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Radiative corrections to hot quantum electrodynamics plasma

R.A. PÉREZ MARTÍNEZ
Departamento de Matemáticas
Centro de Investigación y de Estudios Avanzados del IPN
Apartado postal 14–740, 07000 México, D.F., México

H. PÉREZ ROJAS* AND L. VILLEGAS LELOVSKY
Departamento de Física
Centro de Investigación y de Estudios Avanzados del IPN
Apartado postal 14–740, 07000 México, D.F., México

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ABSTRACT. The problem of the radiative corrections to the thermodynamic potential in statistical quantum electrodynamics in the limit of temperatures of the order of the electron rest mass is revisited. It is argued that only longitudinal modes can contribute to the so-called correlation term of the thermodynamic potential, in agreement with the pioneering calculations made by Akhiezer-Peletminskii (*Sov. Phys. JETP* **11** (1960) 1316), and disagreement with the more recent calculations made by Leermakers/van der Weert (*Phys. Lett.* **135** (1984) 118). Analogy with the Bose-Einstein condensate and consequences for the effective potential which arises in the discussion of the phase transitions which occur in the standard model are discussed.

RESUMEN. Se revisa el problema de las correcciones radiativas al potencial termodinámico en la electrodinámica cuántica estadística, en el límite de temperaturas del orden de la masa del electrón en reposo. Probamos que solamente los modos longitudinales pueden contribuir el así llamado término de correlación del potencial termodinámico, en conformidad con los primeros cálculos, hechos por Akhiezer-Peletminskii (*Sov. Phys. JETP* **11** (1960) 1316), y en desacuerdo con los cálculos más recientes, realizados por Leermakers-van der Weert (*Phys. Lett.* **135** (1984) 118). Además se examina la analogía con el condensado de Bose-Einstein y las consecuencias para el potencial efectivo que se obtiene en la discusión de la transición de fase que ocurre en el modelo estándar.

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1. INTRODUCTION

The present paper is a continuation of a previous one [1], where two of the present authors investigated the problem of the photon mass in statistical quantum electrodynamics (QED). In the present paper, we want to continue that work by investigating some points

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* Permanent address: Grupo de Física Teórica, ICIMAF, Academia de Ciencias de Cuba, Calle E No. 309, Vedado, La Habana 4, Cuba.

related to the contribution of the photon mass to the radiative corrections to the thermodynamic potential, as well as some specific features and analogies of the high temperature photon spectrum.

According to statistical quantum electrodynamics, electromagnetic radiation field can be in thermodynamic equilibrium with the electron-positron field at very high temperature $T \geq mc^2$, where m is the electron mass (an scenario which may hold in a very hot star). Such equilibrium between the photon gas and the electron-positron plasma is characterized by the temperature T and the equation for the chemical potentials $\mu_{e^-} + \mu_{e^+} = \mu_\gamma$, where $\mu_\gamma = 0$ and in consequence $\mu_{e^-} = -\mu_{e^+}$. In the case of electrical neutrality, as in the blackbody radiation, also $\mu_{e^-} = 0$, which means equal densities for electrons and positrons. This equilibrium is preserved by a continuous balance in the processes of pair creation and annihilation, and by scattering of electrons and positrons by photons. Usually an ionic background is supposed to exist, which interacts also with the electromagnetic field. This background provides a positive charge to balance the electron charge in the case of $\mu_e \neq 0$.

These high temperature processes lead, for the electromagnetic radiation propagating inside the plasma, to a drastic departure from the light cone dispersion equation. That behavior comes from the more complex structure of the photon self-energy in a medium, as compared with that of vacuum. We refer especially to the appearance of the plasmon mass and Debye screening [2, 15, 1]. We have also the effect of temperature radiative corrections to the expression for the thermodynamic potential Ω, in other words, the QED corrections to the Planck formula. This calculation was performed the first time by Akhiezer-Peletminskii [4]. They used in their calculations the one-loop approximation of the polarization operator $\Pi_{\mu\nu}$, (*i.e.*, to order e^2).

In recent years, the so-called correlation term in QED (and QCD) has been calculated again by some authors, as Kapusta [5] and by Leermakers-van der Weert [6]. The importance of clarifying the QED case lies precisely in the fact that it helps to understand better the more involved cases of non-abelian theories. We mention especially the infrared problem in high temperature QCD and the hot electroweak phase transition.

The authors of Ref. [6] argued that the results obtained by Ref. [4] in QED were wrong concerning the correlation term, which must be corrected by some nonvanishing contribution of the transverse modes. The initial arguments of Ref. [6] were motivated by previous claims by Kislinger and Morley [8], concerning the role of the plasmon mass effect in connection to the symmetry restoration by temperature suggested by Kirzhnitz and Linde (see Ref. [9]). But in calculations made afterwards by Toimela [7] in QCD it was shown that the claims made in Ref. [6] were not justified (the arguments of Ref. [7] are equally valid for QED), and that the terms these authors considered as nonzero, actually vanish. In a later review paper by Landsman and van Weert [10], the results of Ref. [7] are only very briefly commented in connection with the QCD (but not QED) problem.

The correlation term was calculated more recently by Pisarski [11] in QCD, leading to conclusions in agreement with that of Ref. [7]. More recently, Parwani and Coriano [12] calculated the thermodynamic potential in QED up to e^5 order. Their results correspond to taking $\Pi_{\mu\nu}$ up to order e^4, and coincide (up to terms e^2, e^3), with those of Akhiezer-Peletminskii.

In the present paper we give alternative arguments in favor of the calculations made by Akhiezer-Peletminskii (the only correction to be made is the insertion of a factor of 3 due probably to a misprint). We consider as essential to point out that the static (infrared) limit to be used in the temperature (sometimes named imaginary-time) formalism is $k_4 = 0$, $\lim \mathbf{k} \to 0$, where $k_4 = 2\pi n\beta$ is the Matsubara frequency. We shall start by recalling some features of the photon spectrum in statistical QED and on the definition of the exchange and correlation terms of the thermodynamic potential.

