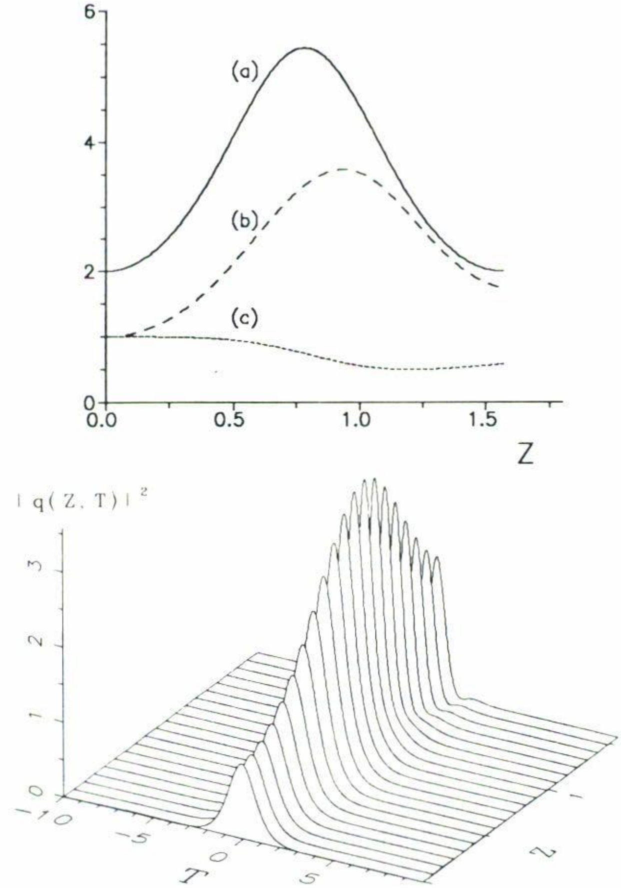

FIGURE 2. Propagation of an initial first order soliton through the first period of the gain profile of Fig. 1b, with $g_0 = 2$ and $L = \pi/2 = Z_0$. In (a) we show the temporal pulse evolution and in (b) the spatial evolution of the pulse energy, $W(Z)$, the pulse peak intensity, $I_p(Z)$ and the pulse width, $\sigma(Z)$.

Then, we denote by

$$\tilde{G}(Z) = \exp\left[\int_0^Z g(s)\, ds\right] - G_0, \tag{6}$$

where G_0 is the spatial average of the energy transferred to the pulse by the gain profile $g(Z)$, and it is given by

$$G_0 = \frac{1}{L}\int_0^L \exp\left[\int_0^Z g(s)\, ds\right] dZ. \tag{7}$$

Using Eq. (6), we can rewrite Eq. (5) as

$$i\frac{\partial Q}{\partial Z} = \frac{1}{2}\frac{\partial^2 Q}{\partial T^2} + G_0\,|Q|^2\,Q + \tilde{G}(Z)\,|Q|^2\,Q. \tag{8}$$

Let us now assume that for small enough L compared to the dispersion length $L_{\rm d}$ the last term in the RHS of Eq. (8) can be treated as a perturbation to the otherwise exact NLSE. Quantitatively speaking, this occurs if $|g_0|L \ll 2\pi$, which can be interpreted as a generalization of the condition $L \ll L_{\rm d}$ given in Ref. [7]. Without the perturbative term, Eq. (8) becomes the NLSE and hence it accepts the soliton solution [9]

$$Q(Z,T) = N\kappa\,\mathrm{sech}(\kappa T)\exp[-i\kappa^2 Z/2], \tag{9}$$

where N, the order of the soliton, is an integer satisfying $\sqrt{G_0} = N + a$ with $|a| < 1/2$, and $\kappa = 1 + (2a/N)$ is the soliton form factor. On the other hand, the influence of the perturbative term in Eq. (8) can be estimated using perturbation theory [10]. There, ϕ, the correction to the phase of the soliton solution, is given by

