## On the problem of the neutron mean square intrinsic charge radius

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ABSTRACT. The value of the neutron mean square intrinsic charge radius (MSICR)  $\langle r_{\rm in}^2 \rangle_N$  is discussed. The experimental data table of the n-e scattering length values  $a_{ne}$  is presented. The experiments can be divided into two groups:  $\langle r_{\rm in}^2 \rangle_N > 0$  and  $\langle r_{\rm in}^2 \rangle_N < 0$ . A possible reason for the discrepancy between the results of the Garching (Germany) and Dubna (Russia) determination of  $a_{ne}$  by the transmission method is discussed. It is shown that introduction into  $\sigma_{\rm tot}$  of energyindependent interresonance interference terms does not affect the result obtained in Dubna. The results of  $\langle r_{\rm in}^2 \rangle_N > 0$  are in contradiction with modern theory and the results of  $\langle r_{\rm in}^2 \rangle_N < 0$  are in confirmation of modern theory.

RESUMEN. Se discute el valor del radio de carga cuadrático medio intrínseco del neutrón (MSICR)  $\langle r_{\rm in}^2 \rangle_N$ . Se presentan datos experimentales de las longitudes de dispersión neutrón-electrón,  $a_{ne}$ . Los experimentos pueden dividirse en dos grupos:  $\langle r_{\rm in}^2 \rangle_N > 0$  y  $\langle r_{\rm in}^2 \rangle_N < 0$ . Se discute una razón posible para explicar la discrepancia entre los resultados obtenidos por el método de transmisión para  $a_{ne}$  de los grupos de investigación de Garching (Alemania) y Dubna (Rusia). Se muestra que el resultado de Dubna no se ve afectado al introducir, en  $\sigma_{\rm tot}$ , términos de interferencia interresonantes independientes de la energía. Los resultados  $\langle r_{\rm in}^2 \rangle_N > 0$  ( $\langle r_{\rm in}^2 \rangle_N < 0$ ) contradicen (confirman) teorías modernas de la estructura intrínseca del neutrón.

PACS: 14.20.Dh; 13.40.Fn; 25.40.Dn

#### 1. INTRODUCTION

In recent years the issue concerning the actual value of the mean square intrinsic charge radius (MSICR) ( $\langle r_{in}^2 \rangle_N$ ) related to the internal structure of the neutron have been widely discussed [1–10]. What is the history of the problem, and, ultimately, what is  $\langle r_{in}^2 \rangle_N$  equal to?

It is well known that in the limiting case of low energies the relation between the mean square charge neutron radius and neutron charge form factor  $G_E(q^2)$  is expressed by equation

$$\langle r_E^2 \rangle_N = 6 \left( \frac{dG_E}{dq^2} \right)_{q^2 = 0},\tag{1}$$

where  $q^2$  is a squared four-momentum transfer, or because the charge form factor is

$$G_E(q^2) = F_1(q^2) + \frac{q^2\hbar^2}{4M^2c^2}\mu_n F_2(q^2),$$
(2)

where  $F_1(q^2)$  is the Dirac form factor describing the spatial distribution of a nuclear charge and associated with the Dirac magnetic moment,  $F_2(q^2)$  is the Pauli form factor associated with the spatial distribution of an anomalous magnetic moment,  $\mu_n$  is the neutron magnetic moment in nuclear magneton, it can be expressed by

$$\langle r_E^2 \rangle_N = 6 \left( \frac{dF_1}{dq^2} \right)_{q^2 = 0} + \frac{3}{2} \frac{\mu_n \hbar^2}{M^2 c^2}.$$
 (3)

The first term in Eq. (3) arises from the nuclear internal structure and it is directly connected with the behaviour of the Dirac form factor  $F_1$  as a function of  $q^2$ . If  $\langle r_{\rm in}^2 \rangle_N$  is the neutron MSICR connected with the neutron internal structure, then

$$\langle r_{\rm in}^2 \rangle_N = 6 \left( \frac{dF_1}{dq^2} \right)_{q^2 = 0} \tag{4}$$

As for the second term in Eq. (3), it is of a magnetic nature associated with the "trembling" or "dancing" (*zitterbewegung*) of the neutron which satisfies the Dirac equation and has an anomalous magnetic moment.

Since the neutron is the Dirac particle one should expect analogous effects for it. Thus if the neutron has an electromagnetic structure, the apparent extent of the charge will arise from the inner extent and additional "smash" associated with the "trembling". In order to derive information concerning the structure of the neutron from the experimental data from the n-e interaction, the contribution of the trembling effect should be determined.

More than 40 years ago Feshbach demonstrated [11] that the scattering of electrons at energies of the order of magnitude of several tens of MeV ( $qR \ll 1$ , where  $q = 2k \sin \theta/2$  is the recoil wave number) makes possible only the measurement of a sole parameter providing information on the size of the nucleus, namely of the MSICR determined by the expression

$$\langle r_{\rm in}^2 \rangle = \int \rho(\vec{r}) r^2 \, d^3 \vec{r}. \tag{5}$$

At about the same time Foldy (see review of Ref. [12]) found the relation between  $\langle r_{in}^2 \rangle_N$  and  $a_{ne}$ , the measurable scattering length of a slow neutron on an electron (the so-called n-e interaction):

$$\langle r_{\rm in}^2 \rangle_N = 6 \left( \frac{dF_1}{dq^2} \right)_{q^2 = 0} = \frac{3\hbar^2}{Me^2} (a_{ne} - a_{\rm F}),$$
 (6)

where  $a_{\rm F} = \mu_n e^2/(2Mc^2) = -1.468 \times 10^{-3}$  fm is the Foldy scattering length related to a free neutron satisfying the Dirac equation and exhibiting an anomalous magnetic moment. The Foldy effect depends on a combination of known constants, and to determine  $\langle r_{\rm in}^2 \rangle_N$  it must be subtracted from the quantity  $a_{ne}$ .

It should be pointed out that in principle the information on the MSICR of the neutron can be obtained from the experiments on the scattering of high-energy electrons (of a few hundred MeV or more) on protons and deuterons providing the information on the form factor  $G_E(q^2)$ . The arising uncertainty level, however, are fairly high in such experiments and the study of the low-energy neutron scattering is still the only direct source of information on the MSICR of the neutron [9].

Besides the Foldy effect, however, there may exist a more interesting kind of interaction between the neutron and the electron [13]. This interaction [the first term in Eq. (3)] is a consequence of the meson theory of nuclear forces. The neutron is surrounded by a "meson cloud" ("fir coat") which has a size of the order of magnitude of  $\hbar/(m_{\pi}c)$ , so in the immediate vicinity of the neutron the presence of an electric field may be expected. If a neutron and an electron come sufficiently close to each other, electrostatic interaction forces are to arise between them, and these forces should be short-ranged. Such an interaction will influence the quantities  $a_{ne}$  and consequently,  $\langle r_{in}^2 \rangle_N$ .

Since  $a_{ne}$  and  $a_{\rm F}$  are both of the same order of magnitude, the determination of  $\langle r_{\rm in}^2 \rangle_N$  will require very precise measurements. Such measurements can be performed within the framework of studies of the interaction of low-energy neutrons with heavy atoms.

As it has been established, the experimentally observed n-e interaction is mainly due to Foldy effect. Moreover it has not been clear for a long time how essential the role is of the internal interaction considered by Fermi between the neutron and electron, and how strong it is.

