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Identification of the electrical parameters of a Blumlein circuit used in the excitation of N_2 lasers

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ABSTRACT. A parametric optimization method is proposed to obtain the average values for the nonmeasurable quantities of inductance and resistance in the laser gas discharge and in the spark gap of a N_2 laser excited with a Blumlein circuit. The application of this method require only the knowledge of the voltage wave form appearing on the laser electrodes and can be applied to any laser excited with a Blumlein circuit.

RESUMEN. En este trabajo se propone el uso de un método de optimización paramétrica para determinar los valores promedio de las inductancia y resistencia no lineales de la descarga láser y del interruptor de chispa en un láser de N_2 excitado con un circuito Blumlein. La aplicación de este método requiere sólo del conocimiento de la forma de onda del voltaje entre los electrodos de láser y puede ser aplicado a cualquier láser excitado con un circuito tipo Blumlein.

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1. INTRODUCTION

In recent years efforts have been made to explain the electrical behavior of the circuits used to excite N_2 lasers [1–8], because they control the laser emission. The knowledge of the electrical behavior of the these circuits is also important because they are widely used to excite many other lasers like excimers, CO_2, Xe, etc. For many years voltage measurements have been reported, where the high voltage probe Tektronix P6015 and capacitative dividers are widely used [4, 5, 7, 9]. Because in most N_2 laser configurations flat capacitors and conductor are used to minimize inductances, good current measurements are achieved with linear coils [8]. It has been shown, when the spark gap and the laser head in a N_2 laser are represented by a constant resistance and a constant inductance connected in series, that the differential equation of all voltages and currents in the circuit is the same, but with different initial conditions [2]. It has been shown also that the solution of this differential equation is given by the superposition of two underdamped oscillations with different amplitudes, damping parameters, frequencies and phases [2]. So, the study of only one of the measured voltages or currents in the laser should be enough to evaluate the amplitudes, damping parameters, frequencies and phases of the solution of its equivalent circuit and to give the corresponding average resistance and inductance of the spark gap and the laser.

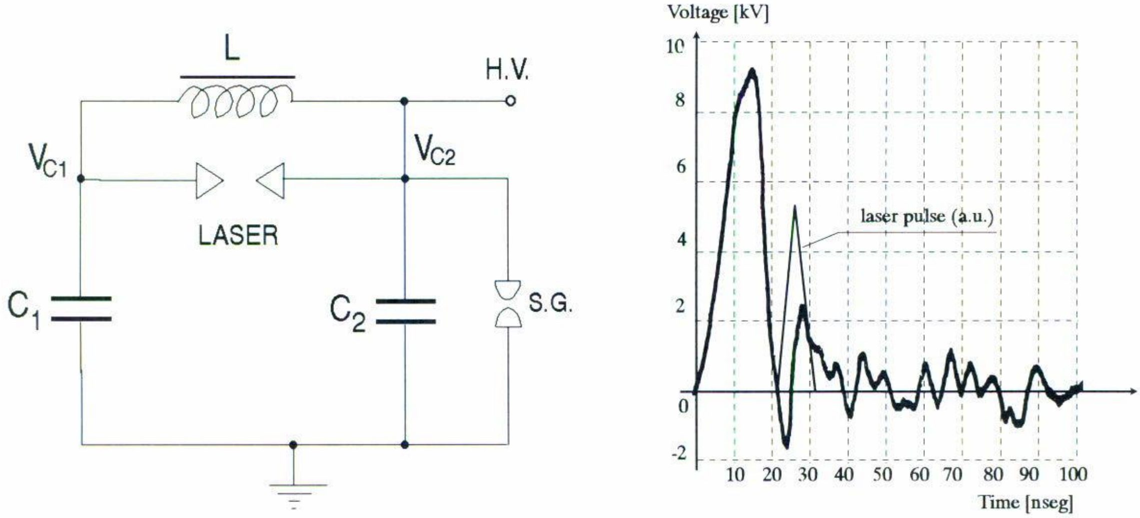

FIGURE 1. (a) Schematic diagram of the Blumlein circuit. (b) Experimentally obtained laser voltage $V(t)$ and radiation emission at 60 hPa N_2 and H.V. = 8 kV.