$$\frac{d\phi}{dZ} = \int_{-\infty}^{\infty} \mathrm{sech}(\kappa T)\big[1 - (\kappa T)\tanh(\kappa T)\big]\,\mathrm{Im}(\delta Q)\,dT, \tag{10}$$

where, for the specific case of Eq. (8), δQ can be approximated as

$$\delta Q \approx -i\tilde{G}(Z)\,|Q(0,T)|^2\,Q(0,T)\,dZ. \tag{11}$$

Substituting δQ into Eq. (10) and performing the resulting integral we obtain

$$\frac{d\phi}{dZ} = -N^3\kappa^2\tilde{G}(Z). \tag{12}$$

However, $\int_0^L \tilde{G}(Z)\,dZ = 0$, as can be noticed from Eq. (6), and hence the net phase changes on $Q(Z,T)$ caused by the perturbation term in one period become null and the lowest order solution given in Eq. (9) becomes the general solution to our problem.

Because of its great importance, we concentrate our attention on the fundamental first-order ($N = 1$) soliton of Eq. (9), which occurs for $1/2 < \sqrt{G_0} < 3/2$, $a = \sqrt{G_0} - 1$ and $\kappa = 1 + 2a$. For these cases, the resulting first order soliton will also contain a linear or non-soliton component which will be eventually lost of the pulse by the fiber dispersive effect. As we will see later, such a process is responsible of the expected transient effects. Nevertheless, the theoretical result of Eq. (9) confirms the physically argumented condition given in Ref. [7] for the existence of soliton solutions, but also generalizes such a condition because it establishes that the asymptotic soliton solution will not have, in general, the same width than the initial pulse. In fact, the width of the asymptotic soliton $1/\kappa$ depends on G_0 and in order to get some insight on how G_0 is affected by the parameters of the gain profiles, let us compute G_0 for the specific gain profiles depicted in Fig. 1.

For the gain profile of Fig. 1a we have

$$\int_0^Z g(s)\,ds = \begin{cases} g_0 Z & \text{if } 0 \le Z \le L_a, \\ g_0 L_a\left[1 - \dfrac{Z - L_a}{L - L_a}\right] & \text{if } L_a < Z \le L, \end{cases}$$

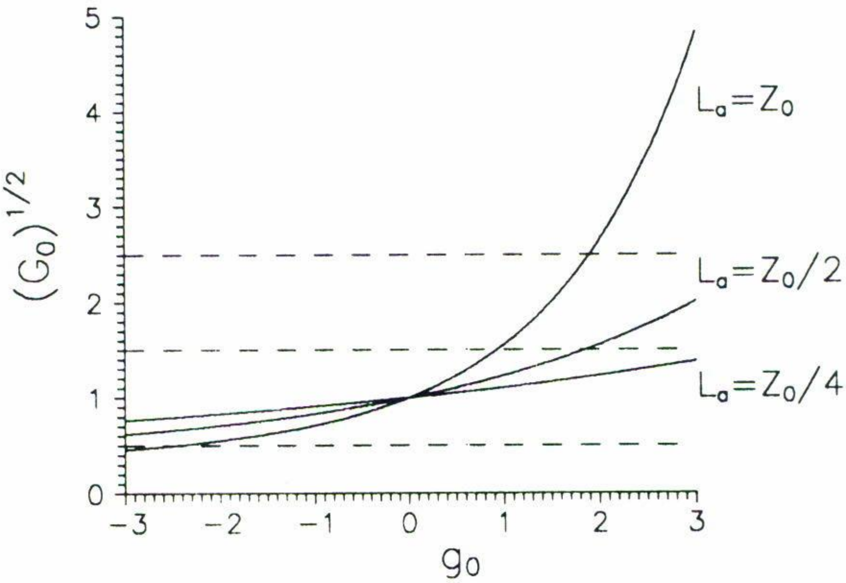

FIGURE 3. Behavior of $\sqrt{G_0}$ for the gain profile of Fig. 1a. The three curves corresponds to the amplification lengths indicated in the figure in soliton units. According to Eq. (1), a soliton period Z_0 corresponds to a distance $Z = \pi/2$.