The MSICR is a fundamental characteristic of the neutron, and its measurements permit verification of modern theoretical ideas concerning nucleons (for instance, of the quark-bag model, Skyrme model, Numbu-Jona-Lasinio model and others).

### 2. Theoretical analysis of N-E interaction manifestations

The amplitude of the Dirac particle scattering by weak, slow-changing pure electrical potential  $\phi(r)$  was obtained by Foldy [12] from the generalized Dirac equation

$$\gamma_{\mu}\frac{\partial\Psi}{\partial x_{\mu}} + \frac{Mc}{\hbar}\Psi - \frac{1}{\hbar c}\sum_{m=0}^{\infty} \left[\epsilon_{m}\gamma_{\mu}\Box^{m}A_{\mu} + \frac{1}{2}\mu_{m}\gamma_{\mu}\gamma_{\nu}\Box^{m}\left(\frac{\partial A_{\mu}}{\partial x_{\nu}} - \frac{\partial A_{\nu}}{\partial x_{\mu}}\right)\right] = 0, \quad (7)$$

where the electromagnetic field is described by a four-dimensional vector potential  $A_{\mu}(x) \equiv (A(r,t); i\phi(r,t)), x \equiv (r, it), \gamma_{\mu,\nu}$  is the Dirac matrix,  $\Box = \Delta - 1/c^2 \partial^2/\partial t^2$  is the D'Alembert operator, and the coefficients  $\epsilon_m$  and  $\mu_m$  characterize the inner electromagnetic structure of the nucleon. In particular  $\epsilon_0$  is the total charge of the Dirac particle and  $\mu_0$  is the anomalous magnetic moment for the Dirac particle. Other terms (m = 1, 2, 3, ...) describe higher radial moment in the distribution of the electric charge of the particle and the current. The coefficient  $\epsilon_1$  is thus connected with the second radial moment of the charge distribution or with the MSICR of the neutron:

$$\epsilon_1 = \frac{e}{6} \langle r_{\rm in}^2 \rangle_N = \frac{1}{6} \int r^2 \rho(\vec{r}) \, d^3 \vec{r}. \tag{8}$$

At m = 0 Eq. (7) is reduced to the usual Dirac equation with electromagnetic potentials, the last two terms of which have the form

$$-\frac{\mu e\hbar}{2Mc}\vec{\sigma}\vec{H} - \frac{i\mu e\hbar}{2Mc}\vec{\alpha}\vec{E}.$$
(9)

The term that contains H is the interaction energy of the magnetic dipole of the neutron,  $\mu e\hbar/(2Mc)$ , and magnetic field, H, which causes the magnetic interaction. The second term is the Foldy interaction which is due to the trembling of the Dirac particle with the magnetic moment  $\mu e\hbar/(2Mc)$ . In the case of low momentum transfer  $\hbar \vec{k}$  only the terms with m = 0 and 1 are important and in the first Born approximation the scattering amplitudes from Eq. (7) has the form

$$f_0(\vec{k}) = -\frac{M\epsilon_0}{2\pi\hbar^2} \int \exp(-i\vec{k}\vec{r})\phi(\vec{r})\,d\vec{r},\tag{10}$$

$$f_1(\vec{k}) = -\frac{M}{2\pi\hbar^2} \left[ \epsilon_1 + \frac{\hbar}{2Mc} \mu_0 + \frac{1}{2} \left( \frac{\hbar}{2Mc} \right)^2 \epsilon_0 \right] \exp(-i\vec{k}\vec{r}) \nabla^2 \phi(\vec{r}) \, d\vec{r}. \tag{11}$$

For the neutron  $\epsilon_0 = 0$ ,  $\mu_0 = \mu_n e\hbar/(2Mc)$ , and at  $k \to 0$  one obtains the n-e scattering length

$$a_{ne} = \frac{2Me}{\hbar^2} \left( \epsilon_1 + \frac{e\hbar^2}{4M^2c^2} \,\mu_n \right). \tag{12}$$

In this relation  $\epsilon_1$  describes the radial extent of the charge distribution in the neutron. The term with  $\mu_n$  represents the Foldy contribution due to the trembling of the particle with an anomalous magnetic moment  $\mu_n$ .

Using Eq. (8) expression (12) can be rewritten in the form (6). Note that the sign of  $\langle r_{in}^2 \rangle_N$  for an overall neutral object could be both positive and negative; it depends mainly on the sign of a peripheral charge.

Taking into account Eq. (8) and comparing Eqs. (1), (3) with (12) we find that

$$\left(\frac{dG_E}{dq^2}\right)_{q^2=0} = 14.41 \, a_{ne},\tag{13}$$

where  $a_{ne}$  is given in fm. Thus the study of the n-e scattering allows one to obtain the information on  $(dG_E/dq^2)_{q^2=0}$ .

To conclude, we may say that two effects contribute to the experimentally investigated n-e interaction: one of them which is due to the Foldy scattering can be calculated, while the other being of great importance and caused by the neutron inner structure has to be estimated experimentally.

#### 3. EXPERIMENTAL METHODS TO STUDY THE N-E INTERACTION

In the interaction of very slow neutrons with atoms when the process can be considered to be pure elastic, the total n-e scattering length may be written in the form

$$a(\theta) = a_{ne} Z f(\sin \theta / \lambda), \tag{14}$$

where  $a_{ne}$  is the n-e scattering length,  $f(\sin \theta / \lambda)$  is the atomic form factor.

Precise measurements of the n-e interaction were performed by the middle of the 40s and also in succeeding years. Those attempts were either based on asymmetry observations in the scattering of thermal neutrons or on studies of the energy dependence of the total cross section in the electronvolt region.

The differential cross section for the coherent scattering of slow neutrons with the wavelength of the order of the size of an atom is described by the relation

$$\sigma(\theta) = |a + a_{\rm F} + Zf(\sin\theta/\lambda) a_{ne}|^2, \tag{15}$$

where a is the coherent nuclear scattering length ( $\simeq 10$  fm) and

$$a_{\rm F} = \frac{1}{2} \frac{Z\mu_n e^2}{Mc^2} = Z \times 1.468 \times 10^{-3} \,\,{\rm fm},$$
 (16)

where  $a_{\rm F}$  is the Foldy term determined in this case by the relativistic effect produced by the interaction of the anomalous magnetic moment of the neutron with the electric field of the nucleus with the charge Ze.