2. THEORETICAL CONSIDERATIONS

Figure 1a shows a schematic diagram of the Blumlein circuit. The circuit is composed of a spark gap (S.G.) as a fast switch, the laser head and two capacitors. When high voltage is applied, both capacitors are equally charged until the breakdown voltage across S.G. is reached. At this potential, the S.G. fires and C_2 begins to discharge through S.G. As capacitor C_2 discharges, a very fast rising high voltage difference appears across the laser head until the laser breakdown voltage is reached and the discharge takes place. Figure 1b shows the voltages and radiation emission waveforms appearing in the system. The mechanical construction of the laser is reported elsewhere [9].

The voltages V_{C1} and V_{C2} were measured with two equal high voltages probes (Tektronix P6015, rise-time < 4.5 ns) combined with a 300 MHz bandwidth oscilloscope (Tektronix 2440). The voltage in the laser head (Fig. 1b) is the voltage difference $V_{C1} - V_{C2}$, which was automatically given by the oscilloscope and is the average of 16 discharges. Stable operation of the laser was achieved at voltages ranging from 6 to 12 kV, pressures between 60 and 130 hPa and frequencies up to 20 Hz. The pulse-to-pulse fluctuations of the laser head voltage were less than 5%.

To analyze the circuit, each discharge taking place in the circuit is simulated by an inductance and a resistance connected in series (see Fig. 2). R_1 and L_1 stand for the inductance and resistance associated with the laser head loop, respectively, and R_2 and L_2 stand for the analogous parameters of the laser loop. The differential equation governing the performance of the voltages after the breakdown of the laser head is given by the relation [4]

$$\frac{d^4V(t)}{dt^4} + K_1\frac{d^3V(t)}{dt^3} + K_2\frac{d^2V(t)}{dt^2} + K_3\frac{dV(t)}{dt} + K_4V(t) = 0, \tag{1}$$

where

$$V(t) = V_{C_1}(t) - V_{C_2}(t), \tag{2}$$

$$\begin{aligned} K_1 &= \frac{R_2}{L_2} + \frac{R_1}{L_1}, \\ K_2 &= \frac{1}{L_2C_2} + \frac{1}{L_1C_2} + \frac{1}{L_1C_1} + \frac{R_2}{L_2}\frac{R_1}{L_1}, \\ K_3 &= \frac{R_2}{L_2}\frac{1}{L_1C_1} + \frac{R_2}{L_2}\frac{1}{L_1C_2} + \frac{R_1}{L_1}\frac{1}{L_2C_2}, \\ K_4 &= \frac{1}{L_1L_2C_1C_2}. \end{aligned} \tag{3}$$

The solution of Eq. (1) when R_1, R_2, L_1, L_2 are considered to be constants is obtained in Ref. [7]:

$$V(t) = Ae^{-\alpha_1 t}\cos(\omega_1 t) + Be^{-\alpha_2 t}\cos(\omega_2 t + \phi), \tag{4}$$

where

$$\begin{aligned} \alpha_1 &= \tfrac{1}{4}\left(K_1 + \sqrt{C_A}\right), \\ \alpha_2 &= \tfrac{1}{4}\left(K_1 - \sqrt{C_A}\right), \end{aligned} \tag{5}$$

$$\begin{aligned} \omega_1 &= \sqrt{-\frac{C_K}{2} + \frac{\sqrt{C_B}}{2} - \frac{K_1}{4}\frac{\sqrt{C_A}}{2}}, \\ \omega_2 &= \sqrt{-\frac{C_K}{2} - \frac{\sqrt{C_B}}{2} + \frac{K_1}{4}\frac{\sqrt{C_A}}{2}}, \end{aligned} \tag{6}$$

