

## Polarizability of the neutron

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**ABSTRACT.** The concept of the neutron electric polarizability (NEP) is discussed. The best result for the NEP coefficient ( $< 0.5 \times 10^{-3} \text{ fm}^3$ ) was obtained by the Dubna-Garching-Riga collaboration (time of flight and neutron resonance technique methods, bismuth, natural lead and nuclei of  $^{208}\text{Pb}$ ). Concerning the job performed by Schmiedmayer *et al.* (the Vienna-Oak Ridge collaboration), it is shown that this experiment should have given rise to doubt. The discussion of this experiment led to the assumption that obtained data only allowed the determination of the upper limit of about  $2 \times 10^{-3} \text{ fm}^3$  for the NEP. It was also shown that the NEP determined by neutron transmission depends on the neutron mean square intrinsic charge radius.

**RESUMEN.** Se discute el concepto de la polarización eléctrica del neutrón (NEP). La colaboración establecida por grupos de investigación de Dubna, Garching y Riga, utilizando métodos y técnicas de tiempo de vuelo y resonancia de neutrones sobre bismuto, un nucleido de  $^{208}\text{Pb}$ , han obtenido los mejores resultados para el coeficiente de polarización eléctrica del neutrón ( $< 0.5 \times 10^{-3} \text{ fm}^3$ ). Los resultados para el coeficiente de polarización en el trabajo experimental de Schmiedmayer *et al.*, (colaboración entre grupos de investigación de Viena y Oak Ridge) deberían ser puestos en duda. La discusión de este experimento conduce a la suposición que los datos obtenidos permiten únicamente la determinación de un límite superior de alrededor  $2 \times 10^{-3} \text{ fm}^3$  para el coeficiente de polarización eléctrica del neutrón. Asimismo se muestra que la determinación del coeficiente de polarización eléctrica del neutrón mediante transmisión de neutrones depende del radio de carga intrínseca cuadrático medio del neutrón.

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### 1. NOTION OF POLARIZABILITY

Polarizabilities (electric and magnetic) are fundamental structure constants of a particle introduced to describe interactions of elementary particles more adequately. They are as important as other constants: the charge, the magnetic dipole moments, the charge radius and so on, but the polarizabilities are not as well known.

The notion of the polarizability of nucleons has emerged from the study of neutron scattering by the Coulomb field of a heavy nucleus as considered by Alexandrov, Bondarenko, Barashenkov and Stakhanov [1, 2], and also (independently and simultaneously) with the question of photon scattering and the photoproduction of pions on nuclei, by Klein [3] and Baldin [4].

The effect of polarizability reflects the possibility for particles to acquire induced electric and magnetic moments in the presence of electric and magnetic fields. It is equal to zero if a particle is point-like or of a hard structure.

The electric polarizability (EP)  $\alpha$  is defined as

$$\vec{d} = \alpha \vec{E}, \quad (1)$$

where  $\vec{d}$  is the induced electric dipole moment (EDM), and  $\vec{E}$  is an external and static electric field.

The magnetic polarizability (MP)  $\beta$  is defined as

$$\vec{d}_m = \beta \vec{H}, \quad (2)$$

where  $\vec{d}_m$  is the induced magnetic dipole moment (MDM), and  $\vec{H}$  an is external and static magnetic field.

To consider the effect of an electric field on a neutron we should take into account all the virtual excited states of the neutron. In the second order of approximation from the perturbation theory we obtain the expression

$$\alpha = 2 \sum_n \frac{\langle 0 | d_z | n \rangle^2}{\omega_n}, \quad (3)$$

where  $d_z$  are the operators of the EDM components, and  $\omega_n$  is the transition frequency from the state of  $|n\rangle$  to the state of  $\langle 0|$ .