Estimations show that the ratio  $Za_{ne}f(\sin\theta/\lambda)/(a+a_{\rm F})$  may amount to approximately 1% for heavy nuclei and therefore can be measured. For the neutron total cross section, using the generally accepted S-matrix of scattering,

$$S_{nn} = \left(1 - i \sum \frac{\Gamma_n}{\Delta E + \frac{i}{2}\Gamma}\right) \exp(2i\delta_{\text{pot}}),\tag{17}$$

which does not take into account the small interresonance interference, and using the optical theorem one can obtain for the case of the nuclear s-scattering:

$$\frac{\sigma_{\text{tot}}}{4\pi} = \frac{\text{Im}\,f(0)}{k} = \frac{1}{k^2}\sin\delta_0\sin(\delta_0 + 2\eta_0) - \frac{1}{2k}\sin(2\delta_0 + 2\eta_0)\Sigma_1 + \frac{1}{4}\Sigma_2 \times \cos(2\delta_0 + 2\eta_0) + \frac{1}{4}\Sigma_3 \times \cos(2\delta_0 + 2\eta_0),$$
(18)

where  $\delta_0$  is the phase shift of nuclear *s*-scattering,  $\eta_0 = -ka_{ne}F$  is the phase shift of n-e scattering,  $F = \frac{1}{2} \int_0^{\pi} f(\sin \theta / \lambda) \sin \theta \, d\theta$  is the angular integrated atomic form factor,

$$\Sigma_1 = \sum_j \frac{g_j \Gamma_{nj} \Delta E_j}{k(\Delta E_j^2 + \Gamma_j^2/4)},$$

$$\Sigma_2 = \sum_j \frac{g_j \Gamma_{nj}^2}{k^2 (\Delta E_j^2 + \Gamma_j^2/4)},$$
  

$$\Sigma_3 = \sum_j \frac{g_j \Gamma_{nj} \Gamma_{\gamma j}}{k^2 (\Delta E_j^2 + \Gamma_j^2/4)}.$$
(19)

The additional phase shift  $\eta_0$  was calculated using the first Born approximation. The calculations performed using more accurate methods than the Born approximation method have shown that the Born approximation fits our energy region adequately.

There are two old methods of measuring the n-e interaction. One of them, originally used by Fermi and Marshall in 1947 [14] depends upon the fact that in the scattering of slow neutrons an asymmetric angular distribution due to  $f(\sin \theta/\lambda)$  is observed.

The main disadvantage of experiments of this kind is the necessity to correct for the effect of the thermal motion of atoms in a gas. The main contribution to the correction is made by the neutrons of large wavelengths in the very region where deviations from the Maxwellian distribution are expected. In the most precise experiments the correction was determined experimentally by performing measurements in argon and neon with the insignificant n-e scattering.

Precise measurements following the Foldy and Marshall method were performed at the Argonne National Laboratory by Krohn and Ringo in 1965–72 [15]. The noble gases: xenon, krypton, and argon were used. Measurements in neon were conducted to check the calculated value for the asymmetry due to thermal atomic motion. The measured value for the correction exceeded the sought-for effect for the xenon by four times and for the krypton by 10 times.

Measures were taken to remove admixtures, especially light ones, because even in small amounts they may greatly distort the result of the experiment.

As a result it was obtained that

$$a_{ne} = (-1.33 \pm 0.03) \times 10^{-3} \text{ fm.}$$
 (20)

In Ref. [9] the possibility of errors was noticed, now present in Ref. [15]. The reasons for them to arise are mainly the following:

- 1. Very weak asymmetry of the neutron scattering on noble gases in comparison with the strong symmetry of neutron-nuclear interactions (so in Ref. [15] 0.5 per cent of the asymmetry effect of the n-e interaction is measured with the error of  $\pm 2.5\%$ ).
- 2. Since the effect under measurement is so weak, experimentators must be absolutely sure that no side effects affect it (*e.g.*, caused by p-resonances, admixtures of light gases, etc.).
- 3. Large values of corrections introduced in the experiment. So the neutron energydependent correction for the scattering asymmetry caused by gas thermal motion exceeds the measured effect for xenon by a factor of 4, for krypton by a factor of 10, etc.

The second method of studying the n-e interaction was used by Havens *et al.* [16]. It consists in observing the dependence of the total scattering cross section on the neutron wavelength near 0.1 nm. The nuclear scattering must remain constant, while the form factor  $f(\sin \theta/\lambda)$  is the cause for the change in the total scattering cross section with  $\lambda$ . In Ref. [16] molten lead and bismuth were used as scattering materials. The total cross section was measured at  $\lambda = 0.03-0.13$  nm.

The most exact value for  $a_{ne}$  obtained by this method is [17]

$$a_{ne} = (-1.56 \pm 0.05) \times 10^{-3} \text{ fm.}$$
 (21)

The error is statistical. The correction for the Schwinger scattering as well as that for the contribution for the resonance scattering was not included.

# 4. The current situation in the study of neutron MSICR. Two groups of experimental data

In order to study the n-e scattering and the polarizability of the neutron, in 1976-86 Koester *et al.* [5] (Garching, Germany) carried out very precise measurements of the neutron coherent scattering length using a gravitational neutron refractometer by the method of reflection of neutrons from bismuth and natural lead mirrors. This interesting apparatus was proposed by Maier-Leibnitz and was built at the FRM reactor in Garching by Koester [18].

The basic equation for the measurements of coherent scattering lengths using the neutron gravity refractometer is

$$b_{\rm coh} = \frac{gm^2 h_{\gamma}}{2\pi N\hbar^2},\tag{22}$$

where N is atomic density (atoms per cm<sup>3</sup>),  $b_{\rm coh}$  is the neutron coherent scattering length,  $h_{\gamma}$  is a height of falling of the neutron.

All quantities in Eq. (22) either are well-known fundamental constants or can be precisely measured. Thus it allows the high-accuracy determination of  $b_{\rm coh}$  which is virtually limited by the experimental errors of the measurements of  $h_{\gamma}$  and N only.

For liquid bismuth and liquid natural lead it was obtained

$$b_{\rm Bi} = 8.5307\,(25) \,\,{\rm fm},$$
 (23)

$$b_{\rm Pb} = 9.4017\,(20)$$
 fm. (24)

The obtained results were compared by Koester *et al.* with the data from measurements of cross-sections for bismuth and natural lead at neutron energies above several electronvolts [5]. The total cross-sections were measured by transmission through melted lead and bismuth at neutron energies corresponding to the resonances of rhodium (1.26 eV), silver (5.19 eV), tungsten (18.8 eV) and cobalt (132 eV). The measurements were carried out with a continuously operating resonance detector consisted of rotating discs made from

resonance-absorbing foils. The upper sections of the discs were activated in the neutron beam, while the activity of the diametrically opposite sections was recorded with a  $\beta$  detector. This arrangement ensured a high statistical accuracy for the measurements. The combination of two identically rotating foils was used. The first foil in the beam counts the sum of resonance and nonresonace activation whereas the second one only spoils the nonresonance activation. Thus the difference of the counting rates is proportional to the neutron current of the resonance energy.

These measurements were repeated later at two energies: 1.97 keV and 143 keV. The neutrons of 1.97 keV were obtained with the help of filters using the method of the double-resonance scattering. The foil from the  $^{63}$ Cu isotope serves as a resonance scatterer near the reactor core. The neutron emerges with an average energy of 1.97 keV at the angle of  $\pi/4$  through the beam tube. After the flight path of about 7 m the beam is scattered again by the  $^{80}$ Se target at the resonance energy of 1.97 keV. Initially scattered neutrons with other energies are suppressed by the filter combination of Sc, B<sub>4</sub>C and Co in the beam line.

Cross section measurements at a median energy of 143 keV were performed in the silicon filtered fission neutron beam of the converter facility [19] at FRM.

The obtained results should be corrected for the elastic incoherent scattering, the Schwinger scattering and the solid-state effect in order to account for the effects connected with the state of a sample under measurement and the scattering energy dependence caused by resonances.