The point of view of the present authors is that the knowledge of the analytic properties of the photon spectrum in the ω, k plane must be taken essentially into account in the investigation of the radiative corrections to the thermodynamic potential. To that end, we consider a more simple model of a massive frequency-dependent boson field which reproduces the infrared and ultraviolet properties of the transverse and longitudinal high temperature QED modes *at the points corresponding to the discrete Matsubara frequencies.* Such model has different infrared ($\omega = 0$) and high frequency masses and we show that the "correlation" term M^3T is due the infrared mass contribution. We apply that result to the QED case. For the transverse modes, being infrared massless (but high frequency massive) their correlation term vanishes, and for the longitudinal mode, having different (nonvanishing) infrared and high frequency masses, it is the infrared Debye mass which contributes to the M^3T correlation term. Here M^2 is roughly equivalent to the exact non-perturbative mass, and can be taken as corresponding to a sum of loops $\sum_n \Pi^n(0)$ of order e^{2n}. We also discuss some analogy existing between the spectrum of massive charged boson gas at the tree level and the transverse photon spectrum for $\omega \geq 0$. The infrared behavior of the thermodynamic potential shows a remarkable analogy also. Finally we discuss some consequences of the previous results.

2. Photon spectrum and masses in statistical QED

Starting from the properties of the spectrum of the high temperature gas [15] it is possible to draw some insights about the contribution of the transverse and longitudinal modes to the radiative corrections to blackbody thermodynamic potential.

Let us consider the zeros of the photon inverse Green function [2], in statistical QED, which can be obtained from the temperature QED effective action functional as $D^{-1}_{\mu\nu} = \delta^2\Gamma/\delta\bar{A}_\mu\bar{A}_\nu$ (where $\bar{A}_\lambda$ means average fields), and is, in consequence, gauge-invariant. As $\det D^{-1}_{\mu\nu}$ is zero ($\det D_{\mu\nu}$ is singular), we shall add some gauge fixing term to it. We shall see that the spectrum is independent of this gauge fixing term, as it must be. We have

$$D^{-1}_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) k^2 - \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) Q(k) - \Pi^s_{\mu\nu} + \alpha k_\mu k_\nu = 0, \tag{1}$$

where α is the gauge fixing parameter,

$$Q(k) = \frac{e^2}{12\pi^2}(\mathbf{k}^2 + k_4^2)^2 \int_{4m^2}^{\infty} dx\, \frac{\left(1 + \frac{2m^2}{x}\right)\left(1 - \frac{4m^2}{x}\right)^{1/2}}{x(x + \mathbf{k}^2 + k_4^2)} \tag{2}$$

and

$$\Pi^s_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{\mathbf{k}^2}\right) A(\mathbf{k}^2, k_4^2) + \Pi_{44}\frac{k_\mu k_\nu k_4^2}{\mathbf{k}^4}, \tag{3}$$

$$\Pi_{\mu 4} = \Pi_{4\mu} = -\Pi_{44}\frac{k_\mu k_4}{\mathbf{k}^2}, \tag{4}$$

where $\mu, \nu = 1, 2, 3$ in the last equation. The explicit expressions for A and Π^s_{44} at finite temperature and nonvanishing electron chemical potential in the one-loop approximation can be found *e.g.* in Ref. [2]. It is important to state here, however, some perturbative-independent properties,

$$\lim_{\substack{k_4=0,\\ \mathbf{k}\to 0}} A = 0, \qquad \lim_{\substack{k_4=0,\\ \mathbf{k}\to 0}} \Pi_{44} = \lambda_{\mathrm{D}}^{-2},$$

λ_{D} being the Debye length.

To obtain the dispersion equation we must solve

$$\det D^{-1}_{\mu\nu}(\alpha) = 0, \tag{5}$$

or explicitly

$$\alpha k^2 (k^2 - Q - A)^2 \left(k^2 - \frac{k^2 \Pi^s_{44}}{\mathbf{k}^2} - Q\right) = 0. \tag{6}$$

One obtains one massless mode, two transverse and one longitudinal massive modes. It can be shown that when calculating the thermodynamic potential corresponding to these modes, the Faddeev-Popov determinant k^4 not only cancels the term k^2 in Eq. (6), but also introduces a subtractive zero mass mode, to keep the adequate counting of degrees of freedom in the limit $e \to 0$, since in the limit we have only the contribution of the two transverse mass less modes.

In this way we get two independent dispersion equations for the photon: $k^2 = A + Q$, for the two transverse modes and $k^2 = \frac{k^2 \Pi^s_{44}}{\mathbf{k}^2} + Q(k)$, for the longitudinal mode [15], and we observe (see below) that Eq. (6) contains, in addition to the unphysical zero mass mode, the contribution of three massive modes, without violating the gauge invariance of the QED Lagrangian.

We will consider here the high temperature infrared limit of A and Π_{44}. At the one-loop level, the previous infrared limit (2) leads to $A \sim |\mathbf{k}| \to 0$, and the solution to the dispersion equation for the transverse modes is

$$k^2 = 0, \tag{7}$$

whereas in that limit, for the longitudinal mode we have

$$\mathbf{k}^2 = -\lambda_{\mathrm{D}}^{-2}. \tag{8}$$

Thus, $\lambda_{\mathrm{D}}^{-2} = \Pi_{44}(0)$ plays the role of an infrared mass of the longitudinal modes. In the high temperature limit $T \gg m_e$ it is $\lambda_{\mathrm{D}}^{-2} = e^2 T^2/3$.