and from Eq. (7) G_0 is given by

$$G_0 = \int_0^{L_a} \exp[g_0 Z]\, dZ + \exp[g_0 L_a] \int_{L_a}^{Z} \exp\left[-\frac{(g_0 L_a)(Z - L_a)}{(L - L_a)}\right] dZ$$

$$= \frac{1}{g_0 L_a}(\exp[g_0 L_a] - 1). \qquad (13)$$

Figure 3 shows the behavior of $\sqrt{G_0}$ as the amplification amplitude g_0 es varied for three different amplification lengths L_a. As it can be seen, the graphs of $\sqrt{G_0}$ basically follow an exponential profile, with a growing rate decreasing as the amplification length L_a decreases. Therefore, large amplification amplitudes can produce final first-order solitons only if small amplification lengths are used. Note that negative values of g_0, which make $\sqrt{G_0} < 1$, can also produce first-order solitons provide $\sqrt{G_0} \geq 1/2$. For these cases, the amplification-absorption process is reversed to absorption-amplification, and the asymptotic soliton width becomes greater than unity.

On the other hand, for the harmonic gain profile of Fig. 1b we have

$$\int_0^{Z} g(s)\, ds = \frac{g_0}{\Omega}\, [1 - \cos(\Omega Z)].$$

Substitution of this expression in Eq. (7), followed by a power series expansion of the

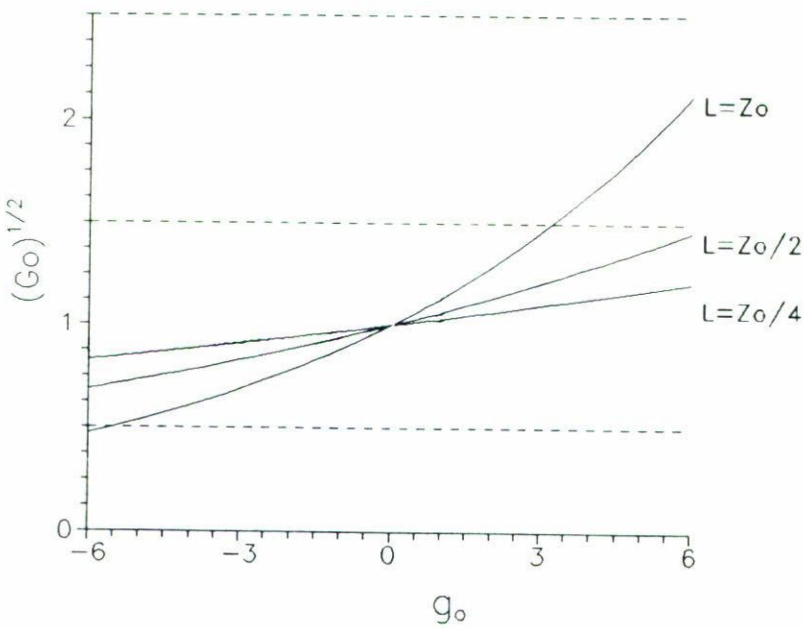

FIGURE 4. Behavior of $\sqrt{G_0}$ for the sinusoidal gain profile of Fig. 1b. The three curves correspond to the period lengths indicated in the figure in soliton units.

resulting exponential function, leads to

$$G_0 = \frac{\exp(g_0/\Omega)}{\Omega L} \int_0^{2\pi} \left(1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \left(\frac{g_0}{\Omega}\right)^n \cos^n y\right) dy,$$

$$= \exp(g_0/\Omega) \left[1 + \sum_{n=1}^{\infty} \frac{1}{(n!)^2} \left(\frac{g_0}{2\Omega}\right)^{2n}\right].$$

For a small g_0/Ω,which agrees with the assumed condition $g_0 L \ll 2\pi$, we can truncate the series after the second power term, and use

$$G_0 \approx \exp(g_0/\Omega)\left[1 + \tfrac{1}{4}(g_0/\Omega)^2 + \tfrac{1}{64}(g_0/\Omega)^4\right] \tag{14}$$

as a good approximation to G_0. Figure 4 shows the behavior of $\sqrt{G_0}$ as the amplitude g_0 is varied for three different periods L.