The total neutron cross section  $\sigma_{tot}$  may be written according to Ref. [5] as

$$\sigma_{\text{tot}} = 4\pi |\text{Re}\,b(E) + \text{Im}\,b(E)|^2 + \sigma_{\text{in}}(E) + \sigma_{\text{Sch}}(E) + \sigma_l(E) + \sigma_{\text{sol}}(E), \tag{25}$$

where  $\sigma_{in}$  is the nuclear incoherent,  $\sigma_{Sch}$  is the spin-orbital Schwinger scattering,  $\sigma_{sol}$  is the solid state and  $\sigma_l$  is the angular momentum interaction l > 0 cross sections.

The real part represents the coherent scattering amplitude:

$$\operatorname{Re} b(E) = -R'e(E) + b_{\mathrm{R}}(E) + b_{ne}Z[\bar{f}(E) - \bar{h}(E)] + b_{\mathrm{p}}\bar{g}(E), \qquad (26)$$

where R' is the nuclear potential radius,  $e(k) = 1 - (kR')^2/6 + (kR')^4/120 - \cdots$ ,  $b_{ne} = -a_{ne}A/(A+1)$ ,  $\bar{f}(E)$  is the angular averaged atomic form factor,  $\bar{h}(E) = 1 - (kR_N)^2/5 + 2(kR_N)^4/135 + \cdots$ ,  $b_p\bar{g}(E)$  is the neutron electric polarizability scattering amplitude,  $\bar{g}(E) = 1 - \pi (kR_N)/3 + (kR_N)^2/3 - \cdots$ ,  $R_N = 0.12027 A^{1/3} 10^{-12}$  cm is the charge radius of nuclei,  $b_R(E)$  is the amplitude of contribution of all resonances. The authors of Ref. [5] believe that the Im b(E) yields only an absorption cross section. It is not quite correct.

The obtained  $\sigma_{tot}$  should be corrected for the scattering energy dependence of  $b_{\rm R}$  caused by resonances which may be calculated. For Bi resonance data are available only up to about 260 keV, for the isotopes of Pb, up to 1–2 MeV, some bound level parameters (at negative energies) are also given [20]. In order to reduce the uncertainty caused by the lack of information on other bound levels and on data for the high energy region the authors of Ref. [5] calculated the resonance scattering term  $b_{\rm R} \sim \sum_j g_j \Gamma_{nj} \Delta E_j / (k(\Delta E_j^2 + \Gamma_j^2/4))$ using the information on known levels and changing  $\Gamma_{nj}^0$  by  $S_0 \langle D_0 \rangle$  and  $E_j$  by  $x \langle D_0 \rangle$ , where  $S_0$  is the strength function,  $\langle D_0 \rangle$  is the mean level distance, x is the integer number. This part of the processing procedure does not seem to be sufficiently correct. We will discuss this question a little bit later. As a result the following value for the n-e scattering length was obtained in Ref. [5] for natural lead and bismuth:

$$a_{ne} = (-1.32 \pm 0.04) \times 10^{-3} \text{ fm.}$$
 (27)

However, a processing procedure which is not exactly correct casts some doubt upon this value.

In this situation it would be very useful (as it was noted in Ref. [7]) to measure the neutron transmission for the double-magic <sup>208</sup>Pb isotope which has very rare resonances. The <sup>208</sup>Pb isotope provides by far the best properties for a heavy isotope to separate the potential scattering from the resonance scattering contribution. It has a negligible thermal absorption cross section  $\sigma_{\gamma} = 0.48(3) \times 10^{-3} \times 10^{-24}$  cm<sup>2</sup>. In <sup>208</sup>Pb there are only *p*-wave and *d*-wave resonances below 500 keV. Preliminary results of the previous <sup>208</sup>Pb measurements are published in Ref. [21].

Let us consider the work of the Dubna group [7]. Precise measurements of the total neutron cross section of bismuth in the electronvolt energy region were carried out at the IBR-30 pulsed reactor in JINR. They covered the energy region from 1 to 90 eV and were performed by the time-of-flight method over the flight path of 60 m using both a liquid sample and a solid sample 18 mm thick. The background measured with the help of rhodium, silver, and tungsten plates (resonance energies of 1.26, 5.19, and 18.93 eV, respectively) placed in the beam, was 0.3-0.4% at 1-6 eV, and no more than 1.5% at about 20 eV. The energy dependence of the total cross section for the interaction between neutrons and bismuth is shown in Fig. 1. The same figure shows the values for  $\sigma_{tot}$  measured at Garching [5].

To obtain information on the n-e scattering length the corrections for the Schwinger scattering and solid state effects were introduced into  $\sigma_{tot}$ ; they did not exceed 0.8%. The data were processed using the following expression:

$$y = \frac{\sigma_{\text{tot}}(E')}{4\pi} - a_{\text{coh}}^2(E) = a^2 (Z^2 - 2ZF') - 2aa_{\text{coh}}(E)(Z - F') + (\Sigma_1 - \Sigma_1') [a_{\text{coh}}(E) - a(Z - F')] + \frac{1}{4} (\Sigma_1)^2 - \frac{1}{2} \Sigma_1 \Sigma_1' + \frac{1}{4} \Sigma_2' + \frac{\sigma_\gamma(E')}{4\pi},$$
(28)

where  $a_{\rm coh}(E) = -b_{\rm coh}(E)A/(A+1)$ ,  $a = -a_{ne}$ ,  $\Sigma_1$  and  $\Sigma_2$  are expressed using (19); the electric polarizability of the neutron is taken equal to zero. The numerical value for  $b_{\rm coh} = 8.5307(20)$  fm is taken the same as in Ref. [5].

In the energy range  $E \ll E_i$  and  $\Gamma_i \ll \Delta E_i$  for the term  $\Sigma_1 - \Sigma_1'$  containing resonances one can use the following expansion into E'/E series:

$$p_1 = \Sigma_1 - \Sigma_1' = E' \sum_i \frac{g_i \Gamma'_{ni}}{k'_i E_i^2} + (E')^2 \sum_i \frac{g_i \Gamma'_{ni}}{k'_i E_i^3} + \dots = \frac{E' k' \sigma_\gamma(E')}{\pi \langle \Gamma_\gamma \rangle},$$
(29)

$$p_2 = \frac{1}{4} (\Sigma_1)^2 - \frac{1}{2} \Sigma_1 \Sigma_1' + \frac{1}{4} \Sigma_2' = \frac{1}{4} \sum_i \frac{g_i \Gamma_{ni}^2}{k_i^2 E_i^2} - \frac{1}{4} \left( \sum_i \frac{g_i \Gamma_{ni}}{k_i E_i} \right)^2.$$
(30)



FIGURE 1. Dependence of  $\sigma_{tot}$  of Bi on the neutron energy  $E: \bullet$ , Ref. [7];  $\circ$ , Ref. [5]. Curves 1 and 2 are calculated for two groups of parameters: 1,  $a_{ne} = -1.6 \times 10^{-3}$  fm,  $\alpha_n = -4.5 \times 10^{-3}$  fm<sup>3</sup>; 2,  $a_{ne} = -1.6 \times 10^{-3}$  fm,  $\alpha_n = 7 \times 10^{-3}$  fm<sup>3</sup>.

