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# The polynomial zeros of degree 2 of the 9–$j$ coefficient

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ABSTRACT. We generate and present, for the first time, the polynomial zeros of degree 2 of the 9–$j$ coefficient from its representation as a triple hypergeometric series. The first 355 degree 2 zeros of the 9–$j$ coefficient, upto $\sigma \leq 22$ are tabulated.

RESUMEN. Generamos y presentamos por primera vez los ceros polinomiales de grado 2 del coeficiente 9–$j$ a partir de su representación como una serie hipergeométrica triple. Se tabulan los primeros 355 ceros de segundo grado del coeficiente 9–$j$, hasta $\sigma \leq 22$.

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## 1. INTRODUCTION

The study of "non-trivial" zeros of angular momentum coupling (3–$j$) and recoupling (6–$j$) coefficients started with the tabulation of the first 1420 zeros of the 6–$j$ coefficient, with any one of the six angular momenta being < 18.5, by Koozekanani and Biedenharn [1], in 1974. Varshalovich *et al.* [2] gave a listing of the zero-valued 3–$j$ coefficients. Observing that neither of these contributions took into account the Regge [3] symmetries of the 3–$j$ and the 6–$j$ coefficients, Bowick [4] tabulated the Regge inequivalent zeros of these coefficients and naturally, Bowick's tables are much shorter.

Since the 3–$j$ coefficient can be related to the Hahn or dual Hahn polynomial [5] (cf. Smorodinskii and Suslov [6]), and the 6–$j$ coefficient can be related to the Racah polynomial (cf. Wilson [7]; Askey and Wilson [8]), we prefer to call the zeros of these coefficients as polynomial zeros and Srinivasa Rao and Rajeswari [9] classified these zeros by their (polynomial) degree.

The triple sum series for the 9–$j$ coefficient, due to Jucys and Bandzaitis [10], is the simplest known algebraic form for that recoupling coefficient. This triple sum series has been identified as a special case of the formal triple hypergeometric series of Lauricella-Saran-Srivastava [11], by Srinivasa Rao and Rajeswari [9]. This identification immediately

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led Srinivasa Rao and Rajeswari [9] to show that there exist polynomial zeros for the 9–$j$ coefficient and to study them, for the first time.

For a review of the closed form expressions for degree 1 zeros of $3n$–$j$ coefficients; algorithms for generating the degree 1 zeros; the connection between degree 1 zeros of $3n$–$j$ coefficients and multiplicative Diophantine equations [12]; the connection between polynomial zeros and exceptional Lie algebras, we refer the interested reader to Srinivasa Rao and Rajeswari [13]. In this article, we are concerned with the tabulation of the first 355 polynomial zeros of degree 2 of the 9–$j$ coefficient. In Sect. 2, we define the degree of the polynomial zero of the 9–$j$ coefficient and discuss the method of generating the degree 2 zeros; while in Sect. 3 we present the tables of these zeros and in Sect. 4 we discuss the results and scope for further studies.

## 2. TRIPLE SUM SERIES AND POLYNOMIAL ZEROS

The simplest known algebraic form for the 9–$j$ coefficient, due to Jucys-Bandzaitis [10] is the triple sum series given by

$$\begin{Bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{Bmatrix} = (-1)^{x5} \frac{(dag)(beh)(igh)}{(def)(bac)(icf)}$$
$$\times \sum_{x,y,z} \frac{(-1)^{x+y+z}}{x!y!z!} \frac{(x1-x)!(x2+x)!(x3+x)!}{(x4-x)!(x5-x)!}$$
$$\times \frac{(y1+y)!(y2+y)!}{(y3+y)!(y4-y)!(y5-y)!} \frac{(z1-z)!(z2+z)!}{(z3-z)!(z4-z)!(z5-z)!}$$
$$\times \frac{(p1-y-z)!}{(p2+x+y)!(p3+x+z)!}, \tag{1}$$

where

$$\begin{aligned} 0 \le x \le \min(x4, x5) &= XF, \\ 0 \le y \le \min(y4, y5) &= YF, \\ 0 \le z \le \min(z4, z5) &= ZF, \end{aligned} \tag{2}$$

and

$$\begin{aligned} &x1 = 2f, & &y1 = -b+e+h, & &z1 = 2a, \\ &x2 = d+e-f, & &y2 = g+h-i, & &z2 = -a+b+c, \\ &x3 = c-f+i, & &y3 = 2h+1, & &z3 = a+d+g+1, \\ &x4 = -d+e+f, & &y4 = b+e-h, & &z4 = a+d-g, \\ &x5 = c+f-i, & &y5 = g-h+i, & &z5 = a-b+c, \\ &p1 = a+d-h+i, & &p2 = -b+d-f+h, & &p3 = -a+b-f+i, \end{aligned} \tag{3}$$

$$(abc) = \left[\frac{(a-b+c)!(a+b-c)!(a+b+c+1)!}{(-a+b+c)!}\right]^{1/2} \tag{4}$$

Replacing the factorials by Pochammer symbols, and akin to Appel's procedure [13] to get double series, with the following replacements for the three pairs of products:

$$\begin{aligned} (1+p2)_x(1+p2)_y &\text{ by } (1+p2)_{x+y}, \\ (1+p3)_x(1+p3)_z &\text{ by } (1+p3)_{x+z}, \\ (-p1)_y(-p1)_z &\text{ by } (-p1)_{y+z}, \end{aligned} \tag{5}$$

where

$$(\alpha)_n = \alpha(\alpha+1)(\alpha+2)\cdots(\alpha+n-1) = \Gamma(\alpha+n)/\Gamma(\alpha), \tag{6}$$

is the Pochammer symbol, Srinivasa Rao and Rajeswari [9] were led to identifying the triple series as a special case of the triple hypergeometric series of Lauricella, Saran and Srivastava [11].

The triple sum series does not exhibit any one of the 72 symmetries possessed by the $9\text{–}j$ coefficient, which manifest when it is expressed as a sum over the projections of a product of six $3\text{–}j$ coefficients (cf. Edmonds [14]). We define the degree of the polynomial zero of the $9\text{–}j$ coefficient as that given by the value of $XF+YF+ZF$. Srinivasa Rao and Rajeswari [9] studied the polynomial zeros of degree 1 of the $9\text{–}j$ coefficient (Viz. $XF+YF+ZF=1$) by generating them using either closed form expressions or through sets of homogeneous multiplicative Diophantine equations of degree 3. Here we are concerned with generating and tabulating the polynomial zeros of degree 2, *viz.* $XF+YF+ZF=2$.

Six cases arise. Of these, the three cases corresponding to: $XF=2$, $YF=ZF=0$; $XF=0$, $YF=2$, $ZF=0$; $XF=YF=0$, $ZF=2$, give rise to the $9\text{–}j$ coefficient being expressible in terms of either a $3\text{–}j$ (or Clebsch-Gordan) coefficient or a single non-zero term [15]. This would imply that the $9\text{–}j$ coefficient even if it is a zero, it is only as a consequence of the polynomial zeros of the $3\text{–}j$ coefficient. These cases are not of interest to us here and will be dealt with elsewhere.

The three remaining cases corresponding to $XF=YF=1$, $ZF=0$; $XF=1$, $YF=0$, $ZF=1$ and $XF=0$, $YF=ZF=1$, are the ones which will give rise to genuine degree 2 polynomial zeros of the $9\text{–}j$ coefficient. For instance, $ZF=0$, when substituted in the triple sum series for (1), yields

$$\begin{Bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{Bmatrix} = (-1)^{x5}\frac{(dag)(beh)(igh)}{(def)(bac)(icf)}\frac{z1!z2!}{z3!z4!z5!}S_{x,y}, \tag{7}$$

where $S_{x,y}$ is a double sum series which can be looked upon as a product of two ${}_4F_3(1)$ series with one parameter contracted, as in the case of the single variable Appell series

$F_3$. Explicitly

$$\begin{aligned}
S_{x,y} &= \sum_{x,y} \frac{(-1)^{x+y}}{x!y!} \frac{(x1-x)!(x2+x)!(x3+x)!}{(x4-x)!(x5-x)!} \\
&\times \frac{(y1+y)!(y2+y)!}{(y3+y)!(y4-y)!(y5-y)!} \frac{(p1-y)!}{(p2+x+y)!(p3+x)!} \\
&= \frac{x1!x2!x3!}{x4!x5!} \frac{y1!y2!}{y3!y4!y5!} \frac{p1!}{p2!p3!} \frac{1}{x!y!} \frac{(1+x2)_x(1+x3)_x(-x4)_x(-x5)_x}{(-x1)_x(1+p3)_x} \\
&\times \frac{(1+y1)_y(1+y2)_y(-y4)_y(-y5)_y}{(1+y3)_y(-p1)_y} \frac{1}{(1+p2)_{x+y}} \\
&= \frac{x1!x2!x3!}{x4!x5!} \frac{y1!y2!}{y3!y4!y5!} \frac{p1!}{p2!p3!} \\
&\times F^{0,4}_{1,2}\left[\begin{matrix} - & : 1+x2,\ 1+x3,\ -x4,\ -x5 & ; 1+y1,\ 1+y2,\ -y4,\ -y5 & ; \\ 1+p2 & : -x1,\ 1+p3 & ; 1+y3,\ -p1 & ; \end{matrix} 1,1\right], \quad (8)
\end{aligned}$$

where we have used the compact notation for the Kampé de Fériet function [15] devised by Burchnall and Chaundy [16], *viz.*,

$$F^{A:B}_{C:D}\left[\begin{matrix}(a) : (b) \ ; \ (b') \ ; \\ (c) : (d) \ ; \ (d') \ \end{matrix} X, Y\right] = \sum_{x,y} \frac{\prod_{j=1}^{A}(a_j)_{x+y} \prod_{j=1}^{B}(b_j)_x(b'_j)_y}{\prod_{j=1}^{C}(c_j)_{x+y} \prod_{j=1}^{D}(d_j)_x(d'_j)_y} \frac{X^x Y^y}{x!y!}, \quad (9)$$

where, in the shortened notation due to Barnes [17], it is understood that there are $A$ of the $a$ parameters, $B$ of the $b, b'$ parameters, etc.

If in Eq. (7) we further stipulate $XF = 1 = YF$, then the double sum reduces to

$$S_{x,y} = 1 - \frac{N_1}{D_1} - \frac{N_2}{D_2} + \frac{N_1 N_2 (1+p2)}{D_1 D_2 D_3} \quad (10)$$

and obviously, degree 2 zeros occur whenever the nine angular momenta $a, b, \ldots, i$ satisfy the condition:

$$((D_1 - N_1)D_2 - N_2 D_1)D_3 + N_1 N_2 (1+p2) = 0 \quad (11)$$

where

$$\begin{aligned}
N_1 &= (1+x2)(1+x3)x4x5, \quad D_1 = x1(1+p2)(1+p3), \\
N_2 &= (1+y1)(1+y2)y4y5, \quad D_2 = (1+y3)p1(1+y2), \quad (12) \\
D_3 &= p2+2.
\end{aligned}$$

Equation (11) stands for a set of eight conditions, since $ZF = 0$ represents $z4 = 0$ or $z5 = 0$; $XF = 1$ represents $x4 = 1$, $x5 \geq 1$ or $x4 \geq 1$, $x5 = 1$; $YF = 1$ represents $y4 = 1$, $y5 \geq 1$ or $y4 \geq 1$, $y5 = 1$.