There are no qualitative differences with Fig. 3, as expected by virtue of the physical meaning of G_0. Note that Fig. 4 also indicates that if the amplification and absorption regions of the harmonic gain profile are interchanged; that is, if g_0 becomes negative, a first-order soliton, broader than the initial pulse, should also be obtained.

It is necessary to remark that the asymptotic soliton solution of Eq. (1) is formed by substitution of Eq. (9) into Eq. (4). Therefore even when transient effects are vanished, the soliton amplitude will develop an oscillating behavior during the amplification-absorption

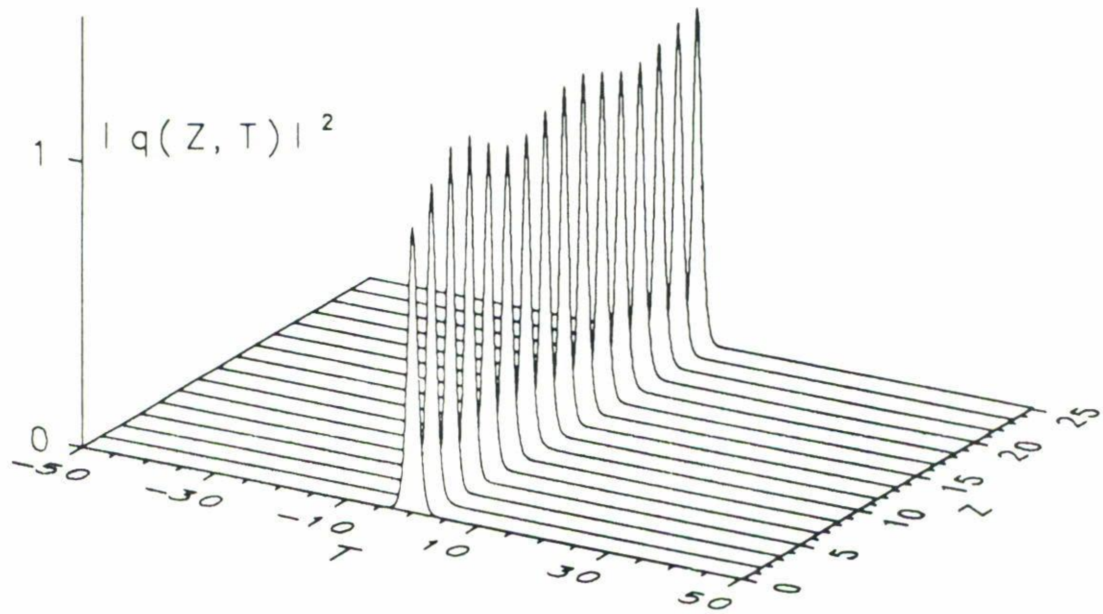

FIGURE 5. Temporal behavior of an initial first order soliton as it propagates through a fiber possessing the periodical gain profile of Fig. 1b with $g_0 = 1$ and $L = Z_0/2$. The total propagation distance is $32L$, and each curve is taken at the distance interval of $2L$.

process, but at the end of each period L it will recover its predicted value $N\kappa$. Note also that once the profile of the asymptotic soliton is reached, its width $1/\kappa$ will remain unchanged although this oscillation on its amplitude.