The relativistic analysis of polarizability effects in the Compton scattering of photons, carried out by Petrunkin [5] and Shekhter [6], has shown that the dynamic (or Compton) EP,  $\bar{\alpha}$ , and MP,  $\bar{\beta}$  in the presence of an external and oscillating electromagnetic field of photons are

$$\bar{\alpha} = \alpha + \Delta\alpha, \quad (4)$$

$$\bar{\beta} = \beta + \Delta\beta, \quad (5)$$

where  $\alpha$  and  $\beta$  are defined by expressions of type (3) and  $\Delta\alpha$  and  $\Delta\beta$  cannot be interpreted as coefficients of polarizability. For example,

$$\Delta\alpha = \frac{e^2}{3M} \langle r_E^2 \rangle + \frac{e^2 \mu^2}{4M^3}, \quad (6)$$

where  $\mu$  is the magnetic moment and  $\langle r_E^2 \rangle$  is the mean square charge radius of the particle [6]. For the proton,  $\Delta\alpha \simeq 3.9 \times 10^{-4} \text{ fm}^3$ , which amounts to about 50% of the  $\alpha$  value. For charged pions and kaons, the value of  $\Delta\alpha$  is larger than that of  $\alpha$  by more than a factor of two. For the neutron  $\Delta\alpha = 0$ .

## 2. THEORETICAL ESTIMATES OF THE POLARIZABILITIES

Nucleon polarizabilities may be considered using either dispersion relations or quark models. The dispersion relation approach, which is a consequence of the causality principle, appears to be the most strict, universal and model-independent at present. It follows that the dispersion sum rules used in the calculations should be obtained. Such sum rules can be written as

$$\bar{\alpha} + \bar{\beta} = \frac{1}{2\pi^2} \int_{\omega_1}^{\infty} \frac{\sigma_{\gamma}(\omega) d\omega}{\omega^2}, \quad (7)$$

where  $\sigma_{\gamma}(\omega)$  is the total photoabsorption cross section and  $\omega_1$  is the photoabsorption threshold.

Baldin [4] was the first to interpret the left-hand side of this equation for the case of nucleons. The value  $\bar{\alpha}_p + \bar{\beta}_p$  for the proton was calculated with Eq. (7) by substituting the well-known values of the proton-photoabsorption cross sections obtained from measurements at low energy and extrapolated at high energy:

$$\bar{\alpha}_p + \bar{\beta}_p = (14.2 \pm 0.3) \times 10^{-4} \text{ fm}^3. \quad (8)$$

For the neutron these cross sections cannot be measured directly but can be estimated theoretically from the cross sections measured for the deuteron. As a result [7]

$$\bar{\alpha}_n + \bar{\beta}_n = (15.8 \pm 0.5) \times 10^{-4} \text{ fm}^3. \quad (9)$$

EP and MP of the nucleon can be qualitatively understood in terms of the simple valence quark model. Positive values of about  $10 \times 10^{-4} \text{ fm}^3$  were obtained for the nucleon EP. These calculations have been made, *e.g.*, in Ref. [7].

Nucleon polarizabilities may be also obtained within the cloudy bag model (CBM) (see, *e.g.*, [8,9]). It appears that the polarizability value is essentially due to the pion cloud distortion. The calculated polarizability values are in good agreement with the experimental ones. It should be noted that all theoretical results have substantial uncertainties and are not always consistent with one another, especially the differences between the proton and neutron polarizabilities obtained with the different models.

## 3. MEASUREMENTS OF THE POLARIZABILITIES BY COMPTON SCATTERING

The scattering of photons by particles with a spin equal to 1/2 and an anomalous magnetic moment (the Compton effect on nucleons) was considered by Gell-Mann and Goldberger [10], Klein [3], Baldin [4], Petrunin [5] and others [11,12]. These processes, connected with structural characteristics of the nucleon, (see Fig. 1(a,b)) are of importance to this effect. The angular distribution of photons is proportional to  $\bar{\alpha}_p + \bar{\beta}_p$  in the forward direction and to  $\bar{\alpha}_p - \bar{\beta}_p$  in the backward direction. Therefore,  $\bar{\alpha}_p$  and  $\bar{\beta}_p$  can be obtained from these distributions independently from (8).