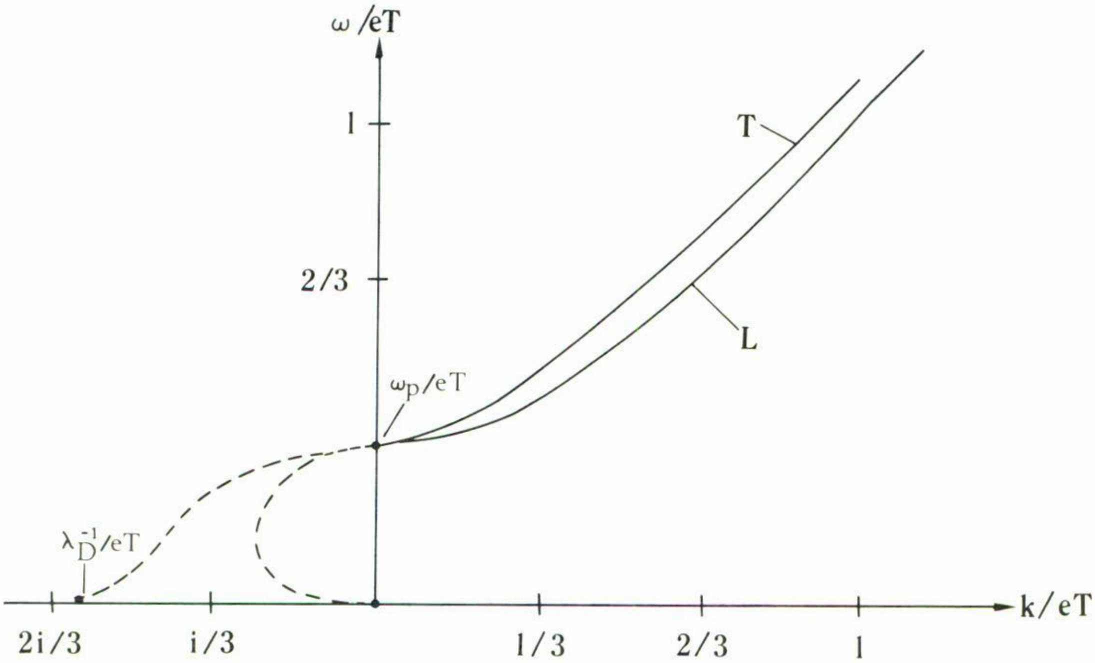

FIGURE 1. Spectra of longitudinal (L) and transverse (T) modes after Weldon [15]. The real k/eT axis is continued to imaginary momenta ik/eT at the left. The infrared (longitudinal) mass, $eT/\sqrt{3}$, is larger than the plasmon mass $M_{\rm p} = eT/3$.

In the limit which we will call ultraviolet (observe that we are dealing with frequencies $\omega \simeq m_e$), $\omega \neq 0$, $|\mathbf{k}| \to 0$, for both transverse and longitudinal modes we obtain as a solution the plasmon mass

$$A(\omega, 0) = -\omega_{\rm p}^2, \tag{9}$$

where $\omega_{\rm p}^2 \simeq e^2T^2/9$ for $T \gg m$.

The detailed study of the behavior of these modes was made on [15]. The approximate picture is given In Fig. 1.

We observe that transverse and longitudinal modes behave as massive at high frequencies, bearing equal plasmon masses. But their behavior at low frequencies is quite different. As indicated by Eq. (7) there is no magnetic screening due to absence of monopoles in QED, whereas (8) means just the existence of Debye screening in the infrared limit.

3. EXCHANGE AND CORRELATION TERMS

Here we reproduce some results of Ref. [4]. The thermodynamic potential of our system of electrons, positrons and photons can be written as

$$\Omega = \Omega_{\rm id} + \Delta\Omega_s + \Delta\Omega_{\rm c}, \tag{10}$$

where Ω_{id} is the thermodynamic potential of the ideal gas of electrons, positrons and photons; the term $\Delta\Omega_s$ is proportional to e^2 and is named the exchange term, and to $\Delta\Omega_{\rm c}$, the correlation term. It contains contributions of higher order, starting from e^3. The expression for Ω_s is

$$\Delta\Omega_s = -\frac{VT}{2(2\pi)^3}\sum_{k_4}\int d^3\,k\frac{1}{k^2}\,\Pi_{\nu\nu}(k), \tag{11}$$

and for $\Omega_{\rm c}$ is

$$\Delta\Omega_{\rm c} = -\frac{VT}{2(2\pi)^3}\sum_{k_4}\int d^3\,k\left[\ln\left(1-\frac{\Pi^s_{44}}{\mathbf{k}^2}\right)+\frac{\Pi^s_{44}}{\mathbf{k}^2}+2\ln\left(1-\frac{A(k_4,\mathbf{k})}{k^2}\right)-\frac{2A(k_4,\mathbf{k})}{k^2}\right]; \tag{12}$$

in the high temperature limit, the result obtained in Ref. [4] for the exchange term was

$$\Delta\Omega_s = \frac{Ve^2}{32}\left[\left(\frac{\partial n}{\partial\mu}\right)^2+\frac{4T^2}{3}\frac{\partial n}{\partial\mu}\right], \tag{13}$$

where

$$n = \frac{1}{(2\pi)^3}\int d^3\,p(n_e-n_p),$$

and the electron-positron distributions are

$$n_{e,p} = [\exp(\epsilon_p \mp \mu)/T+1]^{-1}.$$

The expression for the correlation term was

$$\Delta\Omega_{\rm c} = -\frac{Ve^3T}{12\pi}\left(\frac{\partial n}{\partial\mu}\right)^{3/2}. \tag{14}$$

Here $e^2\frac{\partial n}{\partial\mu} = -\lambda_{\rm D}^{-2}$, $\lambda_{\rm D}$ is the Debye length, which as pointed out previously, in the high T limit is $\lambda_{\rm D}^{-2} = \frac{e^2T^2}{3}$. Thus, in that limit we have

$$\Delta\Omega_s = \frac{5e^2T^4V}{288}, \quad \text{and} \quad \Delta\Omega_{\rm c} = \frac{e^3T^4V}{36\sqrt{3}\,\pi}.$$

The expression for the total thermodynamic potential of electrons, positrons and photons can be written as

$$\Omega = -\frac{11\pi^2T^4}{180}+\frac{5e^2T^4}{288}+\frac{e^3T^4}{36\sqrt{3}\,\pi}, \tag{15}$$

from which the energy can be found from $U = TS + \Omega$, the entropy being $S = -(\partial\Omega/\partial T)_{V=\text{const}}$. In this way we obtain an expression in which the radiative corrections for the energy must be three times the value reported by Ref. [4] (due probably to a misprint).