4. NUMERICAL SIMULATIONS

In order to verify the theoretical predictions of the precedent section we will now proceed to numerically solve Eq. (1) with the sinusoidal gain profile of Fig. 1b and with $q(0,T) = \operatorname{sech} T$ as the initial pulse. We used the standard split-step Fourier numerical method [8] with a uniform temporal grid of 1024 points and, in order to accurately follow the pulse parameters as it tends to its stationary soliton profile given by Eq. (9) with $N = 1$, we used a spatial grid of up to 2000 points. At each propagation step we computed the pulse peak intensity $I_p(Z)$, the relative pulse width $\sigma(Z)$, and the pulse energy $W(Z)$. These parameters will be compared with those predicted by Eq. (9) for the specific values used for the amplitude and the period of the gain profile.

Figure 5 shows the propagation of the initial pulse through a distance equivalent to $Z = 32\,L$. Here the parameters of the gain profile are $g_0 = 1$ and $L = \pi/4$ $(= Z_0/2)$. Each curve on the graph is taken at the end of one gain profile period, and there is a separation of $Z = 2L$ between each one. As it can be seen, the presence of a transient and decaying initial process is evident but a stationarity on the pulse is not obvious. More quantitative and contundent arguments in favor of stationarity is shown in Fig. 6, where we plot the spatial evolution of the pulse energy W, of the pulse peak intensity I_p, and of the

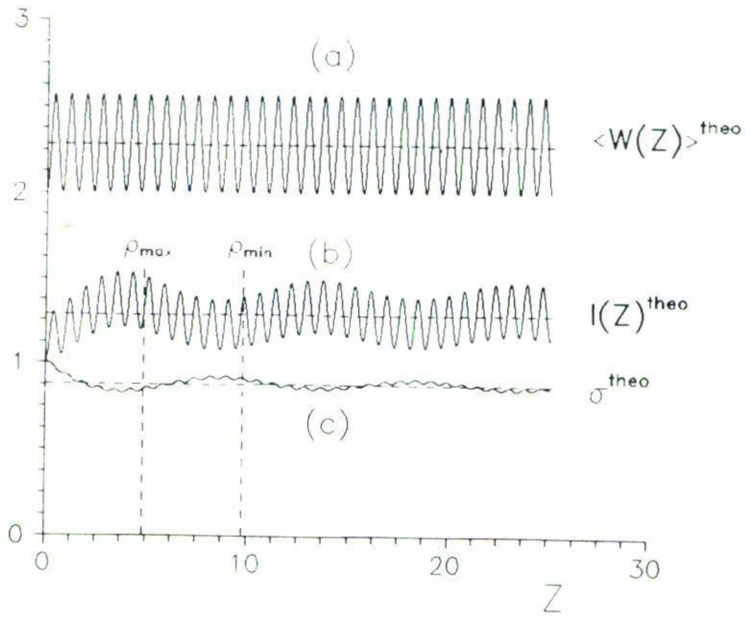

FIGURE 6. Spatial traces of (a) the pulse energy, (b) the pulse peak intensity and (c) the pulse width for the numerical simulation depicted in Fig. 5. The predicted values for the asymptotic soliton parameters are indicated by the corresponding horizontal lines.

normalized pulse width σ corresponding to the numerical simulation of Fig. 5. For the gain parameters used in the simulation, Eq. (14) gives $G_0 = 1.138$, and hence $a = 0.0668$ and the form factor of the predicted asymptotic soliton becomes $\kappa = 1.134$. From Eq. (9) this means that the soliton will posses an intensity peak of $I_{\rm p}^{\rm theo} = \kappa^2 = 1.286$ and a duration of $\sigma^{\rm theo} = 1/\kappa = 0.882$. These theoretical values are also graphed in Fig. 6. There is an excellent concordance with the values at which the numerical results tend. The average over the pulse energy also agrees with the theoretical value of $\langle W(Z)\rangle = W_0 G_0 = 2.28$.