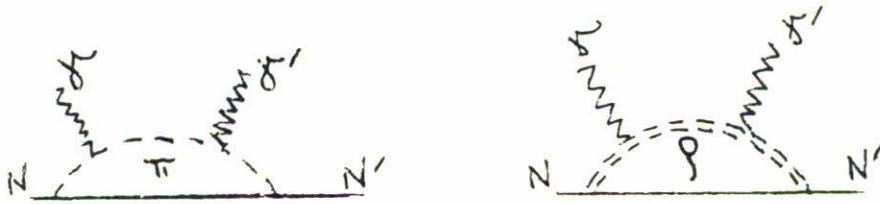


FIGURE 1. A Few diagrams representing the Compton effect on the nucleon.

Direct measurements of the EP of the proton were carried out in 1960 by Goldansky, *et al.*, [13], then by Baranov, *et al.*, [14], by Federspiel, *et al.*, [15] and Zieger, *et al.*, [16]. The best results are [16]

$$\begin{aligned} \bar{\alpha}_p &= (10.7 \pm 1.1) \times 10^{-4} \text{ fm}^3, \\ \bar{\beta}_p &= (-0.7 \pm 1.6) \times 10^{-4} \text{ fm}^3. \end{aligned} \tag{10}$$

It should be noted that this scattering process has a very small cross section (on the order of  $10^{-32} \text{ cm}^2$ ). At energies above the meson production threshold (150 MeV) this process is difficult to separate from the  $\pi$ -meson photoproduction whose cross section is about 100 times larger.

A direct measurement of Compton scattering by free neutrons is impossible but quasi free scattering by the neutron bound in the deuteron can be measured. Analysis of the first measurements ( $E_\gamma = 80\text{--}104 \text{ MeV}$  energy interval) of quasi-free Compton scattering by the neutron bound in the deuteron using the sum rule of Eq. (9), gives the following results [17]:

$$\bar{\alpha}_n = (11.7_{-11.7}^{+4.3}) \times 10^{-4} \text{ fm}^3. \tag{11}$$

The method of determining EP via quasi-free Compton scattering was worked out by the Lebedev Physics group [18].

The possibility of studying the Compton effect on hadrons by measuring the radiation scattering of high-energy hadrons by the Coulomb field of a nucleus has been discussed in the literature [19]. The first experiment was carried out on a beam of charge pions of 40 GeV (Serpukhov, Russia) [20]. It should be noted, however, that this method hardly allowed the determination of the EP of the neutron, since its zero electric charge leads to the absence of interference between an independent from frequency  $\omega$  term and from terms in  $\omega^2$  and  $\omega^3$ . The terms containing the EP appear only at higher powers of  $\omega$  (*e.g.*, fourth, fifth and so on) and will also contain additional unknown parameters. Detailed experimental information about these parameters is not presently available.

#### 4. COULOMB SCATTERING OF NEUTRONS FROM HEAVY NUCLEI

The study of the Coulomb scattering of neutrons in the extremely intense static electric field (up to  $10^{20} \text{ V/m}$ ) near heavy nuclei is still the only direct source of information on the EP of the neutron.

The potential  $V_\alpha$  describing the Coulomb interaction between an induced neutron electric moment and the electric field of the nucleus with a charge  $Ze$  is

$$V_\alpha = -\vec{d}_n \vec{E} = -\frac{1}{2} \alpha_n E^2 = -\frac{\alpha_n Z^2 e^2}{2r^4}. \quad (12)$$

This formula does not account for the screening effect of the atomic electron cloud. Estimates have shown that this effect is reduced to corrections on the order of  $R/a \simeq 10^{-4}$  for polarizability scattering amplitude,  $a$  is the size of the atom. The scattering amplitude caused by EP of the neutron was first calculated by the Born approximation in Ref. [2] as

$$f_p(\phi) = \frac{M\alpha_n}{2R} \left( \frac{Ze}{\hbar} \right)^2 qR \left( \frac{\sin qR}{(qR)^2} + \frac{\cos qR}{qR} + \text{si}(qR) \right), \quad (13)$$

where  $\text{si}(qR) = \int_0^{qR} (\sin x)/x dx - \pi/2$ ,  $\hbar q = 2\hbar k \sin(\phi/2)$  is the momentum transfer. Eq. (13) is valid for  $qR \ll 1$ . The conventional expansion in terms of Legendre polynomials is

$$f_p(\phi) = \frac{1}{2ik} \sum_l (2l+1) (\exp(2i\zeta_l) - 1) P_l(\cos \phi), \quad (14)$$

where

$$\begin{aligned} \zeta_0 &= M\alpha_n (Ze/\hbar)^2 (k/R - \pi k^2/3 + \dots), \\ \zeta_1 &= M\alpha_n (Ze/\hbar)^2 (\pi k^2/15 - Rk^3/9 + \dots). \end{aligned} \quad (15)$$