In Eq. (15), the coupling constant is to be considered as an implicit function of the renormalization scale adopted, let us name it λ. If we include the next order loops, terms of order e^4T^4 and $O(e^4 \ln T/\lambda)$ appear. We can choose $\lambda = T$, eliminating thus any logarithmic dependence on T [12], or either use the perturbative renormalization group invariant coupling constant approximately given by $e^2(T) = e^2[1 + \frac{e^2}{6\pi^2}\ln(T/\lambda)]$. Thus, at the scale of temperatures we are working we may write $\Omega = \Omega(e^2(T), T)$.

At lower temperatures as compared with the electron mass m_e, we usually ignore such dependence of the coupling constant on temperature. We must also subtract from the ideal gas term in the thermodynamic potential some terms dependent on the electron mass which are positive and tend to reduce the entropy, like $m_e^2T^2/12$.

4. The one-loop thermodynamic potential of scalar particles

We now turn to writing alternative expressions for the thermodynamic potential Ω of a massive scalar field, keeping in mind to use the results later for several purposes (*i.e.*, Planck's formula can be obtained by taking the mass $M = 0$, and multiplying by 2, the number of transverse modes). By multiplying by 3 we get the thermodynamic potential of a vector massive gas; by using some modified spectrum (see below), we may construct an approximate model for the boson part of the very hot QED plasma.

The free-particle spectrum $\varepsilon_k = (\mathbf{k}^2 + M^2)^{1/2}$ is given by the poles of the propagator $G^{-1} = 0$, *i.e.*, $k_4 = \pm i\varepsilon_k$.

The field near the massive scalar particles is screened, and this screening is described by the pole $k_4 = 0$, in other words, by $k = \pm iM$, giving the Yukawa force $F \sim e^{-M_r}/r$ (the analog of the Debye screening in the QED case). It is important to stress here the role of this $k_4 = 0$ pole in determining this infrared behavior (see Fig. 2a).

The one-loop thermodynamic potential can be obtained from the propagator $G(x,y)$ [2, 14] as $\Omega = \operatorname{tr}\ln g$ (here $\operatorname{tr} A = (4\pi^3\beta)^{-1}\int d^3p\, A$), or

$$\Omega = -\frac{T}{2(2\pi)^3}\sum_n \int d^3k\, \ln(k_4^2 + \mathbf{k}^2 + M^2), \tag{16}$$

where $k_4 = 2\pi nT$, and $n \in Z$,

Usually Eq. (16) is evaluated by differentiating with respect to M^2, summing over n, and integrating back over M^2. The result is the well-known expression

$$\Omega = \frac{T}{2(2\pi)^3}\int d^3k\, \ln(1 - e^{-\mathcal{E}_k/T}) + \frac{1}{(2\pi)^3}\int d^3k\, \mathcal{E}_k. \tag{17}$$

We can write Eq. (17) as $\Omega = \Omega_p + \Omega_0$ where Ω_p is a temperature dependent term, which is finite, and Ω_0 is a divergent term, which is independent from the temperature