Introducing the numerical values for  $\sigma_{\gamma}$  and  $\langle \Gamma_{\gamma} \rangle$  into Eq. (29) one obtains

$$p_1 = \Sigma_1 - \Sigma_1' = 0.6 \times 10^{-4} \times 10^{-12} E' \text{ cm.}$$
(31)

The estimates show that the contribution of  $p_2$  into y is 10–15%, but the lack of information on resonance levels with negative energies does not allow one to find its exact value. Therefore, this contribution was changed to fit experimental data best and appeared to be equal to  $-0.0023 \times 10^{-24} \text{ cm}^2/\text{sr.}$ 

Experimental data were processed by the least square method. The results are summarized below:

Garching data: 
$$a_{ne} = (-1.57 \pm 0.10) \times 10^{-3}$$
 fm;  
Dubna data:  $a_{ne} = (-1.55 \pm 0.11) \times 10^{-3}$  fm. (32)

The obtained data are in best agreement with the results of the neutron diffraction measurements carried out with a tungsten single crystal [22, 23].

It seems attractive to find a method with a more significant effect under measurement. The most promising direction in the study of the n-e interaction is the investigation of thermal neutron diffraction from single crystals of tungsten which was proposed and developed in Dubna [22-25].

The tungsten isotope, <sup>186</sup>W, is well suited since its neutron scattering length in the thermal energy range is small and negative because of the interference between resonance and potential scattering [24,25]. The coherent scattering length of neutrons from a mixture of tungsten isotopes enriched with <sup>186</sup>W is determined from

$$b_{\rm coh} = R - \frac{\beta \Gamma_n}{2k_0 E_0} \left( 1 + \frac{E}{E_0} \right) + a_{ne} Z f(\sin \theta / \lambda) = a + a_{ne} Z f(\sin \theta / \lambda), \tag{33}$$

where  $\Gamma_n$  is the neutron width of the first resonance of <sup>186</sup>W,  $E_0$  is the neutron energy corresponding to the first resonance of <sup>186</sup>W,  $k_0 = 2\pi/\lambda_0$  is the wave number, and  $\beta$  is the <sup>186</sup>W content in the mixture. Precise measurements of the neutron scattering length using a mixture of tungsten isotopes containing 90.7% of <sup>186</sup>W were performed by the Christiansen filter method on a beam of cold neutrons ( $\langle \lambda \rangle \simeq 1.5$  nm) in Garching and yielded  $b_{\rm coh} = (-0.466 \pm 0.006)$  fm [26], the absolute value of which was an order of magnitude smaller than the corresponding value of  $b_{\rm coh}$  for a natural mixture of isotopes, and it also had the opposite sign. In the diffraction experiments two single-crystal balls made of two different isotopic mixtures, being 5 mm in diameter each, were employed. One mixture contained 90.7% of <sup>186</sup>W ( $b_{\rm coh} = -0.466$  fm), the other ( $b_{\rm coh} = +0.267$  fm) was prepared from the first one by adding 14% of natural tungsten. The experiments were mainly staged at the IBR-30 pulsed reactor and at stationary reactors. At a given wavelength the integral intensities  $I_{(hkl)}$  of eight reflections were measured: (110), (200), (220), (310), (400), (330), (420), (510).

Since tungsten is of paramagnetic nature, the magnetic scattering must not contribute to the Bragg reflection and the integral intensity of the diffraction peak corresponding to an (hkl) reflection is determined from

$$I_{(hkl)} = C \Big[ [a + Z f_{(hkl)} (\sin \theta / \lambda) a_{ne}]^2 + [1 - f_{(hkl)} (\sin \theta / \lambda)]^2 \gamma^2 \cot^2 \theta \Big] A_{(hkl)} \\ \times \frac{\exp[-2B(\sin \theta / \lambda)^2]}{\sin 2\theta},$$
(34)

where C is a constant coefficient and  $A_{(hkl)}$  is the factor taking absorption in the crystal into account. The second term in this equation describes the Schwinger scattering,  $\gamma = 1/2\mu_n Z e^2/(Mc^2)$ . Equation (34) shows that the quantity

$$\left(\frac{I_{(hkl)}\sin 2\theta \exp[2B(\sin\theta/\lambda)^2]}{A_{(hkl)}C} - \gamma^2 \cot^2\theta [1 - f_{(hkl)}(\sin\theta/\lambda)]^2\right)^{1/2} = a + Zf_{(hkl)}a_{ne}$$
$$= b_{\rm coh} \tag{35}$$

is to be a linear function of  $Zf_{(hkl)}$  with a slope determined by  $a_{ne}$ . Further all the experiments performed at various installations have shown that it appears impossible to describe the results obtained for these two mixtures by a linear function of  $Zf_{(hkl)}$  at one and the same value for  $a_{ne}$ . As no simple cause for the deviation of experimental results from Eq. (35) was found, Alexandrov and Ignatovich [27] advanced the hypothesis that additional scattering contributes to the diffraction peaks. The additional scattering is caused by the scattering of neutrons on the domains of ordered magnetic moments which exist in the investigated tungsten sample. Later on this hypothesis was confirmed in other experiments as well. The activation analysis has shown that the tungsten samples under investigation contain a microadmixture of cobalt (several fraction of a per cent). Tungsten atoms form magnetic clusters around cobalt atoms. In other words the tungsten could be in a heterophase state which is characterized by the symmetry properties of both the paramagnetic and ferromagnetic phases simultaneously. It should be noted, however, that

magnetic admixtures are not a necessary condition for the formation of the heterophase states. The heterophase fluctuations which take place over a vide range of temperatures [28] are also important here.

If the magnetic cluster formation phenomenon is taken into account, Eq. (34) will take the form

$$I_{(hkl)} = C \Big( [a + Zf_{(hkl)}(\sin\theta/\lambda)a_{ne}]^2 + [1 - f_{(hkl)}(\sin\theta/\lambda)]^2 \gamma^2 \cot^2\theta + p^2 \Big) A_{(hkl)} \\ \times \frac{\exp[-2B(\sin\theta/\lambda)^2]}{\sin 2\theta},$$
(36)

where  $p^2 = 2/3f_M^2 a_M^2$ , and  $f_M$  and  $a_M$  are the magnetic form factor and the magnetic scattering amplitude, respectively. Thus the problem of determining  $a_{ne}$  from diffraction experiments with tungsten single crystals is reduced to the determination of the dependence of the transferred momentum of the  $f_M$  magnetic form factor. This dependence together with the value for  $a_{ne}$  were found from the available diffraction data. For the latter,

$$a_{ne} = (-1.60 \pm 0.05) \times 10^{-3} \text{ fm},$$
 (37)

which is in agreement with the result (32) obtained by measuring the total cross section of bismuth at the IBR-30 reactor. The results of all measurements are presented in Table I. From this Table it follows that the most accurate experiments fall into two groups: the measurements of Refs. [1, 5, 15] lead, in accordance with Eq. (6) to  $\langle r_{\rm in}^2 \rangle > 0$ , which contradicts the modern theory (see below), and the measurements of Refs. [7, 17, 22, 23] lead to  $\langle r_{\rm in}^2 \rangle < 0$  which confirms it.

Recently, Leeb and Teichtmeister [2] have analyzed the results [5,7] of the low energy (< 150 eV) total neutron-atom cross sections. They have confirmed that the discrepancy between the  $a_{ne}$  values is due to different ways of treatment of the resonance contribution. They believe that the  $a_{ne}$  value which is less negative than the corresponding Foldy value (that is  $\langle r_{in}^2 \rangle > 0$ ) is more favorable.