At small values for  $kR$  the amplitude (13) can be expanded into a series as

$$f_p(\phi) = \frac{M\alpha_n}{R} \left( \frac{Ze}{\hbar} \right)^2 \left( \frac{6}{5} - \frac{\pi}{4} qR + \frac{1}{7} (qR)^2 - \dots \right). \quad (16)$$

From Eq. (16) it follows that the scattering amplitude caused by the EP has a consistent term independent of energy on the order of  $10^{-1}$  fm (about 1% of the nuclear amplitude) at  $Z = 80$  and  $\alpha_n \sim 10^{-3}$  fm<sup>3</sup>. It appears impossible, however, to identify the contribution of the polarizability scattering due to this constant, since there is no exact theory of nuclear scattering at the moment. We may use the  $f_p(\phi)$  dependence of  $q \sim \sqrt{E}$ , such as the second term in Eq. (16). In this case, the sought-for effect is reduced by a factor of  $1/(qR)$ . No uncertainty appears, however, in the  $\alpha_n$  value because of the inexact value of the  $R$  radius, since the second term in Eq. (16) is not dependent on it.

The question was also investigated of what should be understood by the  $\alpha_n$  quantity entering Eqs. (12) and (13) for the amplitude. Bernabeu and Tarrach [21] have shown that  $\alpha_n$  relates to  $\bar{\alpha}_n$  in the following way:

$$\alpha_n = \bar{\alpha}_n + \mu_n \frac{m_n + 2M_{\text{nucl}}}{m_n M_{\text{nucl}}} \left( \frac{e\hbar}{2m_n c^2} \right)^2. \quad (17)$$

The second term in (17) is equal to about 10% of the first term.

Since the scattering due to EP occurs as a result of a long-range interaction, the sought-for effect manifesting itself at neutron energies on the order of a few MeV should be conducted in a small angle scattering range (less than 10 degrees). Apart from the effect related to the EP of the neutron, Schwinger scattering also occurs in the small angle range and can easily be accounted for. The main difficulty in interpreting the experimental data is in taking correct account of nuclear scattering. Since there is no strict theory, one has to resort to various model representations. For example, in the neutron energy range from 0.5 to 14 MeV the results were compared with those calculated within the framework of the optical model. An upper limit of  $10^{-2}$  fm<sup>3</sup> was obtained in this manner by [22, 23].

Experiments on the angular distribution of elastically scattered neutrons by heavy nuclei in the low energy range (below 100 keV) allow the upper limit of the EP to be estimated. If the differential cross section

$$\sigma(\phi) = \frac{\sigma_0}{4\pi} \left( 1 + \sum_{l=0}^{\infty} \omega_l P_l(\cos \phi) \right) \quad (18)$$

and the phase shifts of nuclear scattering  $\delta_l \sim (kR)^{2l+1}$  are used, then

$$\omega_1 = aE + b\sqrt{E}, \quad (19)$$

where  $b \sim \alpha_n$ .

A value for  $\alpha_n$  within the limits

$$-5 \times 10^{-3} \leq \alpha_n \leq 6 \times 10^{-3} \text{ fm}^3, \quad (20)$$

was obtained in this manner in Dubna [24] using the TOF method to measure the angular distribution of neutrons elastically scattered by lead at energies from 0.6 to 26 keV.