Similarly, when $YF = 0$, the 9–$j$ coefficient will be proportional to the function

$$F^{0,4}_{1,2}\left[\begin{matrix} - & : 1+x2,\ 1+x3,\ -x4,\ -x5\ ; & -z3,\ -z4,\ -z5,\ 1+z2\ ; \\ 1+p3 & : \quad -x1,\ 1+p2 \quad ; & -z1,\ -p1 \quad ; \end{matrix}\ 1,1\right], \tag{13}$$

and when $XF = 1 = ZF$, polynomial zeros of degree 2 arise when

$$((D_1 - N_1)D_2 - N_2D_1)D_3 + N_1N_2(1+p3) = 0 \tag{14}$$

where

$$\begin{aligned} N_1 &= (1+x2)(1+x3)x4x5, & D_1 &= x1(1+p2)(1+p3), \\ N_2 &= z3z4z5(1+z2), & D_2 &= z1p1(1+p3), \\ D_3 &= p3+2. \end{aligned} \tag{15}$$

Equation (14) stands for a set of eight conditions, since $YF = 0$ represents $y4 = 0$ or $y5 = 0$; $XF = 1$ represents $x4 = 1$, $x5 \geq 1$ or $x4 \geq 1$, $x5 = 1$; $ZF = 1$ represents $z4 = 1$, $z5 \geq 1$ or $z4 \geq 1$, $z5 = 1$.

Finally, when $XF = 0$, the 9–$j$ coefficient will be proportional to the function

$$F^{0,4}_{1,2}\left[\begin{matrix} - & : 1+y1,\ 1+y2,\ -y4,\ -y5\ ; & -z3,\ -z4,\ -z5,\ 1+z2\ ; \\ -p1 & : \quad 1+p2,\ 1+y3 \quad ; & -z1,\ 1+p3 \quad ; \end{matrix}\ 1,1\right], \tag{16}$$

and when $XF = 1 = YF$, polynomial zeros of degree 2 arise when

$$((D_1 - N_1)D_2 - N_2D_1)D_3 + N_1N_2.p1 = 0 \tag{17}$$

where

$$\begin{aligned} N_1 &= (1+y1)(1+y2)y4y5, & D_1 &= p1(1+p2)(1+y3), \\ N_2 &= z3z4z5(1+z2), & D_2 &= z1p1(1+p3), \\ D_3 &= p1-1. \end{aligned} \tag{18}$$

Equation (17) again stands for a set of eight conditions, since $XF = 0$ represents $x4 = 0$ or $x5 = 0$; $YF = 1$ represents $y4 = 1$, $y5 \geq 1$ or $y4 \geq 1$, $y5 = 1$; $ZF = 1$ represents $z4 = 1$, $z5 \geq 1$ or $z4 \geq 1$, $z5 = 1$.

A simple and straightforward procedure to generate the polynomial zeros of degree 2 of the 9–$j$ coefficient is then to program the set of 24 conditions given by Eqs. (11), (14) and (17). The first degree 2 zero is

$$\left\{\begin{matrix} 3/2 & 2 & 3/2 \\ 3/2 & 1/2 & 1 \\ 2 & 3/2 & 1/2 \end{matrix}\right\}, \tag{19}$$

with $\sigma = a + b + \cdots + i = 12$, being the sum of the nine angular momenta. In this case since $XF = 0$, $YF = ZF = 1$, the degree of this zero is 2, but it has four terms. This can be looked upon as a "4-term" degree 2 zero. Unlike the polynomial zeros of the 3–$j$ and the 6–$j$ coefficient it is to be emphasized that the inherent lack of symmetry of the triple sum series implies that the symmetries of (19) will **not** all be polynomial zeros of degree 2. To illustrate our point, consider the symmetry of (19) :

$$\begin{Bmatrix} 3/2 & 3/2 & 2 \\ 2 & 1/2 & 3/2 \\ 3/2 & 1 & 1/2 \end{Bmatrix}, \tag{20}$$

which has $XF = 0$, $YF = 1$, $ZF = 2$, implying that it is a polynomial zero of degree $(XF + YF + ZF =)$ 3. However, since $p1 = 3$, $p2 = 0$, $p3 = -1$, the constraints on the summation indices

$$y + z \geq p1 \quad \text{and if} \quad p2, p3 < 0, \quad \text{then} \quad x + y \geq |p2|,\ x + z \geq |p3|, \tag{21}$$

restrict $z$ to $1 \leq z \leq 2$. The number of terms which occur in the triple sum series is thus 4, but by definition it is a polynomial zero of degree 3 . Therefore, we may call this a "4–term" degree 3 zero.

## 3. TABULATION OF THE DEGREE 2 ZEROS

The first 355 polynomial zeros of degree 2 of the 9–$j$ coefficient, which correspond to $12 \leq \sigma \leq 22$ are presented in Table I (beginning on p. 187). We restrict $a, b, d, e$ to $\leq 5$ and the total number of degree 2 zeros generated is about 1600. Simple Fortran programs were written to check for the conditions (11), (14) and (17). As stated in Sect. 2, there are 24 conditions and hence 24 Fortran programs were written to generate the zeros. The programs consist essentially of four `DO` loops for $a, b, d$ and $e$ and five more loops for $c, f, g, h$ and $i$ being generated using the triangle inequalities that must be satisfied by the triads

$$(abc),\ (adg),\ (def),\ (beh),\ (cfi),\ \text{and}\ (ghi). \tag{22}$$

The quantities $N_1$, $N_2$, $D_1$, $D_2$ are computed for the given values $a, b, \ldots, i$ and if the condition (11), (14) or (17) is satisfied then the set of values $a, b, \ldots, i$, along with their sum $\sigma$ are stored in a file, until the upper limits for $a, b, d, e$ set as 5 are reached. The 24 output files contain degree 2 zeros with several repetitions. A sort program is used to sort on $\sigma$ and the entries in each of the output files are ordered for increasing $\sigma$. An edit program then weeds out the identical entries, *i.e.*, entries in which $a, b, \ldots, i$ are all the same string of characters. The 24 files containing the repetitions weeded out are now merged to give the Table I presented here. The cases when any one (or more) of the nine angular momenta is a zero are avoided, since it is well-known that a 9–$j$ coefficient in such a case reduces to a 6–$j$ coefficient.

In Table II (beginning on p. 190) are presented the first 102 polynomial zeros of the 9–$j$ coefficient, wherein symmetries of the coefficient are taken into account (and each zero appears only once). Additional useful information is also provided here. The tabulated results are part of the list of 829 zeros generated for values of $a, b, \cdots i$ such that their sum $\sigma$ is $\leq 30$. For the sake of easy readability, in this table, we have given twice the values for all angular momenta: the first nine columns of the table give the values of $2a, 2b, \cdots 2i$, and the remaining five columns provide the values of: $2\sigma$ ($\sigma = a+b+\cdots+i$); ntr gives the minimum number of terms in the triple sum series representation for the coefficient; n6j gives the minimum number of terms that occur in the single sum over the product of three 6–$j$ coefficients formula for the 9–$j$ coefficient; str represents the number of stretchings in the 9–$j$ coefficient and the final column gives the type of stretching (*viz.*, one, two, three or four of the six angular momentum couplings in the 9–$j$ coefficient being stretched) and the final column gives the type of stretching (if $2 \leq$ str $\leq 4$). At the end of Table II, the types A, B, T and Q are explicitly given. For the remaining 9–$j$ coefficients with 2 stretchings (see Sharp [19], denoted as type c, d or e) no zeros can occur since the summation part consists of positive contributions only. For 3 or more stretchings, closed form expressions exist (see formulae (10.8.14)–(10.8.27) of Varshalovich *et al.* [2]), which implies that they will not give rise to a zero of the 9–$j$ coefficient. The tables provided here will be very useful in further analysis of the nature of the polynomial zeros of the 9–$j$ coefficient.

## 4. Concluding remarks

In this article we have studied the polynomial zeros of degree 2 of the 9–$j$ coefficient, arising from the triple-sum series of Jucys-Bandzaitis. The first few hundred of these are generated from the conditions (11), (14) and (17) to be satisfied by the nine angular momenta $a, b, \ldots, i$, for the 9–$j$ coefficient to be a degree 2 zero. It is noted that the triple sum series reduces to a double series in these cases of interest which can be considered as special cases of such series studied by Kampé de Fériet [15] and expressed in a compact notation by Burchnall and Chaundy [16].

An interesting observation made is that due to the inherent lack of symmetry of the triple sum series, the 72 symmetries of a given 9–$j$ coefficient, which is a polynomial zero of degree $n$, will not be all of the same degree $n$. It is perhaps worthwhile to study the relation between the degree of the zero, defined as the sum of the upper limits $XF$, $YF$, $ZF$, and the number of terms which add up to give the degree $n$ zero. The number of terms is always $n+1$, for polynomial zeros of degree $n$, if it is a 3–$j$ or a 6–$j$ angular momentum coefficient, since in these cases, the series is a single sum series.

Another point to be noted is that the Howell [18] procedure for ordering the nine angular momenta cannot be resorted to, to reduce the Table I, by retaining only the "generic" zeros and discarding its symmetries. This is again due to the inherent lack of symmetry of the triple sum series for the 9–$j$ coefficient. It is our sincere hope that the analysis of the polynomial zeros presented here will be the source for further studies to understand their complicated nature.

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TABLE I. Degree 2 zeros of the 9–$j$ coefficient $\begin{Bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{Bmatrix}$. The values of $a, b, \ldots, i$ and the total sum $\sigma = a + b + \cdots + i$ are given.