and

$$\begin{aligned} C_A &= K_1^2 - 4K_2 + 4X_{\mathrm{r}}, \\ C_B &= X_{\mathrm{r}}^2 - 4K_4, \\ C_K &= \frac{K_1^2}{4} - \frac{K_2}{2} - \frac{X_{\mathrm{r}}}{2}; \end{aligned} \tag{7}$$

where X_{r} is a real root of the cubic equation

$$X^3 - K_2X^2 + (K_1K_3 - 4K_4)X + (4K_2K_4 - K_3^2 - K_1^2K_4) = 0. \tag{8}$$

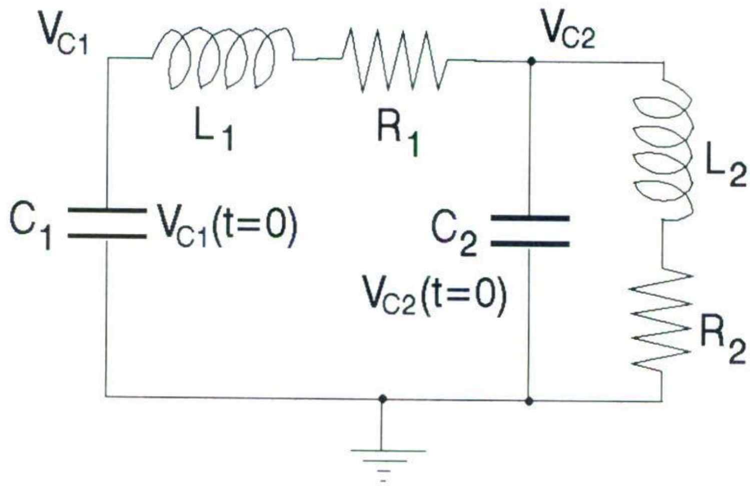

FIGURE 2. Equivalent circuit.

3. Parametric identification

The parametric identification is accomplished through a comparison of the values in the real process and the theoretical model. To do that is necessary to consider n experimental voltage values $V_i(t_i)$ for $i = 1, 2, \ldots, n$, satisfying Eq. (4), so we can write

$$V_i = Ae^{-\alpha_1 t_i} \cos(\omega_1 t_i) + Be^{-\alpha_2 t_i} \cos(\omega_2 t_i + \phi). \tag{9}$$

To simplify the mathematical description, we make the following substitutions:

$$\begin{aligned} a_i &= e^{-\alpha_1 t_i} \cos(\omega_1 t_i), \\ b_i &= e^{-\alpha_2 t_i}, \\ c_i &= \omega_2 t_i. \end{aligned} \tag{10}$$

In this way, Eq. (9) becomes

$$Aa_i + Bb_i \cos(c_i + \phi) = V_i \tag{11}$$

or

$$f_i(A, B, \phi) - V_i = 0, \tag{12}$$

for $i = 1, 2, \ldots, n$, where

$$f_i(A, B, \phi) = Aa_i + Bb_i \cos(c_i + \phi). \tag{13}$$

So, we are looking for A, B, ϕ values to satisfy

$$\forall\, i \in \{1, 2, \ldots, n\}, f_i(A, B, \phi) - V_i = 0. \tag{14}$$

Actually we can only obtain approximate values for these parameters. So, Eq. (14) should be written as

$$f_i(A, B, \phi) - V_i = \epsilon_i, \tag{15}$$

where ϵ_i is an approximation error.

In this way the parametric identification problem becomes an optimization problem, where we require the A, B, ϕ values for which ϵ_i has a minimum, in accordance with the established optimization index. We choose in this case the index

$$J = \sum_{i=1}^{n} (f_i(A, B, \phi) - V_i)^2, \tag{16}$$

so that the problem is reduced to the minimization of

$$\min_{A,B,\phi} \sum_{i=1}^{n} (f_i(A, B, \phi) - V_i)^2. \tag{17}$$

To use Eq. (17) we need the values of R_1, R_2, L_1, L_2, C_1, C_2. The last ones are established by design, but R_1, R_2, L_1, L_2 are the non-measurable, non linear resistance and inductance of the laser and spark gap, respectively. Here we consider them as constants. So, we are looking for the values of R_1, R_2, L_1, L_2 and A, B, ϕ, for which