The most precise results can be obtained from measurements of the energy dependence of  $\sigma_{\text{tot}}$  for the interaction between neutrons and heavy nuclei in the low energy range (below 100 eV). This question was discussed in Dubna (see, *e.g.* Ref. [25]). In this case the additional terms connected with the EP have to appear in the equation for  $y$  (see Eq. (28) of the Ref. [26]):

$$\begin{aligned} y = \frac{\sigma_{\text{tot}}(E')}{4\pi} - a_{\text{coh}}^2(E) &= a^2(Z^2 - 2ZF') - 2aa_{\text{coh}}(E)(Z - F') \\ &+ p_1 \left[ a_{\text{coh}}(E) - a(Z - F') - \frac{1}{3}\pi k' Rf \right] \\ &+ p_2 - \frac{2}{3}\pi k' Ra_{\text{coh}}f - 2afF' + \frac{\sigma_{\gamma}(E')}{4\pi}, \end{aligned} \quad (21)$$

where  $f = \int_0^{\pi} f_p \sin \theta d\theta = \frac{M\alpha_n}{R} \left( \frac{Ze}{\hbar} \right)^2$  (see Eq. (16)).

Precise measurements of the total neutron cross section of bismuth in the electronvolt energy region were carried out on the pulsed reactor of the Joint Institute for Nuclear

Research (JINR) [27]. They covered the region from 1 to 90 eV and were performed by the TOF method over a 60 m flight path using both a liquid sample and a solid sample 18 mm thick. The background, measured with the help of plates of rhodium, silver, and tungsten (resonance energies 1.26, 5.19, and 18.83 eV, respectively) placed in the beam, was 0.3–0.4 per cent at 1–6 eV, and not more than 1.5 per cent at about 20 eV. The energy dependence of the total cross section for the interaction of neutrons with bismuth is grown in Fig. (see Fig. 1 [26]). The same figure shows the values for  $\sigma_{\text{tot}}$  measured at Garching (Germany) by Koester, *et al.*, [28].

To obtain information of the values of  $\alpha_n$  and  $a_{ne}$  the experimental data were processed by the method described above. Before this was done, however, corrections for Schwinger scattering, the solid state effects were introduced into  $\sigma_{\text{tot}}$ ; they did not exceed 0.8%.

The obtained value for  $a_{ne}$  coincides within experimental error with the result of independent neutron diffraction measurements on a single crystal of tungsten ( $a_{ne} = (-1.60 \pm 0.05) \times 10^{-3}$  fm) [29,30]. Making use of this value we can obtain

$$\alpha_n = (1.5 \pm 2.0) \times 10^{-3} \text{ fm}^3. \quad (22)$$

In 1976–88 Koester, *et al.*, [28] carried out precise measurements of  $b_{\text{coh}}$  and  $\sigma_{\text{tot}}$  (see previous report [26]). As a result, in addition to the  $a_{ne}$  value the following estimate for the  $\alpha_n$  was obtained:

$$\alpha_n = (0.8 \pm 1.0) \times 10^{-3} \text{ fm}^3. \quad (23)$$

As I mentioned above, part of processing procedure (see [28]) does not seem to be sufficiently correct, in particular, resonance scattering is not fully taken into account.

In 1994 (April 26–28) at the II International Seminar on Interaction of Neutrons with Nuclei (ISINN-2), which was in Dubna, it was reported that from experimentally measured data, obtained using enriched  $^{206,207,208}\text{Pb}$  targets and neutrons in the energy region between 1 eV and 2 keV, the conclusion was as follows [31]:

$$\alpha_n = \begin{cases} (-0.3 \pm 0.5) \times 10^{-3} \text{ fm}^3 & \text{if } b_{ne} = -1.32 \times 10^{-3} \text{ fm,} \\ (-1.3 \pm 0.5) \times 10^{-3} \text{ fm}^3 & \text{if } b_{ne} = -1.59 \times 10^{-3} \text{ fm,} \end{cases} \quad (24)$$

or from new, more accurate data:

$$\alpha_n = (0.0 \pm 0.5) \times 10^{-3} \text{ fm}^3 \text{ if } b_{ne} = -1.32 \times 10^{-3} \text{ fm.} \quad (25)$$