| $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ | $i$ | $\sigma$ |
|---|---|---|---|---|---|---|---|---|---|
| 1.5 | 2.0 | 1.5 | 1.5 | 0.5 | 1.0 | 2.0 | 1.5 | 0.5 | 12 |
| 1.5 | 1.0 | 0.5 | 2.5 | 1.5 | 1.0 | 2.0 | 1.5 | 1.5 | 13 |
| 2.0 | 2.5 | 1.5 | 1.5 | 1.0 | 0.5 | 1.5 | 1.5 | 1.0 | 13 |
| 2.0 | 1.5 | 1.5 | 1.5 | 0.5 | 1.0 | 2.5 | 1.0 | 1.5 | 13 |
| 2.0 | 1.5 | 1.5 | 1.5 | 1.0 | 0.5 | 2.5 | 1.5 | 1.0 | 13 |
| 1.5 | 2.0 | 1.5 | 0.5 | 1.5 | 1.0 | 1.0 | 2.5 | 1.5 | 13 |
| 1.5 | 2.0 | 1.5 | 1.0 | 1.5 | 0.5 | 1.5 | 2.5 | 1.0 | 13 |
| 1.5 | 1.0 | 2.5 | 1.0 | 0.5 | 1.5 | 1.5 | 1.5 | 2.0 | 13 |
| 0.5 | 1.5 | 1.0 | 1.0 | 2.5 | 1.5 | 1.5 | 2.0 | 1.5 | 13 |
| 1.0 | 1.5 | 0.5 | 2.5 | 2.0 | 1.5 | 1.5 | 1.5 | 1.0 | 13 |
| 2.0 | 1.5 | 0.5 | 2.5 | 1.0 | 1.5 | 1.5 | 1.5 | 2.0 | 14 |
| 2.5 | 2.0 | 0.5 | 2.0 | 1.0 | 1.0 | 1.5 | 2.0 | 1.5 | 14 |
| 1.5 | 2.0 | 1.5 | 2.0 | 1.0 | 1.0 | 2.5 | 2.0 | 0.5 | 14 |
| 1.5 | 2.5 | 2.0 | 1.5 | 0.5 | 1.0 | 2.0 | 2.0 | 1.0 | 14 |
| 1.5 | 2.0 | 1.5 | 2.0 | 0.5 | 1.5 | 2.5 | 1.5 | 1.0 | 14 |
| 1.0 | 1.5 | 2.5 | 1.5 | 0.5 | 2.0 | 1.5 | 2.0 | 1.5 | 14 |
| 1.0 | 1.5 | 0.5 | 2.0 | 1.5 | 2.5 | 1.0 | 2.0 | 2.0 | 14 |
| 1.5 | 2.0 | 0.5 | 2.5 | 1.5 | 2.0 | 1.0 | 1.5 | 1.5 | 14 |
| 0.5 | 1.5 | 1.0 | 2.5 | 1.5 | 2.0 | 2.0 | 2.0 | 1.0 | 14 |
| 2.0 | 3.0 | 2.0 | 1.5 | 1.0 | 0.5 | 1.5 | 2.0 | 1.5 | 15 |
| 2.0 | 1.5 | 1.5 | 2.0 | 0.5 | 1.5 | 3.0 | 1.0 | 2.0 | 15 |
| 2.0 | 1.5 | 1.5 | 2.0 | 1.5 | 0.5 | 3.0 | 2.0 | 1.0 | 15 |
| 1.5 | 2.0 | 1.5 | 0.5 | 2.0 | 1.5 | 1.0 | 3.0 | 2.0 | 15 |
| 1.5 | 2.0 | 1.5 | 1.5 | 2.0 | 0.5 | 2.0 | 3.0 | 1.0 | 15 |
| 3.0 | 2.5 | 0.5 | 2.5 | 1.5 | 1.0 | 1.5 | 2.0 | 1.5 | 16 |
| 1.5 | 2.0 | 1.5 | 2.5 | 1.5 | 1.0 | 3.0 | 2.5 | 0.5 | 16 |
| 1.5 | 3.0 | 2.5 | 1.5 | 0.5 | 1.0 | 2.0 | 2.5 | 1.5 | 16 |
| 1.5 | 2.0 | 1.5 | 2.5 | 0.5 | 2.0 | 3.0 | 1.5 | 1.5 | 16 |
| 1.5 | 2.0 | 1.5 | 2.5 | 1.0 | 1.5 | 3.0 | 2.0 | 1.0 | 16 |
| 1.0 | 1.5 | 0.5 | 2.5 | 1.5 | 3.0 | 1.5 | 2.0 | 2.5 | 16 |
| 0.5 | 1.5 | 1.0 | 3.0 | 1.5 | 2.5 | 2.5 | 2.0 | 1.5 | 16 |
| 1.5 | 1.0 | 0.5 | 3.5 | 2.0 | 1.5 | 3.0 | 2.0 | 2.0 | 17 |
| 2.0 | 3.0 | 2.0 | 2.0 | 0.5 | 1.5 | 3.0 | 2.5 | 0.5 | 17 |
| 2.0 | 3.5 | 2.5 | 1.5 | 1.0 | 0.5 | 1.5 | 2.5 | 2.0 | 17 |
| 3.0 | 3.5 | 1.5 | 2.0 | 1.5 | 0.5 | 2.0 | 2.0 | 1.0 | 17 |
| 2.0 | 1.5 | 1.5 | 2.5 | 0.5 | 2.0 | 3.5 | 1.0 | 2.5 | 17 |
| 2.0 | 1.5 | 1.5 | 2.5 | 1.0 | 1.5 | 3.5 | 1.5 | 2.0 | 17 |
| 2.0 | 1.5 | 1.5 | 2.5 | 1.5 | 1.0 | 3.5 | 2.0 | 1.5 | 17 |
| 2.0 | 1.5 | 1.5 | 2.5 | 2.0 | 0.5 | 3.5 | 2.5 | 1.0 | 17 |
| 3.0 | 2.0 | 2.0 | 1.5 | 0.5 | 1.0 | 3.5 | 1.5 | 2.0 | 17 |
| 3.0 | 2.0 | 2.0 | 1.5 | 1.0 | 0.5 | 3.5 | 2.0 | 1.5 | 17 |
| 1.5 | 2.0 | 1.5 | 0.5 | 2.5 | 2.0 | 1.0 | 3.5 | 2.5 | 17 |
| 1.5 | 2.0 | 1.5 | 1.0 | 2.5 | 1.5 | 1.5 | 3.5 | 2.0 | 17 |
| 1.5 | 2.0 | 1.5 | 1.5 | 2.5 | 1.0 | 2.0 | 3.5 | 1.5 | 17 |
| 1.5 | 2.0 | 1.5 | 2.0 | 2.5 | 0.5 | 2.5 | 3.5 | 1.0 | 17 |
| 2.0 | 3.0 | 2.0 | 0.5 | 1.5 | 1.0 | 1.5 | 3.5 | 2.0 | 17 |
| 2.0 | 3.0 | 2.0 | 1.0 | 1.5 | 0.5 | 2.0 | 3.5 | 1.5 | 17 |
| 3.0 | 3.5 | 1.5 | 1.0 | 1.5 | 0.5 | 2.0 | 3.0 | 1.0 | 17 |
| 2.0 | 1.5 | 3.5 | 1.0 | 0.5 | 1.5 | 2.0 | 2.0 | 3.0 | 17 |
| 0.5 | 1.5 | 1.0 | 1.5 | 3.5 | 2.0 | 2.0 | 3.0 | 2.0 | 17 |
| 1.5 | 2.0 | 0.5 | 3.5 | 3.0 | 1.5 | 2.0 | 2.0 | 1.0 | 17 |
| 1.5 | 2.5 | 1.0 | 2.0 | 3.5 | 1.5 | 1.5 | 2.0 | 1.5 | 17 |
| 2.0 | 1.5 | 0.5 | 2.0 | 1.5 | 2.5 | 3.0 | 2.0 | 3.0 | 18 |
| 2.5 | 2.0 | 0.5 | 1.5 | 1.5 | 2.0 | 3.0 | 2.5 | 2.5 | 18 |
| 2.5 | 2.0 | 0.5 | 3.0 | 1.5 | 1.5 | 2.5 | 2.5 | 2.0 | 18 |
| 1.5 | 2.0 | 1.5 | 2.5 | 3.0 | 0.5 | 2.0 | 3.0 | 2.0 | 18 |
| 1.5 | 2.5 | 2.0 | 2.0 | 2.5 | 0.5 | 1.5 | 3.0 | 2.5 | 18 |
| 3.5 | 3.0 | 0.5 | 3.0 | 2.0 | 1.0 | 1.5 | 2.0 | 1.5 | 18 |
| 1.5 | 1.5 | 2.0 | 2.0 | 2.5 | 0.5 | 2.5 | 3.0 | 2.5 | 18 |
| 1.5 | 1.5 | 2.0 | 2.5 | 2.0 | 0.5 | 3.0 | 2.5 | 2.5 | 18 |
| 1.5 | 2.0 | 2.5 | 1.5 | 2.0 | 0.5 | 2.0 | 3.0 | 3.0 | 18 |
| 2.0 | 1.5 | 2.5 | 2.0 | 1.5 | 0.5 | 3.0 | 2.0 | 3.0 | 18 |
| 1.5 | 2.0 | 1.5 | 3.0 | 2.0 | 1.0 | 3.5 | 3.0 | 0.5 | 18 |
| 1.5 | 3.5 | 3.0 | 1.5 | 0.5 | 1.0 | 2.0 | 3.0 | 2.0 | 18 |
| 3.0 | 3.5 | 1.5 | 2.0 | 1.0 | 1.0 | 3.0 | 2.5 | 0.5 | 18 |
| 1.5 | 2.0 | 1.5 | 3.0 | 0.5 | 2.5 | 3.5 | 1.5 | 2.0 | 18 |
| 1.5 | 2.0 | 1.5 | 3.0 | 1.0 | 2.0 | 3.5 | 2.0 | 1.5 | 18 |
| 1.5 | 2.0 | 1.5 | 3.0 | 1.5 | 1.5 | 3.5 | 2.5 | 1.0 | 18 |
| 3.0 | 3.0 | 2.0 | 1.5 | 0.5 | 1.0 | 3.5 | 2.5 | 1.0 | 18 |
| 1.5 | 2.0 | 1.5 | 2.0 | 3.0 | 2.0 | 0.5 | 3.0 | 2.5 | 18 |
| 2.0 | 2.5 | 1.5 | 2.5 | 3.0 | 1.5 | 0.5 | 2.5 | 2.0 | 18 |
| 2.0 | 3.0 | 2.0 | 1.5 | 2.0 | 1.5 | 0.5 | 3.0 | 2.5 | 18 |
| 2.5 | 3.0 | 1.5 | 2.0 | 2.5 | 1.5 | 0.5 | 2.5 | 2.0 | 18 |
| 1.5 | 2.0 | 1.5 | 2.0 | 0.5 | 2.5 | 2.5 | 2.5 | 3.0 | 18 |
| 1.5 | 2.5 | 2.0 | 1.5 | 0.5 | 2.0 | 2.0 | 3.0 | 3.0 | 18 |
| 1.5 | 0.5 | 2.0 | 1.5 | 2.5 | 2.0 | 2.0 | 3.0 | 3.0 | 18 |
| 2.0 | 0.5 | 2.5 | 1.5 | 2.0 | 1.5 | 2.5 | 2.5 | 3.0 | 18 |
| 1.5 | 1.5 | 3.0 | 2.0 | 0.5 | 2.5 | 2.5 | 2.0 | 2.5 | 18 |
| 1.5 | 2.0 | 0.5 | 3.0 | 2.5 | 2.5 | 1.5 | 2.5 | 2.0 | 18 |
| 0.5 | 2.0 | 1.5 | 2.5 | 2.0 | 1.5 | 3.0 | 3.0 | 2.0 | 18 |
| 0.5 | 2.5 | 2.0 | 2.0 | 1.5 | 1.5 | 2.5 | 3.0 | 2.5 | 18 |
| 1.0 | 1.5 | 0.5 | 3.0 | 1.5 | 3.5 | 2.0 | 2.0 | 3.0 | 18 |
| 0.5 | 1.5 | 1.0 | 3.5 | 1.5 | 3.0 | 3.0 | 2.0 | 2.0 | 18 |
| 0.5 | 1.5 | 2.0 | 2.5 | 1.5 | 2.0 | 3.0 | 2.0 | 3.0 | 18 |
| 0.5 | 2.0 | 2.5 | 2.0 | 1.5 | 1.5 | 2.5 | 2.5 | 3.0 | 18 |
| 2.0 | 1.5 | 1.5 | 0.5 | 2.5 | 2.0 | 2.5 | 3.0 | 2.5 | 18 |
| 2.5 | 2.0 | 1.5 | 0.5 | 2.0 | 1.5 | 3.0 | 3.0 | 2.0 | 18 |
| 2.0 | 1.5 | 0.5 | 3.5 | 2.5 | 1.0 | 3.5 | 3.0 | 1.5 | 19 |
| 2.5 | 2.0 | 0.5 | 3.5 | 1.0 | 2.5 | 2.0 | 2.0 | 3.0 | 19 |
| 3.5 | 3.0 | 0.5 | 2.5 | 1.0 | 1.5 | 2.0 | 3.0 | 2.0 | 19 |
| 2.0 | 3.0 | 2.0 | 2.5 | 1.0 | 1.5 | 3.5 | 3.0 | 0.5 | 19 |
| 2.0 | 3.5 | 2.5 | 2.0 | 0.5 | 1.5 | 3.0 | 3.0 | 1.0 | 19 |
| 2.0 | 4.0 | 3.0 | 1.5 | 1.0 | 0.5 | 1.5 | 3.0 | 2.5 | 19 |
| 3.0 | 4.0 | 2.0 | 2.0 | 1.5 | 0.5 | 2.0 | 2.5 | 1.5 | 19 |
| 2.0 | 3.0 | 2.0 | 2.5 | 0.5 | 2.0 | 3.5 | 2.5 | 1.0 | 19 |
| 2.0 | 1.5 | 1.5 | 3.0 | 0.5 | 2.5 | 4.0 | 1.0 | 3.0 | 19 |
| 2.0 | 1.5 | 1.5 | 3.0 | 1.0 | 2.0 | 4.0 | 1.5 | 2.5 | 19 |
| 2.0 | 1.5 | 1.5 | 3.0 | 2.0 | 1.0 | 4.0 | 2.5 | 1.5 | 19 |
| 2.0 | 1.5 | 1.5 | 3.0 | 2.5 | 0.5 | 4.0 | 3.0 | 1.0 | 19 |
| 3.0 | 2.0 | 2.0 | 2.0 | 0.5 | 1.5 | 4.0 | 1.5 | 2.5 | 19 |
| 3.0 | 2.0 | 2.0 | 2.0 | 1.5 | 0.5 | 4.0 | 2.5 | 1.5 | 19 |
| 1.5 | 2.0 | 1.5 | 0.5 | 3.0 | 2.5 | 1.0 | 4.0 | 3.0 | 19 |
| 1.5 | 2.0 | 1.5 | 1.0 | 3.0 | 2.0 | 1.5 | 4.0 | 2.5 | 19 |
| 1.5 | 2.0 | 1.5 | 2.0 | 3.0 | 1.0 | 2.5 | 4.0 | 1.5 | 19 |
| 1.5 | 2.0 | 1.5 | 2.5 | 3.0 | 0.5 | 3.0 | 4.0 | 1.0 | 19 |
| 2.0 | 3.0 | 2.0 | 0.5 | 2.0 | 1.5 | 1.5 | 4.0 | 2.5 | 19 |
| 2.0 | 3.0 | 2.0 | 1.5 | 2.0 | 0.5 | 2.5 | 4.0 | 1.5 | 19 |
| 3.0 | 3.5 | 1.5 | 1.5 | 2.0 | 0.5 | 2.5 | 3.5 | 1.0 | 19 |