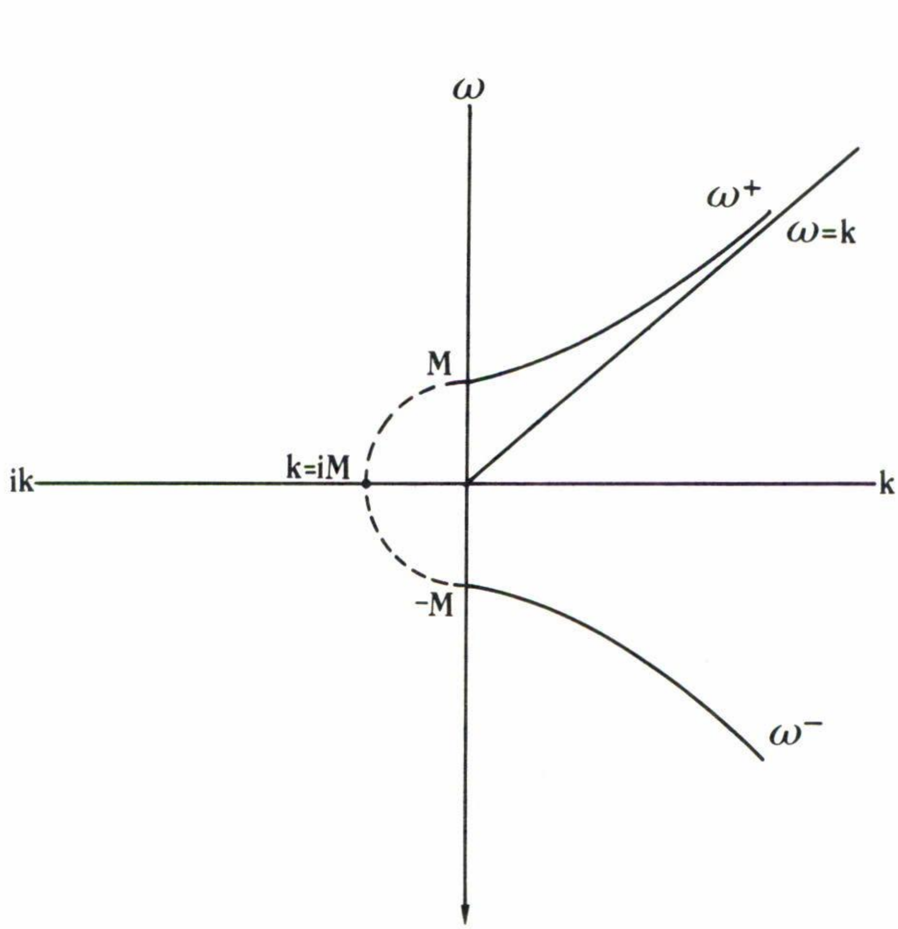

FIGURE 2. a) Usual spectra of free Bose particles have equal infrared and ultraviolet masses for particles and antiparticles.

and comes from the infinite vacuum energy. By expanding the logarithm in series and using the integral representation

$$e^{-s^{1/2}/T} = \frac{1}{2\pi^{1/2}T}\int_0^\infty \frac{d\nu}{\nu^{3/2}}\, e^{-s\nu-1/4\nu T^2},$$

we can write Ω_p as

$$\Omega_p = -\frac{1}{8\pi^2}\int_0^\infty \frac{d\nu}{\nu^3}\, e^{-M^2\nu-n^2/4\nu T^2}. \tag{18}$$

From (18) we can get the asymptotic high temperature expansion of Ω_p [14]:

$$\begin{aligned}\Omega_p = &-\frac{\pi^2T^4}{90}+\frac{M^2T^2}{24}-\frac{M^3T}{12\pi}\\ &-\frac{M^4\ln M^2/T^2}{64\pi}-\frac{cM^4}{64\pi^2}+O(M^6/T^2),\end{aligned} \tag{19}$$

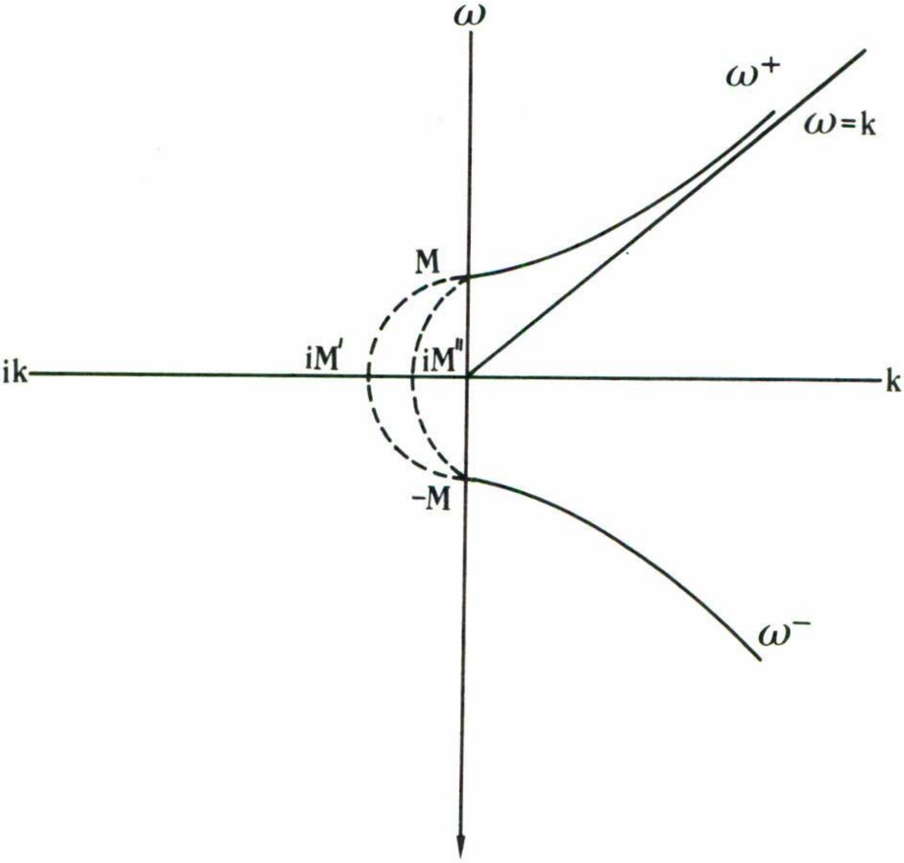

FIGURE 2. b) In our modified scalar spectra, the infrared mass M' or M'', responsible for the screening, differs from the ultraviolet masses M.

where $c = 2\ln 4\pi + 3/2 - 2\gamma$. By comparing Eqs. (16) and (18) we conclude that the term $M^3T/12\pi$, comes just from the term $n = 0$ in Eq. (16). (This point can be better understood if in Eq. (18) we use the integral representation

$$\ln A = -\int_0^\infty \frac{d\nu}{\nu}\,(e^{-A\nu} - e^{-\nu}),$$

and introduce adequate regularizing parameters in the integrals over d^3k of the vacuum terms. We omit these details here.)

It is convenient for later purposes to write also from Ref. [14] the corresponding expression for a gas of massive fermions plus antifermions:

$$\Omega_p = -\frac{7\pi^2T^4}{180} + \frac{M^2T^2}{12} - \frac{M^4\ln M^2/T^2}{16\pi} - \frac{c'M^4}{16\pi^2} + O(M^6/T^2), \tag{20}$$

where $c' = 2\ln\pi + 3/2 - 2\gamma$.