With additional data measured at the neutron energy of 143 keV the result was reported to be [32]

$$\alpha_n = \begin{cases} (-0.06 \pm 0.43) \times 10^{-3} \text{ fm}^3 & \text{if } b_{ne} = -1.32 \times 10^{-3} \text{ fm,} \\ (-1.01 \pm 0.43) \times 10^{-3} \text{ fm}^3 & \text{if } b_{ne} = -1.59 \times 10^{-3} \text{ fm.} \end{cases} \quad (26)$$

In 1988 Schmiedmayer *et al.* [33] (Vienna), studied neutron transmission through lead (with a natural mixture of isotopes) and carbon on the pulsed neutron source Helios at Harwell (UK). The measurements were performed by the TOF method over a flight path

of 150 m at neutron energies from 50 eV to 50 keV. The sample was at a distance of 56 m from the neutron source. Corrections for Schwinger, n-e and resonance scattering were introduced into the measured values. The resonance were accounted for with the help of the parameters obtained during the measurements. Resonances at  $E > 0$  and a level having a negative energy of 36 keV, which belongs to the  $^{207}\text{Pb}$  isotope were taken into account. The measurement for carbon was performed as a test. In the absence of resonance neutron-nucleus scattering the total scattering cross section can be parametrized by

$$\sigma_s(k) = \sigma_s(0) + ak + bk^2 + O(k^4). \quad (27)$$

After corrections for resonance, n-e and Schwinger scattering, one can obtain in the energy range from 50 eV to 20 keV ( $k = 0.0015$  to  $0.031 \text{ fm}^{-1}$ ) for lead

$$\sigma_s = 11.253(5) + 0.60(51)k - 371(27)k^2, \quad (28)$$

and from the term proportional to  $k$ :

$$\alpha_n = (1.2 \pm 1.0) \times 10^{-3} \text{ fm}^3. \quad (29)$$

In 1991 Schmiedmayer *et al.* (Vienna-Oak Ridge collaboration), continued the neutron transmission experiments [34]. The  $^{208}\text{Pb}$   $\sigma_{\text{tot}}$  was measured as a function of neutron energy between 50 eV and 40 keV by the TOF method using the Oak Ridge Electron Linear Accelerator (ORELA). The energy dependence of this cross section was analyzed to give the following results:

$$\sigma_s(k) = 11.508(5) + 0.69(9)k - 448(3)k^2 + 9500(400)k^4, \quad (30)$$

and from the term proportional to  $k$ , the EP of neutron was obtained

$$\alpha_n = (1.20 \pm 0.15 \pm 0.20) \times 10^{-3} \text{ fm}^3, \quad (31)$$

where the first uncertainty is statistical, and the second is systematic (background, multiple scattering, resonance correction, Schwinger scattering and so on). Therefore, for the first time this method gives a nonzero value for  $\alpha_n$ .

But recently it was shown [25,35,36] that the results (29) and (31) should have given rise to doubt (see below). The discussion of Schmiedmayer's experiment led to the assumption that the data reduction in [34] only allowed the determination of an upper limit of about  $2 \times 10^{-3} \text{ fm}^3$  for the neutron EP. I will discuss this question a little bit later.



## 5. SYSTEMATIC ERRORS IN NEUTRON EXPERIMENTS FOR THE DETERMINATION OF THE EP AND MSICR

As stated above, the determination of  $\alpha_n$  and  $a_{ne}$  is based on precise measurements of either the total neutron cross section and scattering length ( $\Delta\sigma/\sigma \simeq \Delta a/a \simeq 10^{-3}-10^{-4}$ ) or the asymmetry of neutron scattering by heavy nuclei ( $\Delta\omega_1 \simeq 10^{-3}$ ). At such accuracies it seems to be difficult to detect and remove the possible sources of systematic errors.

First, reliable methods for background determination must be available. As a rule, the background must not exceed 1–2% of the effective intensity and it must not experience sharp changes depending on the parameter being varied in the experiment (*e.g.*, dependent on neutron energy or scattering angle).

Second, in the measurement of  $\sigma_{\text{tot}}$ , corrections for the detector's miscounts at high-duty-cycle operation must be minimized. As a rule, the dead time of the detector and the electronic system must be less than 0.5  $\mu\text{s}$ .

