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Surface polaritons coupled to charge carriers in anisotropic media

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ABSTRACT. The dispersion relation of surface polaritons in the system anisotropic dielectric–metallic film–vacuum is obtained and the two dimensional limit is discussed. Numerical calculations for α–quartz and films of different thickness and conductivity are presented. The occurrence and behaviour of stop modes is discussed. Stop below modes, previously reported in superlattices, are shown to appear whenever both anisotropy and charge carriers are present.

RESUMEN. Se obtiene la relación de dispersión de los polaritones superficiales que aparecen en el sistema dieléctrico anisotrópico–capa metálica–vacío y se discute el límite bidimensional. Se presentan cálculos numéricos para α–cuarzo y capas de diferente grosor y conductividad. Se muestra que los modos *stop below*, reportados anteriormente en superredes, aparecen en presencia de anisotropía y portadores de carga a la vez.

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1. INTRODUCTION

Surface polaritons coupled to charge carriers (SWC) were first studied by Nakayama [1], who considered a sheet of charge carriers located at the boundary between two dielectric media. In recent years, a special interest in the propagation of surface waves (SW) at boundaries involving superlattices has appeared. Surface polaritons dispersion relations as a function of filling fraction for semi-infinite metal/insulator and metal/metal superlattices have been studied by Apell and Hundery [2]. Two types of stop modes arise: stop below, begining at some $k > k_{\rm b}$ (SB), and stop above, ending at some $k_{\rm a} < \infty$ (SA). The characteristics and origin of SA modes or so called photon-induced excitation surface polaritons are explained in [3] for anisotropic dielectrics. They were discussed first in Ref. [4] for the SW propagating on the surface of α–quartz crystal.

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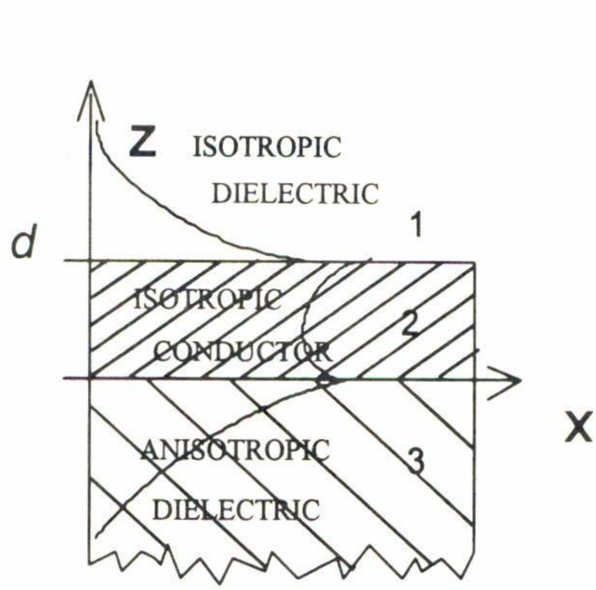

FIGURE 1. An isotropic conductor of finite width d sandwiched between two dielectrics, one of them anisotropic. Evanescent electric fields are schematically indicated by curved lines.

In the present work, SWC in anisotropic media are studied. For this purpose, the model of an isotropic conductor of finite width sandwiched between two dielectrics, one of them anisotropic (Fig. 1), is considered.

2. DISPERSION RELATION

Consider electromagnetic waves with TH-polarization, that is, with electric fields lying on the plane of incidence (or plane xz of Fig. 1).They are of the general form

$$\begin{aligned} \mathbf{E}_1 &= \mathbf{E}_{10} e^{i(kx-\omega t)-\alpha_1(z-d)}, \\ \mathbf{E}_2 &= (\mathbf{E}_{21} e^{-\alpha_2 z} + \mathbf{E}_{22} e^{\alpha_2 z}) e^{i(kx-\omega t)}, \\ \mathbf{E}_3 &= \mathbf{E}_{30} e^{i(kx-\omega t)+\alpha_3 z}. \end{aligned}$$