Nikolenko and Popov [3] have tried to explain the difference between [5] and [7] by the fact that inter-resonance interference terms are neglected in the analyses of Ref. [7]. However, as is shown in Refs. [4,6,29] the result of Ref. [3] cannot be considered sufficiently correct. Though Eq. (28) does not contain any evident terms which do account for the inter-resonance interference, this one has contributed to the  $p_2$  term. The value of  $p_2^{exp} =$  $-2.3 \times 10^{-27}$  cm<sup>2</sup>/sr was determined in Ref. [7] by fitting experimental data and due to this fitting procedure it contains the inter-resonance interference term.

Meanwhile one can evaluate analytically the contribution of the inter-resonance interference effect. There are well-known S matrices that do account for this phenomenon:

1. [30, 31]

$$S_{nn} = \left[1 + i \sum_{\lambda\lambda'} \Gamma_{\lambda n}^{1/2} \Gamma_{\lambda' n}^{1/2} A_{\lambda\lambda'}\right] \exp(-2ikR), \tag{38}$$

TABLE I. The results of measurements of $a_{ne}$ .				
Authors, year	Method	Magnitude of effect, ne/tot	$-a_{ne} \times (10^3) \text{ fm}$	Ref.
P. Dee, 1932	Recoil electron in cloud chamber		< 1000	_
E. Fermi, L. Marschall, 1947	Neutron scattering on noble gases	$\Delta\sigma/\sigma \cong 0.5\%$	$100 \pm 1800$	[14]
W. Havens, <i>et al.</i> , 1947–51	Total neutron cross section on lead and bismuth	$\Delta\sigma/\sigma \cong 1.5\%$	$1.91 \pm 0.36$	[16]
D. Hughes <i>et al.</i> , 1952–53	Neutron total reflection from $O_2$ -Bi mirror	$\Delta \Theta / \Theta \cong 50\%$	$1.39\pm0.13$	—
M. Hamermesh et al., 1952	Neutron scattering on noble gases	$\Delta\sigma/\sigma \cong 0.5\%$	$1.5 \pm 0.4$	_
M. Crouch <i>et al.</i> , 1956	Neutron scattering on noble gases	$\Delta\sigma/\sigma \cong 0.5\%$	$1.43 \pm 0.30$	
E. Melkonian et al., 1959	Total neutron cross section on bismuth	$\Delta\sigma/\sigma \cong 0.5\%$	$1.56 \pm 0.05^{*}$	[17]
V. Krohn, G. Ringo, 1966–73	Neutron scattering on noble gases Total neutron cross	$\Delta\sigma/\sigma\cong 0.5\%$	$1.30\pm0.03$	[15]
L. Koester <i>et al.</i> , 1970-88	section and atomic scattering length on bismuth and lead Neutron diffraction on a	$\Delta\sigma/\sigma \cong 1.2\%$	$1.32\pm0.04$	[5]
Yu. Alexandrov et al., 1974-85	tungsten-186 single crystal	$\Delta\sigma/\sigma \cong 20\%$	$1.60\pm0.05$	[22, 23]
Yu. Alexandrov <i>et al.</i> , 1985	Total neutron cross section on bismuth Total neutron cross	$\Delta\sigma/\sigma \cong 1.2\%$	$1.55\pm0.11$	[7]
S. Kopecki <i>et al.</i> , 1994	section on rediogenic lead $(72.6\%^{208}\text{Pb})$	$\Delta\sigma/\sigma\cong 1.2\%$	$1.35\pm0.04$	[1]

\*Without correction for Schwinger scattering and resonance scattering.

where the reciprocal of A has the components:

$$(A^{-1})_{\lambda\lambda'} = (E_{\lambda} - E)\delta_{\lambda\lambda'} - \frac{i}{2}\sum_{c}\Gamma_{\lambda c}^{1/2}\Gamma_{\lambda' c}^{1/2},$$
(39)

and the c index runs through all channels.

2. [32]

$$S_{nn} = \exp(-2ikR) \left[ 1 + i \sum_{j} \frac{\alpha_{nj} + i\beta_{nj}}{\mu_j - E - i\nu_j} \right], \tag{40}$$

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where  $\sum_{j} (\alpha_{nj} + i\beta_{nj}) = \sum_{j} \Gamma_{nj}$ ,  $\sum_{j} \beta_{nj} = 0$ ,  $\mu_j = \operatorname{Re} \tilde{E}_j$ ,  $\nu_j = -\operatorname{Im} \tilde{E}_j$ ,  $\tilde{E}_j$  is the complex energy of *j*-th resonance (at  $\beta_{nj} = 0$ ,  $\tilde{E}_j = E_j - i\Gamma_j/2$ ). At  $\beta_{nj} = 0$  we have  $\sigma_{\text{tot}} = 2\pi g (1 - \operatorname{Re} S_{nn})/k^2$  as the sum of Breit-Wigner's terms taking into account only the interference between the potential and resonance scattering.

From Eq. (38) one can express [6, 29] the inter-resonance term to  $\sigma_{tot}$ :

$$\frac{\sigma_{\text{int}}}{4\pi} = \frac{g_+}{4k^2} \frac{\sum_i \Gamma_{ni} \Delta E_i \left(\sum_{j \neq i} \frac{\Gamma_j}{\Delta E_j}\right)}{\Delta E_i^2 + \frac{1}{4} \left(\Gamma_i + \Delta E_i \sum_{j \neq i} \frac{\Gamma_j}{\Delta E_j}\right)^2}$$

+ (a similar term for the other spin). (41)

At energies far from the resonance energy, owing to the fact that  $\Gamma = \Gamma_n + \Gamma_\gamma$  the term containing  $\Gamma_{ni}\Gamma_{nj}$  in Eq. (41) does not vary with energy (e.g., for bismuth at energies below 50 eV), the second term containing  $\Gamma_{ni}\Gamma_{\gamma j}$  is much less than the first one (for bismuth it is 40 times less at an energy of 10 eV). Since in Dubna work [7] the  $p_2$  term does not depend on energy either, one cannot affect the result of the  $a_{ne}$  determination in [7] by introducing a constant term,  $\sigma_{int}/(4\pi)$ . Calculations of  $\sigma_{int}/(4\pi)$  based on (41) were performed for bismuth with the known resonances  $0 < E_{0j} < 265$  keV [20]. They have shown that the additional interference term at an energy of about 10 eV makes  $\sigma_{int}/(4\pi) = 0.0086 \times 10^{-24} \text{ cm}^2/\text{sr}$  (the total cross section of bismuth at this energy is  $\sigma_{tot} = 0.76 \times 10^{-24} \text{ cm}^2/\text{sr}$ , *i.e.* nearly 90 times larger).

#### 5. On the controversy about the intrinsic charge radius of the neutron. Discrepancy between the Garching and Dubna results

As you know from the above-mentioned section there is a controversy in the physical community about the value of intrinsic  $\langle r_{in}^2 \rangle_N$  for the neutron. Part of physicists believe that the value of  $\langle a_{ne} \rangle = -1.309 \times 10^{-3}$  fm is true. The other part has another point of view, *i.e.*,  $\langle a_{ne} \rangle = -1.577 \times 10^{-3}$  fm. From the standpoint of an experimentalist the question of the  $\langle a_{ne} \rangle$  value is to be solved by an experiment, *e.g.* by comparing  $\sigma_{tot}$  measured at different energies with  $b_{coh}$  measured at very small energies (like in Ref. [7]). This kind of measurements is carried out at the moment by the Dubna-Germany-Czech Republic collaboration [33, 34].