$$\forall i \in \{1, 2, \ldots, n\}, f_i(R_1, R_2, L_1, L_2, A, B, \phi) - V_i = 0, \tag{18}$$

where

$$f_i(R_1, R_2, L_1, L_2, A, B, \phi) = A a_i^{RL} + B b_i^{RL} \cos(c_i^{RL} + \phi) \tag{19}$$

and

$$\begin{aligned} a_i^{RL} &= e^{-\alpha_1^{RL} t_i} \cos(\omega_1^{RL} t_i), \\ b_i^{RL} &= e^{-\alpha_2^{RL} t_i} \\ c_i^{RL} &= \omega_2^{RL} t_i. \end{aligned} \tag{20}$$

Here α_1^{RL}, α_2^{RL}, ω_1^{RL}, ω_2^{RL} are values depending on R_1, R_2, L_1, L_2.

In this way, Eq. (16) becomes

$$J = \sum_{i=1}^{n} (f_i(R_1, R_2, L_1, L_2, A, B, \phi) - V_i)^2. \tag{21}$$

And so, the determination of all parameters is reduced to the problem of obtaining

$$\min_{R_1,R_2,L_1,L_2,A,B,\phi} \sum_{i=1}^{n} \big(f_i(R_1, R_2, L_1, L_2, A, B, \phi) - V_i\big)^2. \tag{22}$$

4. Proposed algorithm

The following methods are among the most commonly used in parametric identification: cyclical change of the parameter values or coordinate descent method or Gauss-Seidel method; fastest gradient method; fastest start method; shot or Newton-Raphson method; stochastic change of the parameter values.

We have used an algorithm based on the Gauss-Seidel method [10,11], which consists of changing the value of only one parameter, holding the other ones as constants, until the minimal value of the optimization index for this parameter is obtained. The changes in the parameter values are carried out using an increasing or decreasing constant. All the other parameters are treated similarly until a first cycle is completed. A second or more cycles could be done, always with smaller increasing or decreasing constant, until the increasing or decreasing constant is lower than a minimal predetermined value. This minimal predetermined constant gives the accuracy of the calculated values. Appendix A shows the algorithm used.

The algorithm was written in FORTRAN and a PC486DX2 (66 MHz) was used. The solution took from a few minutes to a maximal 30 minutes to obtain, depending on the initial values of the declared parameters. To obtain a graphical representation of the voltage we have used an additional FORTRAN program and the Harvard Graphics software.

5. Results and conclusions

From the experimental voltage (see Fig. 1b) we chose 16 values for calculation (see Table I). After processing with V_i, t_i, for $i, 2, \ldots, 16$, we obtained the parameters values shown in Table II. Finally, Fig. 3 shows the voltage behavior obtained with the parameters from Table II and Eq. (9). When the laser discharge is a glow discharge (the first pulse in Fig. 1b and Fig. 3), a comparison between Fig. 3 and Fig. 1b shows a good fit taking into consideration that a linear mathematical model was used to simulate a nonlinear process. It is worthy to note that when the experimental voltage rises from zero ($t = 0$) to its maximum value, in the laser head does not flow any current at all, but only in the loop formed by C_2, L_2 and R_2. The breakdown in the laser head takes place when the voltage reaches its maximum value. From this time and on the process can be represented by the equivalent circuit of Fig. 2. Here, we have considered the equivalent circuit of Fig. 2 starting from time $t = 0$. After laser emission the laser discharge changes into an arc discharge, changing the inductance and resistance drastically. Because this discharge period of time is not interesting for laser emission it has not been analyzed.

TABLE I. Data from the experimental voltage curve, for $C_1 = C_2 = 3$ nF.