Third, attention must be drawn to effects capable of distorting the energy dependence of the measured values. Thus in the measurement of  $\sigma_{\text{tot}}$  on large flight paths (*e.g.*, in Refs. [33, 34] the distance between the sample and the detector was several meters) the solid angle covered by the detector is small (apparently, on the order to 0.5 degrees) and the energy dependence of  $\sigma_{\text{tot}}$  may be distorted due to possible small-angular scattering of neutrons in the sample (such as  $Nb^2 \exp(-k^2\theta^2 R^2/5)$ , where  $R$  is the size of the inhomogeneities ( $\simeq 2-10$  nm), and  $N$  is the number of atoms in the inhomogeneity). There exist numerous reasons for scattering at small angles to take place (*e.g.*, cluster defects in the structure, magnetic heterophase fluctuations, etc.). This phenomenon was taken into account in the diffraction experiments with tungsten monocrystals [29, 30], and taking it into account resulted in  $a_{ne}$  changing from  $-1.06 \times 10^{-3}$  fm to  $-1.60 \times 10^{-3}$  fm. In any case, the influence of small angle scattering of the neutrons should be investigated.

Fourth, attention should be paid to accurate introduction of the correction for p-wave scattering. The effect of p-wave scattering ( $\sigma_1 = 4\pi/(k^2)3 \sin^2 \delta_1$ ) makes up about 0.3% of s-wave scattering ( $\sigma = 4\pi/(k^2) \sin^2 \delta_0$ ) at the energy of 20 keV. The effect of neutron EP scattering is less than 0.3%. Therefore, the calculations for p-wave scattering have to be executed very accurately even at this energy.

For neutrons, as it is known from Ref. [37],

$$\exp(2i\delta_l) = \frac{G_l(R) - iF_l(R)}{G_l(R) + iF_l(R)}, \quad (32)$$

where  $R$  is the channel radius,  $G_l(R) = -\sqrt{\pi kr/2} N_{l+1/2}(kr)$ ,  $F_l(R) = \sqrt{\pi kr/2} J_{l+1/2}(kr)$ .

At small energies ( $kR \ll 1$ ):

$$\delta_1 \simeq \frac{-(kR)^{2l+1}}{(2l-1)!!(2l+1)!!}. \quad (33)$$

The calculations, carried out by Guseva [36] (Gatchina), have shown that the differences between  $\sigma_1$ , calculated by these two methods, are 10% at energy  $E = 24$  keV, 25% at energy  $E = 45$  keV, and 40% at energy  $E = 145$  keV.

This means that the corrections for p-wave scattering should be made with the help of Bessel function formalism, but not by Eq. (33).

Fifth, in Eq. (27) from Refs. [33,34] there is no term which is proportional to  $k^3$ . Eq. (27), however, can be obtained by expanding in a series the expression for potential scattering cross section  $\sigma_{\text{tot}} = 4\pi/(k^2) \sin \delta_0 \sin(\delta_0 + 2\zeta_0)$  (see Eq. (18) in Ref. [26]). In this case the term proportional to  $k^3$  will appear in Eq. (27). This term is the term proportional to  $k$  as  $2/3(kR)^2$ , which is 7% at energy 20 keV, 10% at energy 45 keV, and 20% at energy 145 keV. Therefore the term proportional to  $k^3$  should be taken into account in calculations.

Sixth, systematic errors may also arise from inaccurate data processing, *e.g.*, in accounting for nuclear resonance scattering. In the analysis of data for  $\sigma_{\text{tot}}$  it is necessary to take into account the influence of resonances located rather far from the energy interval under investigation. In the case of levels with positive energies, this procedure can in principal be carried out for all the resonances known, but in the case of levels with negative energies this is impossible because of the lack of information about these levels. Furthermore, in the data processing performed for a natural mixture of isotopes, if the  $-36$  keV level ( $^{207}\text{Pb}$  isotope) is excluded, the value of  $\alpha_n$  may even change its sign.

Thus, in spite of the high statistical accuracy of the values obtained for  $\sigma_{\text{tot}}$  the values for the  $\alpha_n$  are uncertain. In any case, systematic errors should be increased.

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