The results of the Garching experiments [5] and Dubna experiment [7] are at the center of the controversy. Different ways of data treatment caused a discrepancy of not more than 1.5 uncertainty in values for  $a_{ne}$  in these experiments. Therefore, strictly speaking one should look for contradictions between the works [5] and [22,23] but not between [5] and [7]. Nevertheless, strange as it may seem, the discussion mainly goes around the latter two works. By formulas (25) and (26) one may obtain for the s-wave scattering (at e(k) = 1,  $\Delta E \gg \Gamma/2$  and  $R = \sin 2\delta_0/(2k)$ ):

$$\frac{\sigma_{\text{tot}}}{4\pi} = \frac{\sigma_{\text{coh}} + \sigma_{\text{in}} + \sigma_{\gamma}}{4\pi} \\
= \frac{\sin^2 \delta_0}{k^2} - \frac{\sin \delta_0}{k^2} \left[ \sum_{+} \frac{g_{+}\Gamma_n \Delta E}{\Delta E^2 + \Gamma^2/4} + \sum_{-} \frac{g_{-}\Gamma_n \Delta E}{\Delta E^2 + \Gamma^2/4} \right] \\
+ \frac{1}{4k^2} \left[ \sum_{+} \frac{g_{+}\Gamma_n \Delta E}{\Delta E^2 + \Gamma^2/4} + \sum_{-} \frac{g_{-}\Gamma_n \Delta E}{\Delta E^2 + \Gamma^2/4} \right]^2 + \frac{\sigma_{\text{in}}}{4\pi} + \frac{\sigma_{\gamma}}{4\pi}, \quad (42)$$

where  $g_{+-} = 1/2(2J+1)/(2I+1)$ ,  $J = \pm 1/2$ , I = 9/2 (for Bi). From [7] it follows that

 $\frac{\sigma_{\text{tot}}}{4\pi} = \frac{\sin^2 \delta_0}{k^2} - \frac{\sin \delta_0}{k^2} \left[ \sum_{+} \frac{g_{+} \Gamma_n \Delta E}{\Delta E^2 + \Gamma^2/4} + \sum_{-} \frac{g_{-} \Gamma_n \Delta E}{\Delta E^2 + \Gamma^2/4} \right] + \frac{1}{4k^2} \left[ \sum_{+} \frac{g_{+} \Gamma_n^2}{\Delta E^2 + \Gamma^2/4} + \sum_{-} \frac{g_{-} \Gamma_n^2}{\Delta E^2 + \Gamma^2/4} \right] + \frac{\sigma_\gamma}{4\pi}.$ (43)

The first two and the last terms in Eqs. (42) and (43) coincide, while the others are different. The first reason for this difference is the fact that Eq. (43) was derived on the basis of a generally accepted S-matrix of scattering (17), which does not take into account inter-resonance interference. As it was shown above, however, taking this phenomenon into account cannot influence the result of  $a_{ne}$  determination in Ref. [7].

So, from Refs. [7, 17, 22, 23] it follows that  $\langle r_{in}^2 \rangle_N < 0$ . What kind of error comes into Ref. [5]?

Let us compare the formulas (42) and (43) for bismuth at the energy of 10 eV taking into account resonances with the energy  $E_{0j} > 0$  and the additional inter-resonance term:

$$\frac{1}{4k^2} \left[ \sum_{+} \frac{g_{+}\Gamma_n \Delta E}{\Delta E^2 + \Gamma^2/4} + \sum_{-} \frac{g_{-}\Gamma_n \Delta E}{\Delta E^2 + \Gamma^2/4} \right]^2 + \frac{\sigma_{\rm in}}{4\pi} = (0.0113 + 0.0006) \times 10^{-24} \,\,{\rm cm}^2/{\rm sr}$$
$$= 0.0119 \times 10^{-24} \,\,{\rm cm}^2/{\rm sr}, \tag{44}$$

$$\frac{1}{4k^2} \left[ \sum_{+} \frac{g_{+}\Gamma_n^2}{\Delta E^2 + \Gamma^2/4} + \sum_{-} \frac{g_{-}\Gamma_n^2}{\Delta E^2 + \Gamma^2/4} \right] + \frac{\sigma_{\text{int}}}{4\pi} = (0.0029 + 0.0086) \times 10^{-24} \text{ cm}^2/\text{sr}$$
$$= 0.0115 \times 10^{-24} \text{ cm}^2/\text{sr}. \tag{45}$$

Thus, if the contribution of the  $\sigma_{int}/(4\pi)$  term is taken into account, expressions (44) and (45) give practically the same results (at  $E_{0j} > 0$ ).

There is some difference, however, between work [5] and [7] in their approach to calculation of the contribution of negative energy resonances  $(E_{0j} < 0)$  and unknown resonances

to the total cross section. In Ref. [5] this contribution of one bound and unknown levels has been calculated using the average parameters of *s*-wave scattering: the strength function,  $S_0 = 0.65 \pm 0.15$ , and the mean level distance  $\langle D_0 \rangle = 4.5 \pm 0.6$  keV [20]. In this situation I think an error may easily creep in, since a resonance at  $E_{01} < 0$ , *e.g.*, may be at a distance  $|E_{01}| < \langle D_0 \rangle$  from the point E = 0 and it will hardly be possible to estimate its influence on the term  $b_{\rm R}$  with any accuracy, because the uncertainty in the determination of  $S_0$  is large (on the order of  $\pm 23\%$ ).

In Ref. [7] we have used a more realistic method consisting in varying the  $p_2$  parameter. This is the main reason for the discrepancy between the results of Garching and Dubna obtained for bismuth. The treatment of the experimental data of Ref. [5], taking into account the parameter  $p_2 = -0.0023 \times 10^{-24} \text{ cm}^2/\text{sr}$  found in Ref. [7] by the least squares method, will lead to a 1.2 times increase in the absolute value of  $a_{ne}$ , *i.e.*, to  $a_{ne} = -1.57 \times 10^{-3} \text{ fm}$  [see (32)].

Thus, to my thinking, the values of  $a_{ne}$  obtained in Refs. [5, 15] are not grounded enough, and, consequently, the actual  $\langle r_{in}^2 \rangle < 0$  [if Eq. (6) is correct]. This conclusion is in agreement with the measurements [7,17,22], but it disagrees with the result of the analysis of available data made in Ref. [2] that favors a value of  $a_{ne}$  which is less negative than the Foldy scattering length.

#### 6. INFLUENCE OF RESONANCE SCATTERING

There is a possibility to calculate  $p_2 + \sigma_{int}/(4\pi)$  directly.