5. THERMODYNAMIC POTENTIAL OF SCALAR PARTICLES WITH MODIFIED SPECTRUM

For the purpose of making a simpler and very approximate model for describing the behavior of the transverse and longitudinal electromagnetic modes in our plasma, we will consider in this section the evaluation of the thermodynamic potential of scalar particles with the modified spectrum $\mathcal{E}_k = (k^2 + M^2(\omega))^{1/2}$, in which we have different infrared and ultraviolet masses, defined by

$$M(\omega) = \begin{cases} M, & \text{for } \omega \geq M, \\ \eta M, & \text{for } 0 \leq \omega < M, \end{cases} \tag{21}$$

that is, we have different analytical expressions for the spectrum in the region of positive and negative k^2. Here we keep in mind the correspondence $k_4 = i\omega$, *i.e.*, we are considering at the same time the temperature problem with discrete Matsubara frequencies $k_4 = 2\pi n$, and its analytic continuation to the quantum-kinetical case with real and continuous frequency ω.

We shall write the thermodynamic potential as the sum of three terms:

$$\Omega = \Omega(0) + \Omega_{\text{ex}} + \Omega_{\text{cor}},$$

where $\Omega(0)$ is the $M = 0$ limit of Ω_p,

$$\Omega_{\text{ex}} = -\frac{T}{4\pi} \sum_n \int_0^\infty k^2\, dk\, \frac{M^2(k_4)}{k_4^2 + k^2}$$

and

$$\begin{aligned} \Omega_{\text{cor}} = &-\frac{T}{4\pi^2} \sum_n \int_0^\infty k^2\, dk \left[\ln(k_4^2 + k^2 + M^2) - \frac{M^2}{k_4^2 + k^2} \right] \\ &+ \frac{T}{4\pi^2} \int_0^\infty k^2\, dk \left[\ln(k^2 + M^2) - \frac{M^2}{k^2} \right] \\ &- \frac{T}{4\pi^2} \int_0^\infty k^2\, dk \left[\ln(k^2 + \eta M^2) - \frac{\eta M^2}{k^2} \right]. \end{aligned} \tag{22}$$

These integrals are finite and can be evaluated exactly and after doing it, we get finally that the terms $M^3/12\pi$ and $-M^3/12\pi$ cancel and in the high temperature limit $T \gg M$, we get

$$\Omega_{\text{cor}} = -\frac{\eta^{3/2} M^3 T}{12\pi} - \frac{M^4 \ln M^2/T^2}{64\pi} + O(M^2/T^2). \tag{23}$$

We will use this expression for the calculation of the correlation energy of a massive vector field which models the blackbody radiation in the high temperature limit.

6. THE CORRELATION ENERGY OF OUR APPROXIMATE MODEL

We can now take the high temperature spectrum of the electromagnetic radiation as represented approximately by a massive vector field with frequency-dependent mass as described by Eq. (21) (see Fig. 2b), if we take, for the two transverse modes the plasmon (ultraviolet) mass $M^2 = e^2T^2/9, \eta \to 0$, which leads to an infrared mass $\eta M = 0$, and for the longitudinal mode also $M^2 = e^2T^2/9$ and $\eta = 3$, which gives and infrared mass $\eta M = eT/\sqrt{3}$, e being the electromagnetic coupling constant. (We must remark at this point that the Matsubara sum picks from just the value at $k_4 = 0$ ($\omega = 0$); the next value being at $k_4 = 2\pi T \gg eT/3$. Thus, what is essential for the correlation term is the value of $M(k_4)$ at $k_4 = 0$ in calculating the pole at $k_4 = \pm i(\mathbf{k}^2 + M^2(k_4))^{1/2}$. Any other functional dependence we choose for $M(\omega)$, for $\omega \simeq 0$, leading to the same value at $\omega = 0$, would give the same contribution to the correlation term.) We have then as the only contribution of order e^3, that coming from the longitudinal mode, *i.e.*,

$$\Omega_{\text{cor}} = \frac{e^3T^4}{36\sqrt{3}\,\pi} + \cdots . \tag{24}$$

This corresponds obviously to what is expected physically: the M^3T term in the expansion of the thermodynamic potential comes just from the infrared mass. (The M^2T^2 term in the exchange part of Ω comes from the UV mass). If higher loops of $\Pi_{\mu\nu}$ were considered, we would have to make the correspondence $M^2 = (c_1e^2 + c_2e^4 + \cdots)T^2$, leading to $M^3 = (c_1'e^3 + c_2e^5 + \cdots)T^2$, all odd powers of e corresponding to corrections to the correlation energy.

The behavior of the (vanishing) transverse infrared mass is consistent with the existence of constant magnetic fields inside the plasma, which are just represented by massless infrared transverse photons. (We excluded monopoles or superconductive effects, which would lead to screening of magnetic fields.)

As an additional argument, we want to point out that if we consider the plasmon as a particle with three degrees of freedom and mass $eT/3$, according to the arguments of Ref. [6], it would be more consistent to expect a contribution to the correlation term of order

$$\Delta\Omega_{\text{c}} = -\frac{3VT}{12\pi}M_p^3 = -\frac{3VT}{12\pi}\frac{e^3T^3}{27} = -\frac{e^3T^4}{108\pi}. \tag{25}$$

But even this quantity is *smaller* than the contribution of the longitudinal mode:

$$\frac{e^3T^4}{108\pi} < \frac{e^3T^4}{36\sqrt{3}\,\pi}; \tag{26}$$

as $\Omega = -PV$, the longitudinal mode contributes to the pressure in an amount greater than a gas of particles with mass M whose exact dispersion law were $\omega^2 = M^2 + \mathbf{k}^2$.