TABLE I. (contd.)

| $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ | $i$ | $\sigma$ |
|---|---|---|---|---|---|---|---|---|---|
| 3.0 | 3.5 | 1.5 | 1.0 | 2.0 | 1.0 | 2.0 | 3.5 | 1.5 | 19 |
| 1.0 | 2.5 | 3.5 | 2.0 | 0.5 | 2.5 | 2.0 | 3.0 | 2.0 | 19 |
| 2.5 | 1.0 | 3.5 | 1.5 | 0.5 | 2.0 | 3.0 | 1.5 | 3.5 | 19 |
| 0.5 | 2.0 | 1.5 | 1.0 | 3.5 | 2.5 | 1.5 | 3.5 | 3.0 | 19 |
| 1.0 | 1.5 | 0.5 | 3.5 | 3.5 | 2.0 | 2.5 | 3.0 | 1.5 | 19 |
| 1.0 | 2.0 | 1.0 | 2.0 | 3.5 | 1.5 | 3.0 | 3.5 | 1.5 | 19 |
| 1.5 | 2.0 | 0.5 | 2.5 | 2.0 | 3.5 | 1.0 | 3.0 | 3.0 | 19 |
| 2.0 | 3.0 | 1.0 | 2.5 | 4.0 | 1.5 | 1.5 | 2.0 | 1.5 | 19 |
| 2.0 | 3.5 | 1.5 | 1.0 | 2.0 | 1.0 | 3.0 | 3.5 | 1.5 | 19 |
| 2.5 | 3.0 | 0.5 | 3.5 | 2.0 | 2.5 | 1.0 | 2.0 | 2.0 | 19 |
| 0.5 | 2.0 | 1.5 | 3.5 | 2.0 | 2.5 | 3.0 | 3.0 | 1.0 | 19 |
| 1.5 | 2.5 | 1.0 | 3.5 | 2.0 | 2.5 | 2.0 | 2.5 | 1.5 | 19 |
| 1.0 | 2.0 | 2.0 | 2.5 | 3.0 | 0.5 | 2.5 | 4.0 | 2.5 | 20 |
| 1.0 | 2.5 | 2.5 | 2.0 | 2.5 | 0.5 | 2.0 | 4.0 | 3.0 | 20 |
| 2.0 | 1.5 | 0.5 | 4.0 | 1.5 | 2.5 | 3.0 | 2.0 | 3.0 | 20 |
| 2.5 | 2.0 | 0.5 | 2.5 | 1.0 | 2.5 | 4.0 | 2.0 | 3.0 | 20 |
| 2.5 | 3.0 | 1.5 | 2.0 | 1.5 | 1.5 | 1.5 | 3.5 | 3.0 | 20 |
| 3.0 | 2.5 | 0.5 | 2.0 | 1.0 | 2.0 | 4.0 | 2.5 | 2.5 | 20 |
| 3.5 | 3.0 | 0.5 | 2.5 | 1.5 | 1.0 | 3.0 | 3.5 | 1.5 | 20 |
| 4.0 | 3.5 | 0.5 | 3.5 | 2.5 | 1.0 | 1.5 | 2.0 | 1.5 | 20 |
| 2.0 | 1.0 | 2.0 | 3.0 | 2.5 | 0.5 | 4.0 | 2.5 | 2.5 | 20 |
| 2.5 | 1.0 | 2.5 | 2.5 | 2.0 | 0.5 | 4.0 | 2.0 | 3.0 | 20 |
| 1.5 | 2.0 | 1.5 | 3.5 | 2.5 | 1.0 | 4.0 | 3.5 | 0.5 | 20 |
| 1.5 | 4.0 | 3.5 | 1.5 | 0.5 | 1.0 | 2.0 | 3.5 | 2.5 | 20 |
| 3.0 | 3.5 | 1.5 | 2.5 | 1.5 | 1.0 | 3.5 | 3.0 | 0.5 | 20 |
| 3.0 | 4.0 | 2.0 | 2.0 | 1.0 | 1.0 | 3.0 | 3.0 | 1.0 | 20 |
| 1.5 | 2.0 | 1.5 | 3.5 | 0.5 | 3.0 | 4.0 | 1.5 | 2.5 | 20 |
| 1.5 | 2.0 | 1.5 | 3.5 | 1.0 | 2.5 | 4.0 | 2.0 | 2.0 | 20 |
| 1.5 | 2.0 | 1.5 | 3.5 | 1.5 | 2.0 | 4.0 | 2.5 | 1.5 | 20 |
| 1.5 | 2.0 | 1.5 | 3.5 | 2.0 | 1.5 | 4.0 | 3.0 | 1.0 | 20 |
| 1.5 | 2.5 | 2.0 | 3.0 | 1.5 | 1.5 | 3.5 | 3.0 | 1.5 | 20 |
| 3.0 | 3.0 | 2.0 | 2.0 | 0.5 | 1.5 | 4.0 | 2.5 | 1.5 | 20 |
| 3.0 | 3.0 | 2.0 | 2.0 | 1.0 | 1.0 | 4.0 | 3.0 | 1.0 | 20 |
| 3.0 | 3.5 | 2.5 | 1.5 | 0.5 | 1.0 | 3.5 | 3.0 | 1.5 | 20 |
| 2.5 | 4.0 | 2.5 | 2.0 | 2.0 | 1.0 | 0.5 | 3.0 | 2.5 | 20 |
| 3.0 | 3.5 | 1.5 | 2.5 | 1.5 | 2.0 | 1.5 | 3.0 | 1.5 | 20 |
| 3.0 | 4.0 | 2.0 | 2.5 | 2.5 | 1.0 | 0.5 | 2.5 | 2.0 | 20 |
| 2.0 | 2.0 | 1.0 | 2.5 | 4.0 | 2.5 | 0.5 | 3.0 | 2.5 | 20 |
| 2.5 | 2.5 | 1.0 | 3.0 | 4.0 | 2.0 | 0.5 | 2.5 | 2.0 | 20 |
| 1.0 | 2.0 | 2.0 | 2.5 | 0.5 | 3.0 | 2.5 | 2.5 | 4.0 | 20 |
| 1.0 | 2.5 | 2.5 | 2.0 | 0.5 | 2.5 | 2.0 | 3.0 | 4.0 | 20 |
| 2.0 | 0.5 | 2.5 | 1.0 | 2.5 | 2.5 | 2.0 | 3.0 | 4.0 | 20 |
| 2.5 | 0.5 | 3.0 | 1.0 | 2.0 | 2.0 | 2.5 | 2.5 | 4.0 | 20 |
| 1.5 | 2.5 | 4.0 | 1.5 | 0.5 | 2.0 | 2.0 | 3.0 | 3.0 | 20 |
| 0.5 | 2.5 | 2.0 | 2.5 | 2.5 | 1.0 | 3.0 | 4.0 | 2.0 | 20 |
| 0.5 | 3.0 | 2.5 | 2.0 | 2.0 | 1.0 | 2.5 | 4.0 | 2.5 | 20 |
| 1.0 | 1.5 | 0.5 | 2.5 | 3.0 | 3.5 | 1.5 | 3.5 | 3.0 | 20 |
| 1.5 | 3.0 | 1.5 | 2.0 | 1.5 | 2.5 | 1.5 | 3.5 | 3.0 | 20 |
| 2.5 | 3.0 | 0.5 | 4.0 | 3.0 | 2.0 | 1.5 | 2.0 | 1.5 | 20 |
| 1.0 | 1.5 | 0.5 | 3.5 | 1.5 | 4.0 | 2.5 | 2.0 | 3.5 | 20 |
| 1.0 | 2.0 | 1.0 | 4.0 | 3.0 | 2.0 | 3.0 | 3.0 | 1.0 | 20 |
| 1.5 | 3.0 | 1.5 | 2.5 | 1.5 | 2.0 | 3.0 | 3.5 | 1.5 | 20 |
| 0.5 | 1.5 | 1.0 | 4.0 | 1.5 | 3.5 | 3.5 | 2.0 | 2.5 | 20 |
| 2.0 | 2.0 | 1.0 | 0.5 | 3.0 | 2.5 | 2.5 | 4.0 | 2.5 | 20 |
| 2.5 | 2.5 | 1.0 | 0.5 | 2.5 | 2.0 | 3.0 | 4.0 | 2.0 | 20 |
| 0.5 | 2.0 | 2.5 | 2.5 | 1.0 | 2.5 | 3.0 | 2.0 | 4.0 | 20 |
| 0.5 | 2.5 | 3.0 | 2.0 | 1.0 | 2.0 | 2.5 | 2.5 | 4.0 | 20 |
| 1.5 | 1.0 | 0.5 | 4.5 | 2.5 | 2.0 | 4.0 | 2.5 | 2.5 | 21 |