Point number	Time [ns]	Voltage [V]
1	0	0
2	5.48	3143
3	10.95	9143
4	16.425	6286
5	22.67	−857
6	23.44	−1715
7	24.21	−1200
8	28.45	2571
9	31.92	1429
10	35.38	857
11	39.405	−153
12	39.96	−571
13	40.52	−153
14	43.57	1143
15	46.07	250
16	48.56	857

TABLE II. Results of parametric optimization.

Parameter	Value
A	−10994.07
B	13006.05
ϕ	5.5982 [rd]
R_1	3.068 [Ω]
R_2	1.43 [Ω]
L_1	19.09 [nH]
L_2	8.65 [nH]

Papadopoulous, *et al.* [4] have calculated values of $R_1 = 0.55\ \Omega$, $R_2 = 0.44\ \Omega$, $L_1 = 17$ nH, and $L_2 = 16.7$ nH in a similar experimental arrangement, using some approximations in the circuit equations. Because of the use of a pulse triggered spark gap and preionization conditions in the laser chamber in the Papadopoulous, *et al.* experiment, they obtain lower R_1 and R_2 values than us, however, the laser inductance is similar. So, the proposed parametric optimization method is useful to obtain the average values of inductance and resistance in the laser gas discharge and in the spark gap of a N_2 laser.

A parametric optimization method has been developed to obtain the average values of the inductance and resistance in the loops of a N_2 laser excited with a Blumlein circuit. The method could be used in the analysis of other discharge circuits and only needs the knowledge of the general form of their analytical solution and the measured laser voltage.

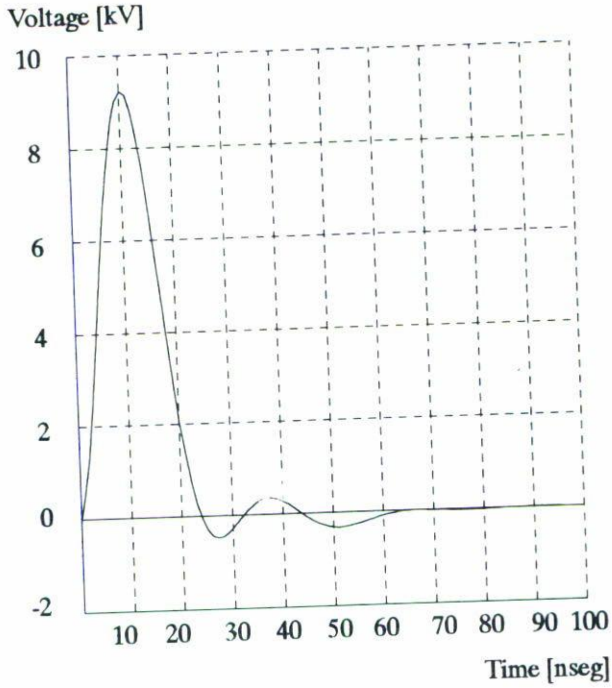

FIGURE 3. Simulation voltage $V(t)$.

The same parametric optimization algorithm, when the parameter values have been identified, can be used to optimize the capacitance C_1 and C_2 if the appropriated laser efficiency index is determinated. This will be the subject of future investigations.

APPENDIX A

Figure 4 shows the proposed algorithm based on the Gauss-Seidel method, where:

- bt, et, mi, mt: small numbers selected by the user which determine the calculation accuracy,
- d1: parametric optimization index value in the present iterative step,
- d: parametric optimization index value in the preceding iterative step,
- dlp, dp: dl and d values beginning the calculation (declared by the user),
- dl: parameter increase or decrease absolute value in the present iterative step,
- dlo: parameter increase or decrease absolute value beginning a cycle of the parameter change,
- dlc: parameter increase or decrease absolute value beginning the calculations (declared by the user),
- N: present changed parameter number,
- N1: number of the parameters for optimization (declared by the user),
- N2: number of the first parameter for change beginning the calculations (declared by the user),