We would like to point out in concluding this section that one can estimate also the exchange contribution as coming from the temperature mass correction due to the three

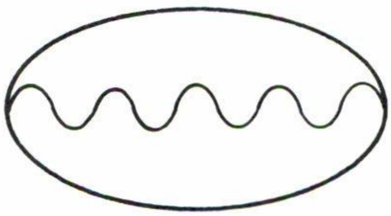

FIGURE 3. Ring diagram of the thermodynamic potential corresponding to the e^2 approximation of the polarization operator $\Pi_{\mu\nu}$.

electromagnetic modes of temperature mass $M_p = eT/3$, plus the two fermion modes, of temperature mass $M_f = eT/\sqrt{8}$ (according to Ref. [19]). This would lead to a term $7e^2T^4/288$, which differs in an amount $e^2T^4/144$ from the value obtained in Ref. [4]. That difference obviously stems from the fact that when one consider the radiative corrections to the gas of photons, electrons and positrons, the electron-positron and photon contributions are not strictly additive, since the interaction is nonlinear; it can be represented (to order e^2) as a ring diagram (Fig. 3) containing both the photon and electron self-energy contributions. The correlation term, which comes from the Matsubara zero frequency, is, however, due only to the photon contribution.

7. DISPERSION CURVE FOR CHARGED MASSIVE RELATIVISTIC BOSON GAS

Let us consider the spectrum of a relativistic charged scalar gas

$$k_4 = \begin{cases} i(\epsilon_p + \mu) & \text{for the particles,} \\ -i(\epsilon_p - \mu) & \text{for the antiparticles,} \end{cases} \tag{27}$$

where μ is the chemical potential.

The dispersion curve $\omega - \mu = \pm\epsilon_p$ in the plane $\omega, \sqrt{\mathbf{k}^2}$ has the form indicated in Fig. 4a.

The upper branch corresponds to the spectrum of particles $\omega^+ = \epsilon_p + \mu$, and the lower, to the antiparticles $\omega^- = -(\epsilon_p - \mu)$. We observe some analogous behavior of the dispersion curve of the present case and that of transverse modes in statistical QED, in the region $\omega \geq 0$.

The transverse mode spectrum in the region $|\mathbf{k}|^2 \leq 0$ closely resembles this picture, as coming from the contribution of the plasmonic branch, whose dispersion law in that region correspond to "particles" of mass $m = M_p/2$ and chemical potential $\mu = m$, and the photonic branch, with spectrum close to $(\mathbf{k}^2 + m^2)^{1/2} - m$, with $\mathbf{k}^2 < 0$ in such region; in other words, near $\omega = 0$, $\mathbf{k}^2 = 0$ we may interpret that there is a condensate of static transverse photons: this means that the equilibrium plasma is compatible with the arising of small constant magnetic fields. The system is even compatible with strong magnetic fields, as theory [18] and astrophysical observations indicate, but the calculations must be modified to include the field intensity from the very beginning.

Let us consider the thermodynamic potential of the relativistic charged scalar gas

$$\Omega = -\frac{T}{8\pi^3}\sum_{p_4}\int d^3p \ln(1 - e^{-(\epsilon_p-\mu)/T})(1 - e^{-(\epsilon_p+\mu)/T}) \tag{28}$$

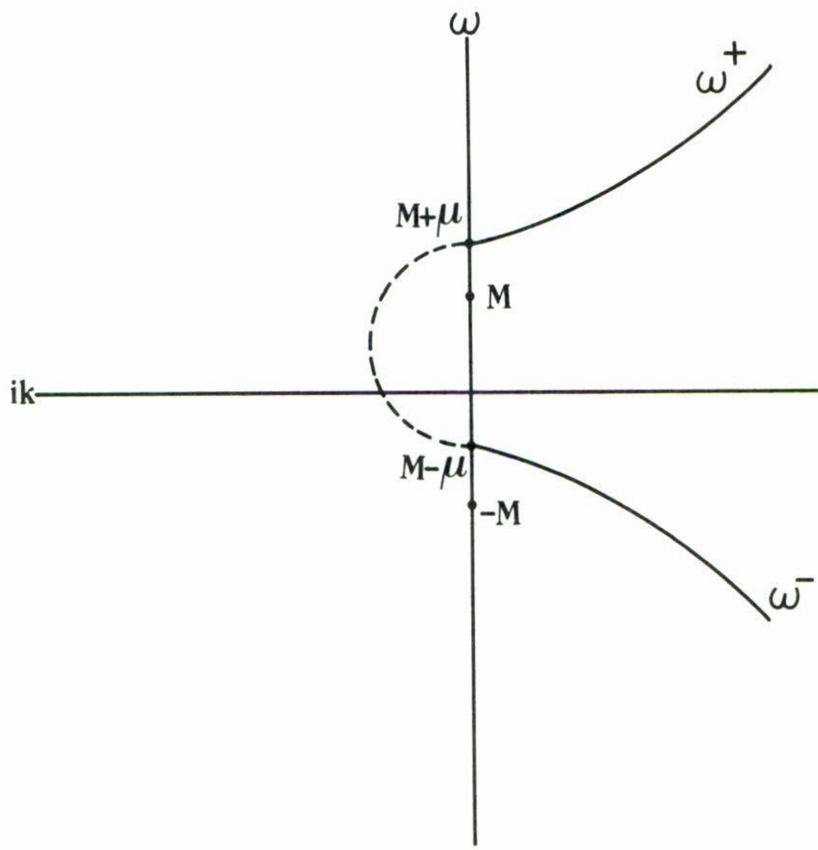

FIGURE 4. a) The effective spectra for a charged scalar free gas indicates different ultraviolet and infrared behavior.

This thermodynamic potential contains both branches of the spectrum.

The particle density is given by $N = -\partial\Omega/\partial\mu$

$$N = \frac{1}{8\pi^3}\int d^3p\left(\frac{1}{e^{(\epsilon_p-\mu)/T}-1} - \frac{1}{e^{(\epsilon_p+\mu)/T}-1}\right), \tag{29}$$

when $\mu \to M$, the ground state density of antiparticles tends to ∞, that is, we have Bose-Einstein condensation (see Fig. 4b). In the dispersion curve for antiparticles this corresponds with the intersection with the axis $\omega = 0$ for $|\mathbf{k}| \to 0$.