| $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ | $i$ | $\sigma$ |
|---|---|---|---|---|---|---|---|---|---|
| 4.0 | 3.5 | 0.5 | 3.0 | 1.5 | 1.5 | 2.0 | 3.0 | 2.0 | 21 |
| 2.0 | 3.0 | 2.0 | 3.0 | 1.5 | 1.5 | 4.0 | 3.5 | 0.5 | 21 |
| 2.0 | 4.0 | 3.0 | 2.0 | 0.5 | 1.5 | 3.0 | 3.5 | 1.5 | 21 |
| 2.0 | 4.5 | 3.5 | 1.5 | 1.0 | 0.5 | 1.5 | 3.5 | 3.0 | 21 |
| 3.0 | 4.5 | 2.5 | 2.0 | 1.5 | 0.5 | 2.0 | 3.0 | 2.0 | 21 |
| 4.0 | 4.5 | 1.5 | 2.5 | 2.0 | 0.5 | 2.5 | 2.5 | 1.0 | 21 |
| 2.0 | 3.0 | 2.0 | 3.0 | 0.5 | 2.5 | 4.0 | 2.5 | 1.5 | 21 |
| 2.0 | 3.0 | 2.0 | 3.0 | 1.0 | 2.0 | 4.0 | 3.0 | 1.0 | 21 |
| 2.0 | 1.5 | 1.5 | 3.5 | 0.5 | 3.0 | 4.5 | 1.0 | 3.5 | 21 |
| 2.0 | 1.5 | 1.5 | 3.5 | 1.0 | 2.5 | 4.5 | 1.5 | 3.0 | 21 |
| 2.0 | 1.5 | 1.5 | 3.5 | 1.5 | 2.0 | 4.5 | 2.0 | 2.5 | 21 |
| 2.0 | 1.5 | 1.5 | 3.5 | 2.0 | 1.5 | 4.5 | 2.5 | 2.0 | 21 |
| 2.0 | 1.5 | 1.5 | 3.5 | 2.5 | 1.0 | 4.5 | 3.0 | 1.5 | 21 |
| 2.0 | 1.5 | 1.5 | 3.5 | 3.0 | 0.5 | 4.5 | 3.5 | 1.0 | 21 |
| 3.0 | 2.0 | 2.0 | 2.5 | 0.5 | 2.0 | 4.5 | 1.5 | 3.0 | 21 |
| 3.0 | 2.0 | 2.0 | 2.5 | 1.0 | 1.5 | 4.5 | 2.0 | 2.5 | 21 |
| 3.0 | 2.0 | 2.0 | 2.5 | 1.5 | 1.0 | 4.5 | 2.5 | 2.0 | 21 |
| 3.0 | 2.0 | 2.0 | 2.5 | 2.0 | 0.5 | 4.5 | 3.0 | 1.5 | 21 |
| 4.0 | 2.5 | 2.5 | 1.5 | 0.5 | 1.0 | 4.5 | 2.0 | 2.5 | 21 |
| 4.0 | 2.5 | 2.5 | 1.5 | 1.0 | 0.5 | 4.5 | 2.5 | 2.0 | 21 |
| 1.5 | 2.0 | 1.5 | 0.5 | 3.5 | 3.0 | 1.0 | 4.5 | 3.5 | 21 |
| 1.5 | 2.0 | 1.5 | 1.0 | 3.5 | 2.5 | 1.5 | 4.5 | 3.0 | 21 |
| 1.5 | 2.0 | 1.5 | 1.5 | 3.5 | 2.0 | 2.0 | 4.5 | 2.5 | 21 |
| 1.5 | 2.0 | 1.5 | 2.0 | 3.5 | 1.5 | 2.5 | 4.5 | 2.0 | 21 |
| 1.5 | 2.0 | 1.5 | 2.5 | 3.5 | 1.0 | 3.0 | 4.5 | 1.5 | 21 |
| 1.5 | 2.0 | 1.5 | 3.0 | 3.5 | 0.5 | 3.5 | 4.5 | 1.0 | 21 |
| 2.0 | 3.0 | 2.0 | 0.5 | 2.5 | 2.0 | 1.5 | 4.5 | 3.0 | 21 |
| 2.0 | 3.0 | 2.0 | 1.0 | 2.5 | 1.5 | 2.0 | 4.5 | 2.5 | 21 |
| 2.0 | 3.0 | 2.0 | 1.5 | 2.5 | 1.0 | 2.5 | 4.5 | 2.0 | 21 |
| 2.0 | 3.0 | 2.0 | 2.0 | 2.5 | 0.5 | 3.0 | 4.5 | 1.5 | 21 |
| 2.5 | 4.0 | 2.5 | 0.5 | 1.5 | 1.0 | 2.0 | 4.5 | 2.5 | 21 |
| 2.5 | 4.0 | 2.5 | 1.0 | 1.5 | 0.5 | 2.5 | 4.5 | 2.0 | 21 |
| 3.0 | 3.5 | 1.5 | 2.0 | 2.5 | 0.5 | 3.0 | 4.0 | 1.0 | 21 |
| 3.0 | 3.5 | 1.5 | 1.0 | 2.5 | 1.5 | 2.0 | 4.0 | 2.0 | 21 |
| 3.0 | 3.5 | 1.5 | 1.5 | 2.5 | 1.0 | 2.5 | 4.0 | 1.5 | 21 |
| 2.5 | 2.0 | 4.5 | 1.0 | 0.5 | 1.5 | 2.5 | 2.5 | 4.0 | 21 |
| 0.5 | 1.5 | 1.0 | 2.0 | 4.5 | 2.5 | 2.5 | 4.0 | 2.5 | 21 |
| 1.0 | 2.5 | 1.5 | 1.5 | 4.0 | 2.5 | 1.5 | 3.5 | 3.0 | 21 |
| 1.0 | 2.5 | 1.5 | 1.5 | 3.5 | 2.0 | 2.5 | 4.0 | 2.5 | 21 |
| 1.5 | 2.5 | 1.0 | 2.5 | 4.5 | 2.0 | 2.0 | 3.0 | 2.0 | 21 |
| 1.5 | 2.5 | 1.0 | 2.5 | 4.0 | 1.5 | 3.0 | 3.5 | 1.5 | 21 |
| 1.5 | 3.5 | 2.0 | 1.0 | 2.5 | 1.5 | 2.5 | 4.0 | 2.5 | 21 |
| 2.0 | 2.5 | 0.5 | 4.5 | 4.0 | 1.5 | 2.5 | 2.5 | 1.0 | 21 |
| 2.5 | 3.5 | 1.0 | 3.0 | 4.5 | 1.5 | 1.5 | 2.0 | 1.5 | 21 |
| 2.5 | 4.0 | 1.5 | 1.5 | 2.5 | 1.0 | 3.0 | 3.5 | 1.5 | 21 |
| 1.5 | 2.0 | 0.5 | 3.0 | 2.0 | 4.0 | 1.5 | 3.0 | 3.5 | 21 |
| 0.5 | 2.0 | 1.5 | 4.0 | 2.0 | 3.0 | 3.5 | 3.0 | 1.5 | 21 |
| 1.0 | 2.0 | 2.0 | 3.5 | 3.0 | 0.5 | 3.5 | 4.0 | 2.5 | 22 |
| 1.0 | 2.5 | 2.5 | 3.0 | 2.5 | 0.5 | 3.0 | 4.0 | 3.0 | 22 |
| 1.5 | 1.0 | 0.5 | 2.5 | 2.5 | 4.0 | 3.0 | 2.5 | 4.5 | 22 |
| 1.5 | 1.0 | 0.5 | 3.0 | 2.0 | 4.0 | 3.5 | 2.0 | 4.5 | 22 |
| 2.0 | 2.0 | 1.0 | 2.0 | 2.0 | 3.0 | 3.0 | 3.0 | 4.0 | 22 |
| 2.5 | 2.0 | 0.5 | 4.0 | 2.0 | 2.0 | 3.5 | 3.0 | 2.5 | 22 |
| 3.0 | 2.5 | 0.5 | 1.0 | 2.5 | 2.5 | 3.0 | 4.0 | 3.0 | 22 |
| 3.0 | 2.5 | 0.5 | 3.5 | 2.0 | 1.5 | 3.5 | 3.5 | 2.0 | 22 |
| 3.0 | 3.0 | 1.0 | 1.5 | 1.5 | 2.0 | 3.5 | 3.5 | 3.0 | 22 |
| 3.5 | 3.0 | 0.5 | 1.0 | 2.0 | 2.0 | 3.5 | 4.0 | 2.5 | 22 |
| 3.5 | 3.0 | 0.5 | 3.5 | 1.0 | 2.5 | 2.0 | 3.0 | 3.0 | 22 |

TABLE I. (contd.)