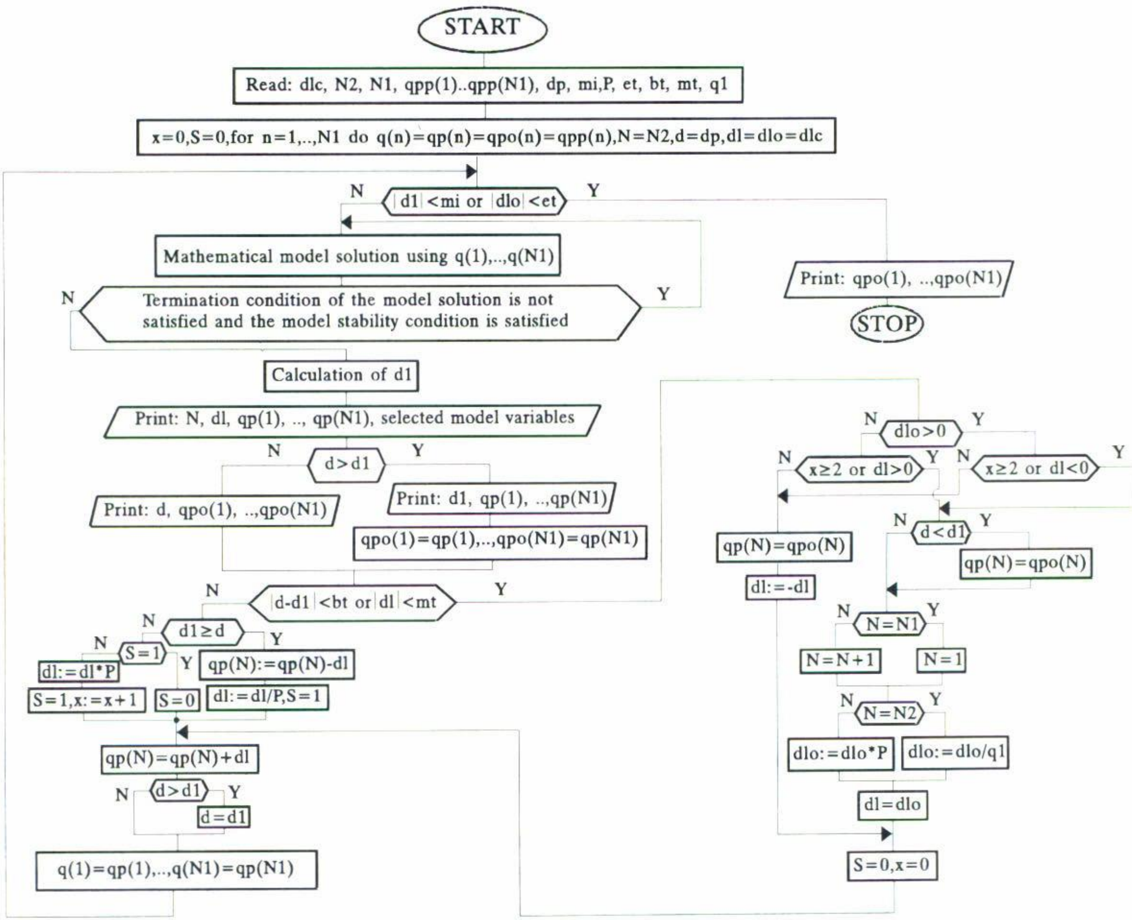

FIGURE 4. Parametric optimization algorithm.

P: integer number (for example P=2), divisor or multiplier used for decrease or increase of the parameter increment or decrement absolute value during a cycle of the parameter changes (declared by the user),

q1: number greater than $2N1^2 * P$, divisor used for decrease of the parameter increment or decrement absolute value finishing a cycle of the parameter changes (declared by the user),

q(N): N-number parameter value during the solution of the mathematical model,

qp(N): initial value of the N-number parameter beginning a iterative cycle of the parameter changes,

qpo(N): optimal value of the N-number parameter,

qpp(N): initial value of the N-number parameter beginning the calculation (declared by the user),

S, x: parametric optimization algorithm flags.

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