It may be shown from Eqs. (30) and (41) that

$$p_{2} + \frac{\sigma_{\text{int}}}{4\pi} = \frac{g_{+}g_{-}}{4} \left[ \sum_{+} \frac{\Gamma_{ni}}{k_{i}E_{i}} - \sum_{-} \frac{\Gamma_{ni}}{k_{i}E_{i}} \right]^{2} + \frac{g_{+}}{4k^{2}} \sum_{i} \frac{\Gamma_{ni}}{\Delta E_{i}} \sum_{j \neq i} \frac{\Gamma_{\gamma j}}{\Delta E_{j}} + \text{(a similar term for the other spin).}$$
(46)

The second and the third terms in Eq. (46) may be negative. Their signs depend on the influence on them of the neighboring levels with  $E_i < 0$ . Thus, there exists no direct argument in favor of excluding the possibility of the negative sign for  $p_2 + \sigma_{int}/(4\pi)$ . For an even-even nucleus  $(g_+ = 1, g_- = 0)$ 

$$p_2 + \frac{\sigma_{\text{int}}}{4\pi} = \frac{1}{4k^2} \sum_i \frac{\Gamma_{ni}}{\Delta E_i} \sum_{\substack{j \neq i}} \frac{\Gamma_{\gamma j}}{\Delta E_j}.$$
(47)

Calculations carried out for E = 1 eV, two known resonances of <sup>208</sup>Pb (507 keV and 1735 keV [20]) and one negative dummy-resonance (-1910 keV) introduced in Ref. [35] give the following result:  $p_2 + \sigma_{\rm int}/(4\pi) \simeq 6.7 \times 10^{-7} \times 10^{-24} \text{ cm}^2/\text{sr}$ . Thus, for nuclei of <sup>208</sup>Pb the contribution of resonance scattering is practically compensated by the contribution of inter-resonance interference scattering. One can also calculate the  $p_1a_{\rm coh}$  term [see

Eqs. (28) and (29)]:  $p_1 a_{\rm coh} \simeq -1.3 \times 10^{-7} \times 10^{-24} \text{ cm}^2/\text{sr}$ , *i.e.*, is also very small. Therefore, the Eq. (28) may be rewritten for the case of <sup>208</sup>Pb as

$$y = \frac{\sigma_{\rm tot}(E')}{4\pi} - a_{\rm coh}^2(E) \simeq -2aa_{\rm coh}(E)(Z - F'), \tag{48}$$

*i.e.*, we can make an important conclusion: in case of <sup>208</sup>Pb the value of  $a_{ne}$  will not be influenced by any resonance scattering.

For bismuth the situation is much more complicated:  $p_2 + \sigma_{int}/(4\pi)$  may be smaller than zero (as it follows from Ref. [7]). One has to be very careful, however, when speaking about  $p_2 + \sigma_{int}/(4\pi)$  as being independent of energy, because the second and the third terms in Eq. (46) depend on energy as  $1/E^{1/2}$ . Comparing values of  $p_2 + \sigma_{int}/(4\pi)$  and  $p_2^{exp}$  at a neutron energy of about 1 eV one can also see that the calculated value is about  $4 \times 10^{-27}$  cm<sup>2</sup>/sr larger than the experimental one. This difference may be explained by the influence of unknown negative energy resonance levels ( $E_j < 0$ ) of bismuth which were not taken into account under the calculations.

# 7. Comparison of measured intrinsic charge radius with its theoretical value

Now about a comparison of the experimental results with modern theoretical ideas which follow from the old meson theory by Yukawa.

The mean square intrinsic charge radius of the neutron is a fundamental characteristic of the neutron, and its measurements permit verification of modern theoretical ideas concerning nucleons. Knowledge of the signs and values of the anomalous magnetic moments of the neutron and proton permits establishing a qualitative picture of the  $\rho(r)$ distribution in the nucleon. This point is illustrated by Fig. 2 [36]. Note that the sign of  $\langle r_{\rm in}^2 \rangle_N$  in the case of an object, which, as a whole, is neutral, may be either positive or negative. This depends mainly on what charge is to be found at the periphery. Thus, for instance, the charge distribution in a neutron, depicted in Fig. 2, should provide for the sign of  $\langle r_{\rm in}^2 \rangle_N$  being negative. This distribution was already known before 1955–57 [36]. In the 50s it was also known that in the old meson theory the process  $n \to p + \pi^-$  gave rise to a negative tail for the intrinsic neutron charge distribution. In all old static models, however, the core of nucleon was not understood and its properties were not calculable.

This problem was solved by modern ideas about the nucleon, *e.g.*, by modern quark models. During the last few years attempts were made to solve the quantum chromodynamics (QCD) equations. In the absence of exact solutions it is natural to rely on phenomenological models, which incorporate features expected from QCD. Of all these models the bag model is the most attractive. The bag model has its beginning in the late 60s, when P.N. Bogoliubov described phenomenologically a system of relativistic massless quarks moving freely inside a spherical volume. The development of Bogoliubov's approach has yielded the MIT (Massachusetts Institute of Technology) model. The main features of the MIT bag model have proven to be essential for the construction of the modern quark model of the nucleon, that is Cloudy Bag Model (CBM) proposed by Thomas, Theberge



FIGURE 2. Expected electric charge distribution inside the nucleon: a) the proton; b) the neutron.

and Miller (see, e.g., Ref. [37]). In this model the nucleon consists of a spherical static cavity with radius R filled with three massless free quarks. The quarks interact with a pion field on the surface of the bag. This surface is the source of a field of negative pions acting at a distance of the order of  $\hbar/(m_{\pi}c) > R$ . In the absence of pions CBM is identical to the MIT model. The latter violates the chiral symmetry, and since chiral symmetry is a property of QCD itself, this gives us quite justifiable concern. By introducing a pion field coupled to the quarks on the bag surface, one can restore the chiral symmetry. The CBM has been developed in response to this difficulty, and in CBM the nucleon is far from being point-like, having a radius of about one fermi. This model has produced a number of remarkable results for the properties of single hadrons, e.g., the magnetic moments of the proton, neutron, and other members of nucleon octet, the form factors, the polarizabilities, the charge radius and so on.

The value  $\langle r_{\rm in}^2 \rangle_N^{\rm exp} > 0$  contradicts the present-day understanding of the neutron not only in CBM but in other theories about the nucleon (see, e.g. Refs. [38–40]), which is essentially based on the old Yukawa meson theory as well. By applying these concepts physicists can precisely calculate within the framework of the static models under the assumption of a motionless (not recoiling) heavy nucleon  $(M \to \infty)$  the value  $\langle r_{\rm in}^2 \rangle_N = \int \rho(\vec{r}) r^2 d^3 \vec{r}$ , to obtain  $\langle r_{\rm in}^2 \rangle < 0$  (see, e.g., Refs. [41–43]). This value cannot include the Foldy term which is equal to zero at  $M \to \infty$ , and it seems to be correct to compare the calculated result with  $\langle r_{\rm in}^2 \rangle_N^{\rm exp}$  obtained after the subtraction of the Foldy scattering length from the measured  $a_{ne}$  value.

Owing to the  $n \to p + \pi^-$  process, there appears a negative tail in  $\rho(\vec{r})$  (see Fig. 2), like in the old static models; by new quark models (e.g. CBM) there also exists a negative  $\pi^-$ -meson tail, which is just what causes the negative sign of  $\langle r_{\rm in}^2 \rangle_N$ . It is practically impossible to obtain  $\langle r_{\rm in}^2 \rangle_N > 0$  following modern concepts. If the results of Refs. [1,5,15] are correct, then a serious revision of our understanding of the structure of nucleon is necessary.

Being a specialist in experimental physics, I do understand that issues of the value of  $a_{ne}$  and, consequently, of the sign of  $\langle r_{in}^2 \rangle_N$  must be studied experimentally. But, honestly, I really do not understand why, from a theoretical point of view, the sign of  $\langle r_{in}^2 \rangle_N$  has to be positive.

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