| $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ | $i$ | $\sigma$ |
|---|---|---|---|---|---|---|---|---|---|
| 1.5 | 2.0 | 1.5 | 3.0 | 4.0 | 1.0 | 2.5 | 4.0 | 2.5 | 22 |
| 1.5 | 3.0 | 2.5 | 2.0 | 3.0 | 1.0 | 1.5 | 4.0 | 3.5 | 22 |
| 2.0 | 2.0 | 1.0 | 4.0 | 4.5 | 0.5 | 3.0 | 3.5 | 1.5 | 22 |
| 2.0 | 2.5 | 1.5 | 3.0 | 4.0 | 1.0 | 2.0 | 3.5 | 2.5 | 22 |
| 2.0 | 3.0 | 2.0 | 2.5 | 3.5 | 1.0 | 1.5 | 3.5 | 3.0 | 22 |
| 2.0 | 4.0 | 3.0 | 2.0 | 2.5 | 0.5 | 1.0 | 3.5 | 3.5 | 22 |
| 2.5 | 2.5 | 1.0 | 4.0 | 4.5 | 0.5 | 2.5 | 3.0 | 1.5 | 22 |
| 2.5 | 4.0 | 2.5 | 2.5 | 3.0 | 0.5 | 1.0 | 3.0 | 3.0 | 22 |
| 3.0 | 3.5 | 1.5 | 2.5 | 2.0 | 1.5 | 1.5 | 3.5 | 3.0 | 22 |
| 4.0 | 3.5 | 0.5 | 3.0 | 2.0 | 1.0 | 3.0 | 3.5 | 1.5 | 22 |
| 4.5 | 4.0 | 0.5 | 4.0 | 3.0 | 1.0 | 1.5 | 2.0 | 1.5 | 22 |
| 1.5 | 1.5 | 2.0 | 2.5 | 3.5 | 1.0 | 3.0 | 4.0 | 3.0 | 22 |
| 1.5 | 1.5 | 2.0 | 3.0 | 3.0 | 1.0 | 3.5 | 3.5 | 3.0 | 22 |
| 1.5 | 1.5 | 2.0 | 3.5 | 2.5 | 1.0 | 4.0 | 3.0 | 3.0 | 22 |
| 1.5 | 2.0 | 2.5 | 3.0 | 2.0 | 1.0 | 3.5 | 3.0 | 3.5 | 22 |
| 1.5 | 2.5 | 3.0 | 1.5 | 2.5 | 1.0 | 2.0 | 4.0 | 4.0 | 22 |
| 2.0 | 1.0 | 2.0 | 3.0 | 3.5 | 0.5 | 4.0 | 3.5 | 2.5 | 22 |
| 2.0 | 1.5 | 2.5 | 2.0 | 3.0 | 1.0 | 3.0 | 3.5 | 3.5 | 22 |
| 2.0 | 2.0 | 3.0 | 1.5 | 2.5 | 1.0 | 2.5 | 3.5 | 4.0 | 22 |
| 2.0 | 2.0 | 3.0 | 2.0 | 2.0 | 1.0 | 3.0 | 3.0 | 4.0 | 22 |
| 2.0 | 2.0 | 3.0 | 2.5 | 1.5 | 1.0 | 3.5 | 2.5 | 4.0 | 22 |
| 2.0 | 3.0 | 4.0 | 1.0 | 1.5 | 0.5 | 2.0 | 3.5 | 4.5 | 22 |
| 2.5 | 1.0 | 2.5 | 2.5 | 3.0 | 0.5 | 4.0 | 3.0 | 3.0 | 22 |
| 2.5 | 1.5 | 1.0 | 2.0 | 2.0 | 3.0 | 3.5 | 2.5 | 4.0 | 22 |
| 2.5 | 1.5 | 1.0 | 2.5 | 1.5 | 3.0 | 4.0 | 2.0 | 4.0 | 22 |
| 2.5 | 1.5 | 3.0 | 2.5 | 1.5 | 1.0 | 4.0 | 2.0 | 4.0 | 22 |
| 2.5 | 2.5 | 4.0 | 1.0 | 1.5 | 0.5 | 2.5 | 3.0 | 4.5 | 22 |
| 2.5 | 2.5 | 4.0 | 1.5 | 1.0 | 0.5 | 3.0 | 2.5 | 4.5 | 22 |
| 3.0 | 2.0 | 1.0 | 1.5 | 2.0 | 2.5 | 3.5 | 3.0 | 3.5 | 22 |
| 3.0 | 2.0 | 4.0 | 1.5 | 1.0 | 0.5 | 3.5 | 2.0 | 4.5 | 22 |
| 3.5 | 2.5 | 1.0 | 1.5 | 1.5 | 2.0 | 4.0 | 3.0 | 3.0 | 22 |
| 1.5 | 2.0 | 1.5 | 4.0 | 3.0 | 1.0 | 4.5 | 4.0 | 0.5 | 22 |
| 1.5 | 4.5 | 4.0 | 1.5 | 0.5 | 1.0 | 2.0 | 4.0 | 3.0 | 22 |
| 2.5 | 4.0 | 2.5 | 2.5 | 0.5 | 2.0 | 4.0 | 3.5 | 0.5 | 22 |
| 3.0 | 3.5 | 1.5 | 3.0 | 2.0 | 1.0 | 4.0 | 3.5 | 0.5 | 22 |
| 3.0 | 4.5 | 2.5 | 1.5 | 0.5 | 1.0 | 2.5 | 4.0 | 2.5 | 22 |
| 3.0 | 4.5 | 2.5 | 2.0 | 1.0 | 1.0 | 3.0 | 3.5 | 1.5 | 22 |
| 3.5 | 4.5 | 2.0 | 1.5 | 0.5 | 1.0 | 3.0 | 4.0 | 2.0 | 22 |
| 1.5 | 2.0 | 1.5 | 4.0 | 0.5 | 3.5 | 4.5 | 1.5 | 3.0 | 22 |
| 1.5 | 2.0 | 1.5 | 4.0 | 1.0 | 3.0 | 4.5 | 2.0 | 2.5 | 22 |
| 1.5 | 2.0 | 1.5 | 4.0 | 1.5 | 2.5 | 4.5 | 2.5 | 2.0 | 22 |
| 1.5 | 2.0 | 1.5 | 4.0 | 2.0 | 2.0 | 4.5 | 3.0 | 1.5 | 22 |
| 1.5 | 2.0 | 1.5 | 4.0 | 2.5 | 1.5 | 4.5 | 3.5 | 1.0 | 22 |
| 1.5 | 3.0 | 2.5 | 3.0 | 1.5 | 1.5 | 3.5 | 3.5 | 2.0 | 22 |
| 3.0 | 3.0 | 1.0 | 3.0 | 0.5 | 2.5 | 4.0 | 2.5 | 2.5 | 22 |
| 3.5 | 3.5 | 1.0 | 2.5 | 0.5 | 2.0 | 4.0 | 3.0 | 2.0 | 22 |
| 3.0 | 3.0 | 2.0 | 2.5 | 0.5 | 2.0 | 4.5 | 2.5 | 2.0 | 22 |
| 3.0 | 3.0 | 2.0 | 2.5 | 1.0 | 1.5 | 4.5 | 3.0 | 1.5 | 22 |
| 3.0 | 3.0 | 2.0 | 2.5 | 1.5 | 1.0 | 4.5 | 3.5 | 1.0 | 22 |
| 3.0 | 4.0 | 3.0 | 1.5 | 0.5 | 1.0 | 3.5 | 3.5 | 2.0 | 22 |
| 2.5 | 3.0 | 1.5 | 2.5 | 2.5 | 1.0 | 4.0 | 4.5 | 0.5 | 22 |
| 2.5 | 4.0 | 2.5 | 3.0 | 3.0 | 1.0 | 0.5 | 3.0 | 2.5 | 22 |
| 3.0 | 3.5 | 1.5 | 2.0 | 2.0 | 1.0 | 4.0 | 4.5 | 0.5 | 22 |
| 3.0 | 4.0 | 2.0 | 3.5 | 3.5 | 1.0 | 0.5 | 2.5 | 2.0 | 22 |
| 1.0 | 2.0 | 2.0 | 1.5 | 3.5 | 3.0 | 0.5 | 4.5 | 4.0 | 22 |
| 1.0 | 2.5 | 2.5 | 1.5 | 3.0 | 2.5 | 0.5 | 4.5 | 4.0 | 22 |
| 1.5 | 2.0 | 1.5 | 2.5 | 4.0 | 2.5 | 1.0 | 4.0 | 3.0 | 22 |
| 1.5 | 2.5 | 2.0 | 2.5 | 3.5 | 2.0 | 1.0 | 4.0 | 3.0 | 22 |

| $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ | $i$ | $\sigma$ |
|---|---|---|---|---|---|---|---|---|---|
| 1.5 | 3.0 | 2.5 | 1.0 | 2.5 | 2.5 | 0.5 | 4.5 | 4.0 | 22 |
| 1.5 | 3.5 | 3.0 | 1.0 | 2.0 | 2.0 | 0.5 | 4.5 | 4.0 | 22 |
| 2.0 | 3.0 | 2.0 | 2.0 | 3.0 | 2.0 | 1.0 | 4.0 | 3.0 | 22 |
| 2.0 | 3.0 | 2.0 | 3.0 | 3.5 | 1.5 | 1.0 | 3.5 | 2.5 | 22 |
| 2.5 | 3.0 | 1.5 | 3.5 | 4.0 | 1.5 | 1.0 | 3.0 | 2.0 | 22 |
| 2.5 | 3.5 | 2.0 | 1.5 | 2.5 | 2.0 | 1.0 | 4.0 | 3.0 | 22 |
| 2.5 | 4.0 | 2.5 | 1.5 | 2.0 | 1.5 | 1.0 | 4.0 | 3.0 | 22 |
| 3.0 | 3.0 | 1.0 | 2.5 | 4.0 | 2.5 | 0.5 | 3.0 | 2.5 | 22 |
| 3.0 | 3.5 | 1.5 | 2.0 | 3.0 | 2.0 | 1.0 | 3.5 | 2.5 | 22 |
| 3.0 | 3.5 | 1.5 | 3.0 | 3.5 | 1.5 | 1.0 | 3.0 | 2.0 | 22 |
| 3.5 | 3.5 | 1.0 | 3.0 | 4.0 | 2.0 | 0.5 | 2.5 | 2.0 | 22 |
| 3.5 | 4.0 | 1.5 | 2.5 | 3.0 | 1.5 | 1.0 | 3.0 | 2.0 | 22 |
| 1.5 | 2.0 | 1.5 | 2.5 | 1.0 | 3.5 | 3.0 | 3.0 | 4.0 | 22 |
| 1.5 | 2.0 | 1.5 | 3.0 | 1.0 | 3.0 | 3.5 | 3.0 | 3.5 | 22 |
| 1.5 | 3.0 | 2.5 | 1.5 | 1.0 | 2.5 | 2.0 | 4.0 | 4.0 | 22 |
| 2.0 | 2.0 | 1.0 | 3.0 | 0.5 | 3.5 | 4.0 | 2.5 | 3.5 | 22 |
| 2.0 | 2.5 | 1.5 | 2.0 | 1.0 | 3.0 | 3.0 | 3.5 | 3.5 | 22 |
| 2.0 | 3.0 | 2.0 | 1.5 | 1.0 | 2.5 | 2.5 | 4.0 | 3.5 | 22 |
| 2.0 | 3.0 | 2.0 | 2.0 | 1.0 | 2.0 | 3.0 | 4.0 | 3.0 | 22 |
| 2.0 | 4.0 | 3.0 | 1.0 | 0.5 | 1.5 | 2.0 | 4.5 | 3.5 | 22 |
| 2.5 | 2.5 | 1.0 | 2.5 | 0.5 | 3.0 | 4.0 | 3.0 | 3.0 | 22 |
| 2.5 | 4.0 | 2.5 | 1.0 | 0.5 | 1.5 | 2.5 | 4.5 | 3.0 | 22 |
| 1.0 | 0.5 | 1.5 | 2.0 | 4.0 | 3.0 | 2.0 | 4.5 | 3.5 | 22 |
| 1.0 | 0.5 | 1.5 | 2.5 | 4.0 | 2.5 | 2.5 | 4.5 | 3.0 | 22 |
| 1.5 | 1.0 | 2.5 | 1.5 | 3.0 | 2.5 | 2.0 | 4.0 | 4.0 | 22 |
| 1.5 | 1.0 | 2.5 | 2.0 | 3.0 | 2.0 | 2.5 | 4.0 | 3.5 | 22 |
| 2.0 | 1.0 | 2.0 | 2.0 | 3.0 | 2.0 | 3.0 | 4.0 | 3.0 | 22 |
| 2.0 | 1.0 | 3.0 | 2.0 | 2.5 | 1.5 | 3.0 | 3.5 | 3.5 | 22 |
| 2.5 | 0.5 | 3.0 | 2.5 | 2.5 | 1.0 | 4.0 | 3.0 | 3.0 | 22 |
| 2.5 | 1.0 | 3.5 | 1.5 | 2.0 | 1.5 | 3.0 | 3.0 | 4.0 | 22 |
| 3.0 | 0.5 | 3.5 | 2.0 | 2.0 | 1.0 | 4.0 | 2.5 | 3.5 | 22 |
| 3.0 | 1.0 | 3.0 | 1.5 | 2.0 | 1.5 | 3.5 | 3.0 | 3.5 | 22 |
| 1.0 | 2.5 | 3.5 | 3.0 | 0.5 | 3.5 | 3.0 | 3.0 | 2.0 | 22 |
| 2.0 | 1.5 | 3.5 | 2.5 | 0.5 | 3.0 | 3.5 | 2.0 | 3.5 | 22 |
| 2.0 | 2.0 | 4.0 | 2.0 | 0.5 | 2.5 | 3.0 | 2.5 | 3.5 | 22 |
| 0.5 | 2.5 | 2.0 | 3.5 | 3.5 | 1.0 | 3.0 | 4.0 | 2.0 | 22 |
| 0.5 | 3.0 | 2.5 | 3.0 | 3.0 | 1.0 | 2.5 | 4.0 | 2.5 | 22 |
| 1.5 | 2.0 | 0.5 | 3.5 | 3.5 | 3.0 | 2.0 | 3.5 | 2.5 | 22 |
| 2.0 | 2.5 | 0.5 | 4.0 | 3.5 | 2.5 | 2.0 | 3.0 | 2.0 | 22 |
| 2.5 | 3.0 | 0.5 | 3.5 | 2.0 | 3.5 | 1.0 | 3.0 | 3.0 | 22 |
| 0.5 | 1.5 | 1.0 | 4.0 | 2.5 | 2.5 | 4.5 | 3.0 | 2.5 | 22 |
| 0.5 | 1.5 | 1.0 | 4.0 | 3.0 | 2.0 | 4.5 | 3.5 | 2.0 | 22 |
| 0.5 | 3.0 | 2.5 | 2.5 | 1.0 | 2.5 | 3.0 | 3.0 | 4.0 | 22 |
| 0.5 | 3.5 | 3.0 | 2.0 | 1.0 | 2.0 | 2.5 | 3.5 | 4.0 | 22 |
| 1.0 | 1.5 | 0.5 | 3.0 | 3.0 | 4.0 | 2.0 | 3.5 | 3.5 | 22 |
| 1.0 | 1.5 | 0.5 | 4.0 | 1.5 | 4.5 | 3.0 | 2.0 | 4.0 | 22 |
| 1.0 | 2.5 | 1.5 | 3.0 | 2.0 | 2.0 | 4.0 | 3.5 | 2.5 | 22 |
| 1.0 | 2.5 | 1.5 | 3.0 | 2.5 | 1.5 | 4.0 | 4.0 | 2.0 | 22 |
| 1.0 | 3.0 | 2.0 | 2.5 | 1.5 | 2.0 | 3.5 | 3.5 | 3.0 | 22 |
| 1.0 | 3.5 | 2.5 | 2.0 | 1.5 | 1.5 | 3.0 | 4.0 | 3.0 | 22 |
| 1.5 | 3.0 | 1.5 | 2.5 | 1.5 | 3.0 | 2.0 | 3.5 | 3.5 | 22 |
| 0.5 | 1.5 | 1.0 | 4.5 | 1.5 | 4.0 | 4.0 | 2.0 | 3.0 | 22 |
| 1.5 | 3.0 | 1.5 | 3.0 | 1.5 | 2.5 | 3.5 | 3.5 | 2.0 | 22 |
| 0.5 | 1.0 | 1.5 | 4.0 | 2.0 | 3.0 | 4.5 | 2.0 | 3.5 | 22 |
| 0.5 | 1.0 | 1.5 | 4.0 | 2.5 | 2.5 | 4.5 | 2.5 | 3.0 | 22 |
| 0.5 | 2.5 | 3.0 | 2.5 | 2.5 | 1.0 | 3.0 | 4.0 | 3.0 | 22 |
| 0.5 | 3.0 | 3.5 | 2.0 | 2.0 | 1.0 | 2.5 | 4.0 | 3.5 | 22 |
| 1.0 | 1.5 | 2.5 | 3.0 | 1.5 | 2.5 | 4.0 | 2.0 | 4.0 | 22 |