The period L defines the duration of the transient, as expected from the approximation assumption. In Fig. 7 we reduce the period of the gain profile to $\pi/8$ $(= Z_0/4)$ but keeping its amplitude the same, $g_0 = 1$. For these parameters the predicted values are $G_0 = 1.066$, $a = 0.0325$, and $\kappa = 1.065$, and therefore the following asymptotic soliton parameters are expected: $I_{\rm p}^{\rm theo} = 1.132$, $\sigma^{\rm theo} = 0.94$ and $\langle W(Z)\rangle = 2.132$. As it can be seen by comparing Figs. 6 and 7, a reduction in L results in a smoothing and reduction of the duration of the transient effects.

Figure 8 shows another example of physical interest, in which the amplification-absorption regions of the gain profile have been inverted. There, $g_0 = -2$, $L = \pi/8$ and, from Eq. (14), the averaged energy given to the pulse is $G_0 = 0.866$. The theoretical form factor is the $\kappa = 0.882$, and the theoretical asymptotic pulse width and peak intensity are $\sigma^{\rm theo} = 1.133$ and $I_{\rm p}^{\rm theo} = 0.778$, respectively. By comparing this figure with Fig. 7 we can notice a resulting broader and smaller soliton output, confirming the theoretical predictions.

In addition to the asymptotic soliton parameters we can also predict some of their transient characteristics. According to Figs. 6–8, the spatial evolution of the peak intensity,

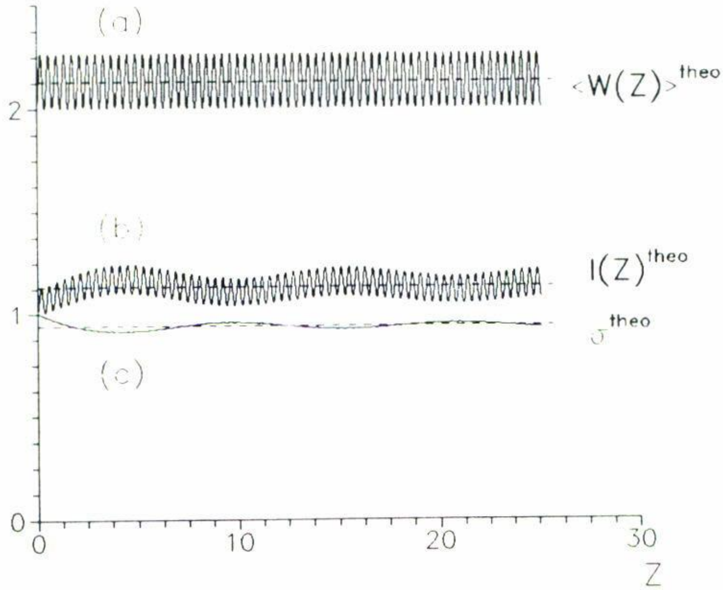

FIGURE 7. Spatial traces of (a) the energy, (b) the peak intensity, and (c) the width of a pulse propagating through a fiber possessing the periodical gain profile of Fig. 1b, with $g_0 = 1$ and $L = Z_0/4$. The theoretically predicted parameters are indicated by the corresponding horizontal lines.