In the asymptotically high temperature limit, the thermodynamic potential (28) has been calculated in Refs. [16] and [17] as

$$\begin{aligned}\Omega_{\mathrm{B}} = {} & \frac{(2\ln 4\pi + \frac{3}{2} - 2\gamma)m^4}{32\pi^2} - \frac{m^4 \ln m^2/T^2}{32\pi^2} - \frac{(m^2-\mu^2)^{3/2}\,T}{6\pi} \\ & - \frac{(2\mu^2 - m^2)T^2}{12} - \frac{(\mu^2 - 3m^2)\mu^2}{24\pi^2} - \frac{\pi^2 T^4}{46} + O(m^6/T^2).\end{aligned} \tag{30}$$

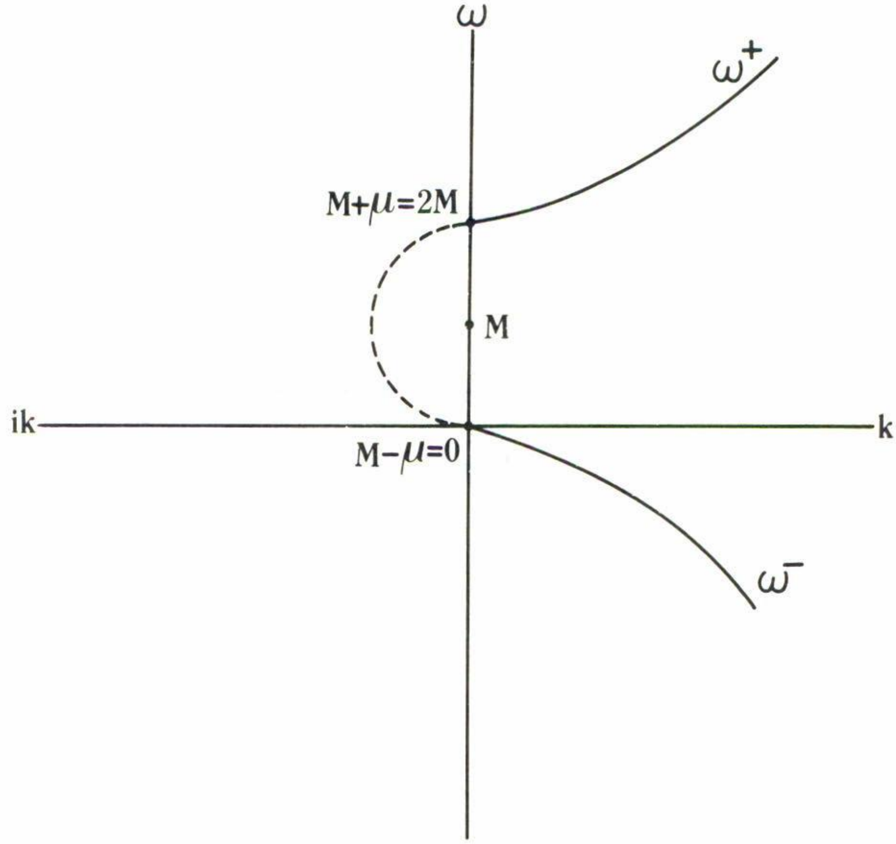

FIGURE 4. b) In the case of antiparticle condensation, their 'effective' infrared mass $M-\mu$ vanishes, leading to a zero pressure contribution to the correlation term.

We observe that the third term corresponds precisely to the correlation term $(-m^3T/6\pi)$ in the limit $\mu \rightarrow 0$. Here $\sqrt{m^2-\mu^2} = \sqrt{(m+\mu)(m-\mu)}$ is the zero momentum limit of the geometric average of the positive end negative frequencies $\lim_{|\mathbf{k}|\rightarrow 0}(-\omega^+\omega^-)^{1/2}$. If $\mu \rightarrow m$ condensation limit of antiparticles, this infrared term tends to zero, and does not contribute to the gas pressure. This is quite reasonable to understand if we interpret $\epsilon_p \pm \mu$ as some effective energy. At zero momentum the effective energy of particles and antiparticles are $\lim_{p\rightarrow 0}(\mu \pm \epsilon_p) = \mu \pm m$, and for $\mu \rightarrow m$, only the antiparticles condensate in the ground state of zero effective energy. The fact that the "correlation" term here tends to zero as $\mu \rightarrow m$ is obviously the physical manifestation of the Bose-Einstein condensation: there is no contribution to pressure from the particles in the ground state. It is not difficult to understand that the analog behavior must be physically expected to occur for the transverse modes: its infrared behavior is compatible just with such a condensate: small constant magnetic fields. It must be emphasized, however, that if condensation occurs, both particles and antiparticles contribute to all other terms in the high temperature expansion of Ω, and changes the sign of the "exchange" term, once for $\mu = m$ it results as $\Delta\Omega = (-m^2T^2/12)$.

8. CONCLUSIONS

We have discussed our reasons for agreement with the results obtained by Akhiezer-Peletminskii [4], (and even recently by Parwani and Coriano [12]), and disagreement with those given by Leermakers-van der Weert [6] for the correlation term in the high temperature radiative corrections to the thermodynamic potential in statistical QED.

Our main argument is that such term comes essentially due to the infrared behavior of the poles of the temperature boson Green function. In the case of the photons, as well as in the case of charged scalar particles, the contribution to the correlation term (understood as the term equivalent to M^3T) is determined by the pole $k_4 = 0$, $\lim |\mathbf{k}| \to 0$. In temperature QED, this means the compatibility of the system with small constant magnetic fields, which suggests the interpretation of a constant magnetic field as a sort of "condensate" (but *not* Bose-Einstein condensate) of transverse photons.

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