TABLE I. (contd.)

| $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ | $i$ | $\sigma$ |
|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 1.5 | 2.5 | 3.0 | 2.0 | 2.0 | 4.0 | 2.5 | 3.5 | 22 |
| 1.0 | 2.0 | 2.0 | 3.0 | 2.0 | 2.0 | 4.0 | 3.0 | 3.0 | 22 |
| 1.0 | 2.0 | 3.0 | 2.5 | 2.0 | 1.5 | 3.5 | 3.0 | 3.5 | 22 |
| 1.0 | 2.5 | 3.5 | 2.0 | 1.5 | 1.5 | 3.0 | 3.0 | 4.0 | 22 |
| 1.0 | 3.0 | 3.0 | 2.0 | 1.5 | 1.5 | 3.0 | 3.5 | 3.5 | 22 |
| 2.0 | 1.0 | 2.0 | 0.5 | 3.5 | 3.0 | 2.5 | 3.5 | 4.0 | 22 |
| 2.0 | 1.5 | 1.5 | 1.0 | 3.0 | 3.0 | 3.0 | 3.5 | 3.5 | 22 |
| 2.0 | 1.5 | 1.5 | 1.0 | 3.5 | 2.5 | 3.0 | 4.0 | 3.0 | 22 |
| 2.5 | 1.0 | 2.5 | 0.5 | 3.0 | 2.5 | 3.0 | 3.0 | 4.0 | 22 |
| 2.5 | 1.5 | 2.0 | 1.0 | 3.0 | 2.0 | 3.5 | 3.5 | 3.0 | 22 |
| 3.0 | 2.0 | 2.0 | 1.0 | 2.0 | 2.0 | 4.0 | 3.0 | 3.0 | 22 |
| 3.0 | 2.0 | 2.0 | 1.0 | 2.5 | 1.5 | 4.0 | 3.5 | 2.5 | 22 |
| 3.0 | 2.5 | 1.5 | 1.0 | 2.5 | 1.5 | 4.0 | 4.0 | 2.0 | 22 |
| 4.0 | 2.5 | 2.5 | 0.5 | 1.5 | 1.0 | 4.5 | 3.0 | 2.5 | 22 |
| 4.0 | 3.0 | 2.0 | 0.5 | 1.5 | 1.0 | 4.5 | 3.5 | 2.0 | 22 |

TABLE II. A listing of the first 102 degree 2 zeros of the 9–$j$ coefficient, along with some relevant details. The first nine columns give the values of the 9 angular momenta. Note that in this table each entry stands for twice the angular momentum. $\sigma$ represents the sum of the nine angular momenta: $a+b+\cdots+i$. ntr stands for the minimum number of terms in the triple sum series. n6j stands for the minimum number of terms in the single sum over a product of three 6–$j$ coefficients expression for the 9–$j$ coefficient. str stands for the number of stretchings int the given 9–$j$ coefficient. By type is meant the type of 9–$j$ coefficient having two (A and B), three (T) or four (Q) stretchings (given at the end of the table), or one of their symmetries.

| $2a$ | $2b$ | $2c$ | $2d$ | $2e$ | $2f$ | $2g$ | $2h$ | $2i$ | $2\sigma$ | ntr | n6j | str | type | deg |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 2 | 4 | 3 | 3 | 3 | 4 | 1 | 24 | 2 | 2 | 4 | Q | 1 |
| 3 | 3 | 2 | 2 | 3 | 1 | 5 | 4 | 3 | 26 | 2 | 2 | 3 | T | 1 |
| 4 | 2 | 4 | 1 | 2 | 3 | 5 | 4 | 3 | 28 | 2 | 2 | 3 | T | 2 |
| 3 | 1 | 4 | 2 | 3 | 3 | 5 | 4 | 3 | 28 | 2 | 2 | 3 | T | 2 |
| 4 | 3 | 3 | 2 | 3 | 1 | 6 | 4 | 4 | 30 | 2 | 2 | 3 | T | 1 |
| 5 | 3 | 4 | 1 | 2 | 3 | 6 | 5 | 3 | 32 | 2 | 2 | 3 | T | 2 |
| 4 | 2 | 4 | 2 | 3 | 3 | 6 | 5 | 3 | 32 | 2 | 2 | 2 | A | 2 |
| 3 | 4 | 3 | 3 | 1 | 4 | 6 | 5 | 3 | 32 | 2 | 2 | 3 | T | 1 |
| 3 | 4 | 1 | 5 | 5 | 4 | 2 | 5 | 3 | 32 | 2 | 2 | 4 | Q | 1 |
| 2 | 3 | 1 | 6 | 7 | 3 | 4 | 6 | 2 | 34 | 2 | 2 | 4 | Q | 1 |
| 4 | 4 | 2 | 3 | 4 | 1 | 7 | 6 | 3 | 34 | 2 | 2 | 3 | T | 1 |
| 5 | 4 | 3 | 2 | 1 | 3 | 7 | 5 | 4 | 34 | 2 | 2 | 3 | T | 1 |
| 4 | 3 | 3 | 3 | 2 | 3 | 7 | 5 | 4 | 34 | 2 | 2 | 2 | A | 1 |
| 1 | 6 | 5 | 4 | 4 | 6 | 3 | 4 | 1 | 34 | 2 | 2 | 4 | Q | 3 |
| 4 | 6 | 4 | 4 | 4 | 2 | 2 | 4 | 4 | 34 | 8 | 3 | 0 | - | 3 |
| 6 | 4 | 4 | 1 | 2 | 3 | 7 | 6 | 3 | 36 | 2 | 2 | 3 | T | 2 |
| 2 | 4 | 2 | 7 | 6 | 3 | 5 | 6 | 1 | 36 | 2 | 2 | 4 | Q | 1 |
| 5 | 3 | 4 | 2 | 3 | 3 | 7 | 6 | 3 | 36 | 2 | 2 | 2 | A | 2 |
| 4 | 5 | 3 | 3 | 1 | 4 | 7 | 6 | 3 | 36 | 2 | 2 | 3 | T | 1 |
| 4 | 2 | 4 | 3 | 4 | 3 | 7 | 6 | 3 | 36 | 2 | 2 | 2 | A | 2 |
| 4 | 5 | 3 | 4 | 1 | 3 | 6 | 6 | 4 | 36 | 2 | 2 | 2 | B | 1 |
| 4 | 5 | 1 | 3 | 3 | 4 | 5 | 6 | 5 | 36 | 2 | 2 | 2 | B | 1 |
| 4 | 2 | 4 | 2 | 3 | 5 | 6 | 5 | 5 | 36 | 2 | 2 | 3 | T | 2 |
| 3 | 4 | 5 | 3 | 1 | 4 | 6 | 5 | 5 | 36 | 2 | 2 | 3 | T | 1 |
| 6 | 5 | 3 | 2 | 1 | 3 | 8 | 6 | 4 | 38 | 2 | 2 | 3 | T | 1 |
| 5 | 4 | 3 | 3 | 4 | 1 | 8 | 6 | 4 | 38 | 2 | 2 | 3 | T | 1 |
| 5 | 4 | 3 | 3 | 2 | 3 | 8 | 6 | 4 | 38 | 2 | 2 | 2 | A | 1 |
| 6 | 5 | 3 | 3 | 2 | 1 | 7 | 7 | 4 | 38 | 2 | 2 | 3 | T | 1 |
| 6 | 4 | 2 | 3 | 3 | 2 | 7 | 7 | 4 | 38 | 2 | 2 | 3 | T | 1 |
| 6 | 7 | 3 | 6 | 6 | 2 | 2 | 3 | 3 | 38 | 8 | 3 | 0 | - | 3 |
| 6 | 7 | 1 | 2 | 5 | 3 | 6 | 4 | 4 | 38 | 2 | 2 | 3 | T | 3 |
| 5 | 1 | 6 | 2 | 4 | 4 | 7 | 5 | 4 | 38 | 2 | 2 | 3 | T | 3 |
| 4 | 3 | 5 | 3 | 2 | 5 | 7 | 5 | 4 | 38 | 2 | 2 | 3 | T | 2 |