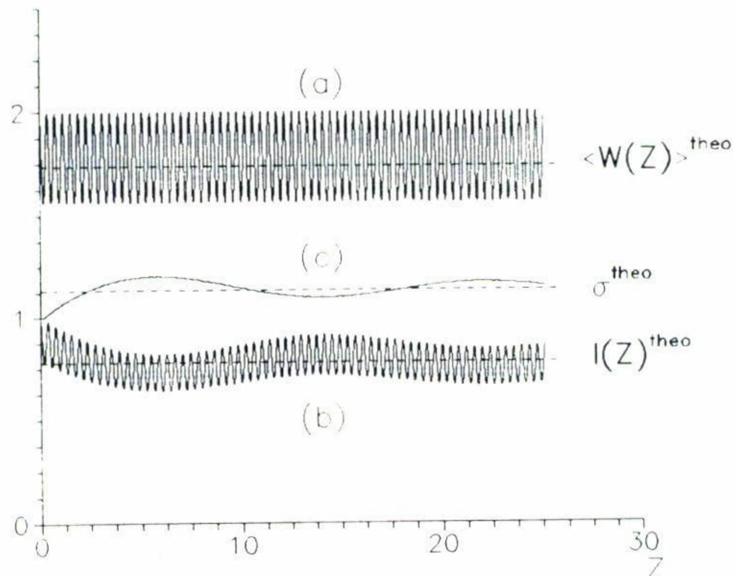

FIGURE 8. Spatial traces of (a) the energy, (b) the peak intensity, and (c) the width of a pulse propagating through a fiber possessing the periodical gain profile of Fig. 1b, with $g_0 = -2$ and $L = Z_0/4$. The theoretically predicted parameters are indicated by the corresponding horizontal lines.

for example, is made of rapid oscillations modulated by a large-scale oscillating function which asymptotically vanish. The rapid variation are due to the periodic characteristic of the gain profile $g(Z)$, and they are established by Eqs. (4) and (9). On the other hand, the large-scale oscillations represent the transient process and we can physically explain them as follows. Assume a given G_0 which satisfy $1 < \sqrt{G_0} < 3/2$. Then $a = \sqrt{G_0} - 1$ is positive and the input pulse of the form $Q(0,T) = \text{sech}(T)$ can be decomposed in two parts. One consisting in a first-order soliton of peak amplitude $\kappa = 1 + 2a$, and the other formed by a linear component of amplitude a, [9]. This linear component will be dispersed away and its amplitude will decrease with the propagation distance as $1/\sqrt{Z}$. But here, the most important fact is that at $Z = 0$ the soliton and the non-soliton components are π radians out of phase, producing a net amplitude $1 + a$. As the pulse propagation takes place, the soliton component develop a linear phase, as it is indicated by Eq. (9), and it will be in phase with the non-soliton component after acquiring an extra π phase shift. When this occurs, both components interfere in a constructive way and produce a maximum in the pulse amplitude. From Eq. (9) the propagation distance at which this first maximum occurs, $Z = \rho_{\text{max}}^{(1)}$, is estimated to be

$$\rho_{\text{max}}^{(1)} = \frac{2\pi}{(1+2a)^2}. \qquad (15)$$

An additional phase shift of π radians int he soliton component will produce a destructive interference pattern with the non-soliton component and, therefore, will produce a minimum in the pulse amplitude. Thus, the propagation distance at which the first minimum in the large-scale oscillation occurs is

$$\rho_{\text{min}}^{(1)} = \frac{4\pi}{(1+2a)^2}. \qquad (16)$$

For the specific case of Fig. 6, where a theoretical $\kappa = 1.134$ was found, the first maximum and minimum in the transient process occur, according to Eqs. (15) and (16), at $\rho_{\text{max}}^{(1)} \approx 4.89$ and at $\rho_{\text{min}}^{(1)} \approx 9.78$, respectively. In Fig. 6 these theoretical values are marked and we can note that they are good enough approximation to the corresponding numerical values. Here the relatively small discrepancies result from the fact that the non-soliton component becomes chirped when dispersed [8]. Then, the linear pulse has an averaged phase shift that reduced the predicted π radians condition to meet in phase or out of phase the soliton component.

5. CONCLUSIONS

We have found the soliton solutions for pulses propagating in a optical fiber possessing periodical gain profiles under conditions of high amplification coefficients. The characteristic parameters of these solitons, *i.e.*, their amplitude and width, depends on the average of the energy given to the pulse by the gain profile. This fact corroborates physically argumented concepts on the stability of high-gain erbium doped fibers, and provides another example of the robustness of the averaged soliton parameters.

6. ACKNOWLEDGEMENTS

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