TABLE II. (contd.)

| $2a$ | $2b$ | $2c$ | $2d$ | $2e$ | $2f$ | $2g$ | $2h$ | $2i$ | $2\sigma$ | ntr | n6j | str | type | deg |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 5 | 4 | 1 | 2 | 3 | 8 | 7 | 3 | 40 | 2 | 2 | 3 | T | 2 |
| 6 | 4 | 4 | 2 | 3 | 3 | 8 | 7 | 3 | 40 | 2 | 2 | 2 | A | 2 |
| 5 | 6 | 3 | 3 | 1 | 4 | 8 | 7 | 3 | 40 | 2 | 2 | 3 | T | 1 |
| 5 | 3 | 4 | 3 | 4 | 3 | 8 | 7 | 3 | 40 | 2 | 2 | 2 | A | 2 |
| 4 | 5 | 3 | 4 | 2 | 4 | 8 | 7 | 3 | 40 | 2 | 2 | 2 | A | 1 |
| 6 | 2 | 6 | 2 | 2 | 4 | 8 | 4 | 6 | 40 | 2 | 2 | 3 | T | 2 |
| 5 | 5 | 2 | 5 | 1 | 4 | 8 | 6 | 4 | 40 | 2 | 2 | 2 | B | 1 |
| 5 | 1 | 6 | 3 | 3 | 4 | 8 | 4 | 6 | 40 | 2 | 2 | 3 | T | 2 |
| 4 | 5 | 1 | 7 | 7 | 4 | 3 | 6 | 3 | 40 | 2 | 2 | 4 | Q | 1 |
| 6 | 7 | 3 | 1 | 3 | 2 | 7 | 6 | 5 | 40 | 2 | 2 | 3 | T | 2 |
| 5 | 7 | 2 | 2 | 3 | 3 | 7 | 6 | 5 | 40 | 2 | 2 | 3 | T | 2 |
| 5 | 3 | 4 | 2 | 3 | 5 | 7 | 6 | 5 | 40 | 2 | 2 | 3 | T | 2 |
| 4 | 5 | 5 | 3 | 1 | 4 | 7 | 6 | 5 | 40 | 2 | 2 | 3 | T | 1 |
| 4 | 2 | 4 | 3 | 4 | 5 | 7 | 6 | 5 | 40 | 2 | 2 | 2 | A | 2 |
| 6 | 7 | 3 | 1 | 3 | 2 | 7 | 6 | 5 | 40 | 2 | 2 | 3 | T | 2 |
| 6 | 3 | 5 | 3 | 3 | 4 | 7 | 6 | 3 | 40 | 3 | 2 | 1 | - | 3 |
| 2 | 5 | 3 | 7 | 6 | 3 | 5 | 7 | 2 | 40 | 2 | 2 | 3 | T | 1 |
| 5 | 6 | 1 | 6 | 4 | 6 | 1 | 6 | 5 | 40 | 2 | 2 | 4 | Q | 1 |
| 5 | 5 | 2 | 4 | 5 | 1 | 9 | 8 | 3 | 42 | 2 | 2 | 3 | T | 1 |
| 7 | 6 | 3 | 2 | 1 | 3 | 9 | 7 | 4 | 42 | 2 | 2 | 3 | T | 1 |
| 6 | 5 | 3 | 3 | 2 | 3 | 9 | 7 | 4 | 42 | 2 | 2 | 2 | A | 1 |
| 5 | 4 | 3 | 4 | 3 | 3 | 9 | 7 | 4 | 42 | 2 | 2 | 2 | A | 1 |
| 6 | 4 | 4 | 3 | 4 | 1 | 9 | 6 | 5 | 42 | 2 | 2 | 3 | T | 1 |
| 5 | 4 | 3 | 4 | 4 | 2 | 9 | 6 | 5 | 42 | 2 | 2 | 2 | A | 1 |
| 6 | 6 | 4 | 2 | 3 | 1 | 8 | 7 | 5 | 42 | 2 | 2 | 3 | T | 1 |
| 5 | 4 | 3 | 5 | 3 | 2 | 8 | 7 | 5 | 42 | 2 | 2 | 3 | T | 1 |
| 5 | 6 | 3 | 3 | 3 | 2 | 8 | 7 | 5 | 42 | 2 | 2 | 2 | A | 1 |
| 4 | 6 | 2 | 4 | 3 | 3 | 8 | 7 | 5 | 42 | 2 | 2 | 3 | T | 1 |
| 7 | 8 | 1 | 3 | 6 | 3 | 6 | 4 | 4 | 42 | 2 | 2 | 3 | T | 3 |
| 6 | 2 | 6 | 2 | 4 | 4 | 8 | 6 | 4 | 42 | 2 | 2 | 2 | A | 3 |
| 5 | 4 | 5 | 3 | 2 | 5 | 8 | 6 | 4 | 42 | 2 | 2 | 3 | T | 2 |
| 5 | 1 | 6 | 3 | 5 | 4 | 8 | 6 | 4 | 42 | 2 | 2 | 3 | T | 3 |
| 4 | 6 | 2 | 5 | 6 | 7 | 1 | 6 | 5 | 42 | 2 | 2 | 4 | Q | 2 |
| 2 | 6 | 4 | 7 | 3 | 6 | 5 | 7 | 2 | 42 | 2 | 2 | 4 | Q | 1 |
| 8 | 6 | 4 | 1 | 2 | 3 | 9 | 8 | 3 | 44 | 2 | 2 | 3 | T | 2 |
| 7 | 5 | 4 | 2 | 3 | 3 | 9 | 8 | 3 | 44 | 2 | 2 | 2 | A | 2 |
| 6 | 7 | 3 | 3 | 1 | 4 | 9 | 8 | 3 | 44 | 2 | 2 | 3 | T | 1 |
| 6 | 4 | 4 | 3 | 4 | 3 | 9 | 8 | 3 | 44 | 2 | 2 | 2 | A | 2 |
| 5 | 6 | 3 | 4 | 2 | 4 | 9 | 8 | 3 | 44 | 2 | 2 | 2 | A | 1 |
| 5 | 3 | 4 | 4 | 5 | 3 | 9 | 8 | 3 | 44 | 2 | 2 | 2 | A | 2 |
| 8 | 9 | 1 | 4 | 4 | 2 | 6 | 7 | 3 | 44 | 2 | 2 | 2 | B | 1 |
| 8 | 9 | 1 | 5 | 5 | 2 | 5 | 6 | 3 | 44 | 2 | 2 | 2 | B | 1 |
| 2 | 9 | 7 | 4 | 6 | 6 | 2 | 5 | 3 | 44 | 2 | 2 | 3 | T | 4 |
| 6 | 2 | 6 | 3 | 3 | 4 | 9 | 5 | 6 | 44 | 2 | 2 | 2 | A | 2 |
| 5 | 1 | 6 | 4 | 4 | 4 | 9 | 5 | 6 | 44 | 2 | 2 | 3 | T | 2 |
| 5 | 6 | 3 | 5 | 2 | 3 | 8 | 8 | 4 | 44 | 2 | 2 | 2 | B | 1 |
| 6 | 7 | 1 | 4 | 2 | 4 | 8 | 7 | 5 | 44 | 2 | 2 | 2 | B | 1 |
| 4 | 6 | 4 | 5 | 2 | 3 | 7 | 8 | 5 | 44 | 2 | 2 | 2 | B | 1 |
| 6 | 4 | 4 | 2 | 3 | 5 | 8 | 7 | 5 | 44 | 2 | 2 | 3 | T | 2 |
| 5 | 6 | 5 | 3 | 1 | 4 | 8 | 7 | 5 | 44 | 2 | 2 | 3 | T | 1 |
| 5 | 2 | 5 | 3 | 3 | 6 | 8 | 5 | 7 | 44 | 2 | 2 | 3 | T | 2 |
| 5 | 3 | 4 | 3 | 4 | 5 | 8 | 7 | 5 | 44 | 2 | 2 | 2 | A | 2 |
| 4 | 4 | 6 | 4 | 1 | 5 | 8 | 5 | 7 | 44 | 2 | 2 | 3 | T | 1 |
| 4 | 5 | 5 | 4 | 2 | 4 | 8 | 7 | 5 | 44 | 2 | 2 | 2 | A | 1 |
| 6 | 5 | 3 | 6 | 2 | 4 | 8 | 7 | 3 | 44 | 2 | 2 | 2 | B | 2 |
| 7 | 8 | 1 | 7 | 6 | 3 | 4 | 6 | 2 | 44 | 2 | 2 | 3 | T | 1 |
| 1 | 8 | 7 | 5 | 5 | 8 | 4 | 5 | 1 | 44 | 2 | 2 | 4 | Q | 4 |
| 4 | 4 | 6 | 4 | 1 | 5 | 8 | 5 | 7 | 44 | 2 | 2 | 3 | T | 2 |
| 6 | 7 | 3 | 2 | 3 | 3 | 8 | 6 | 6 | 44 | 2 | 2 | 2 | A | 2 |
| 6 | 8 | 2 | 4 | 6 | 4 | 4 | 6 | 4 | 44 | 3 | 2 | 1 | - | 1 |
| 5 | 6 | 1 | 5 | 2 | 5 | 8 | 6 | 6 | 44 | 2 | 2 | 2 | B | 2 |
| 5 | 7 | 2 | 3 | 3 | 4 | 8 | 6 | 6 | 44 | 2 | 2 | 3 | T | 1 |
| 3 | 6 | 3 | 8 | 5 | 5 | 5 | 7 | 2 | 44 | 2 | 2 | 4 | Q | 2 |
| 5 | 7 | 4 | 1 | 4 | 3 | 6 | 7 | 7 | 44 | 2 | 2 | 3 | T | 2 |

TABLE II. (contd.)

| $2a$ | $2b$ | $2c$ | $2d$ | $2e$ | $2f$ | $2g$ | $2h$ | $2i$ | $2\sigma$ | ntr | n6j | str | type | deg |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 7 | 3 | 2 | 4 | 4 | 6 | 7 | 7 | 44 | 2 | 2 | 3 | T | 2 |
| 4 | 6 | 2 | 3 | 7 | 6 | 3 | 7 | 6 | 44 | 3 | 2 | 1 | - | 1 |
| 4 | 6 | 2 | 4 | 3 | 5 | 6 | 7 | 7 | 44 | 2 | 2 | 2 | B | 4 |
| 7 | 3 | 6 | 4 | 3 | 5 | 7 | 6 | 3 | 44 | 3 | 2 | 1 | - | 4 |
| 7 | 3 | 6 | 4 | 3 | 5 | 7 | 6 | 3 | 44 | 3 | 2 | 1 | - | 4 |

Note: Of the four types, the types A and B specified in the next to the last column of this table correspond to the types a and b of Sharp (1967) and they are:

$$\text{Type A:}\left\{\begin{matrix} a & b & c \\ d & e & f \\ a+d & b+e & i \end{matrix}\right\} \qquad \text{Type B:}\left\{\begin{matrix} a & b & c \\ d & b+h & f \\ a+d & h & i \end{matrix}\right\}$$

$$\text{Type T:}\left\{\begin{matrix} a & b & c \\ d & e & d+e \\ a+d & b+e & i \end{matrix}\right\} \qquad \text{Type Q:}\left\{\begin{matrix} a & b & a+b \\ d & e & d+e \\ a+d & b+e & i \end{matrix}\right\}$$