Here $k = k(\omega)$ is the wave vector and $\alpha_i = \alpha_i(\omega)$ the attenuation constants of the wave with frequency ω propagating in medium i ($i = 1, 2, 3$). Supposing the principal axes of the anisotropic dielectric in the same direction as the coordinate axes, the dispersion relation obtained for TH modes after applying the usual electromagnetic boundary conditions is

$$e^{-2\alpha_2 d}\left(1 - \frac{\alpha_1\epsilon_2}{\alpha_2\epsilon_1}\right)\left(1 - \frac{\alpha_3\epsilon_2}{\alpha_2\epsilon_{x3}}\right) = \left(1 + \frac{\alpha_3\epsilon_2}{\alpha_2\epsilon_{x3}}\right)\left(1 + \frac{\alpha_1\epsilon_2}{\alpha_2\epsilon_1}\right), \tag{1}$$

where the attenuation constants are given by

$$\alpha_1 = \sqrt{k^2 - \frac{\omega^2}{c^2}\epsilon_1}, \qquad \alpha_2 = \sqrt{k^2 - \frac{\omega^2}{c^2}\epsilon_2}, \qquad \alpha_3 = \sqrt{\frac{\epsilon_{x3}}{\epsilon_{z3}}k^2 - \frac{\omega^2}{c^2}\epsilon_{x3}}.$$

In Eq. (1) ϵ_1 is the dielectric function of medium 1, ϵ_2 describes the isotropic conductor, ϵ_{x3}, ϵ_{z3} are the principal components of the dielectric tensor of medium 3 in the x and z directions and d is the width of the conductor film. These TH modes only exist if all the functions ϵ_1, ϵ_2, ϵ_{x3} do not have the same sign. Moreover, surface modes arise if ϵ_{x3} and ϵ_{z3} have the same sign or $\epsilon_{x3} < 0$ and $\epsilon_{23} > 1$. There are not TE modes in this configuration, that is, there are not modes with electric fields perpendicular to the plane of incidence.

Surface polaritons in medium 3 disappear when α_3 is zero (they become bulk excitations) or α_3 is infinite (the wave does not penetrate medium 3). In the first case stop points (SA) appear at $k = \frac{\omega}{c}\sqrt{\epsilon_{z3}}$ or $\omega = \omega_{\mathrm{L}x}$. The second situation occurs at $\omega = \omega_{\mathrm{L}x}$ (SB) or $\omega = \omega_{\mathrm{T}x}$ (SA).

Equation (1) contains the dispersion relation for three adjacent isotropic dielectrics [5] and reduces to the cases of two media models in the limits $d = 0$ and $d = \infty$. In the limit $d \to 0$, $\sigma d \to \sigma^{\mathrm{s}}$, Eq. (1) becomes

$$\frac{\epsilon_l}{\alpha_l} + \epsilon_{x3}\frac{1}{\alpha_3} - \frac{4\pi\sigma^{\mathrm{s}}}{i\omega} = 0. \tag{2}$$

This is a generalization of the dispersion relation obtained in Ref. [1] to the case when medium 3 is anisotropic. All surface modes described by (2) are stop-like.

3. VACUUM–SILVER–α-QUARTZ SYSTEM

Consider that medium 1 is vacuum, the isotropic conductor is a silver film and the anisotropic dielectric is α–quartz, with the **c** axis perpendicular to the boundary surface. The Drude like function with $\omega_p = 1.367 \times 10^{16}$ rad/s [6]

$$\epsilon_2(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

is taken for the silver layer. In α–quartz the dielectric function is a superposition of optic modes:

$$\epsilon_3(\omega) = \epsilon(\infty) + \sum_i \frac{S_i\omega_{\mathrm{T}i}^2}{\omega_{\mathrm{T}i}^2 - \omega^2}.$$

Using the data of Ref. [7] the dispersion curves shown in Fig. 2 are obtained. The curve labeled sw refers to the SW propagating along the boundary between vacuum and a quartz crystal, the name swc (alone) refers to the SWC in vacuum–silver–α-quartz system using Eq. (2) and taking $d = 10^{-8}$ cm to evaluate σ^{s}. The remaining curves are obtained for $d = 10^{-8}$ cm, $d = 10^{-7}$ cm and $d = 10^{-6}$ cm using Eq. (1).

As d and σ^{s} increase the dispersion relations move to the left as predicted by Nakayama for isotropic crystals. The new result is the existence of both SA and SB modes in systems involving anisotropy and coupling to charge carriers. The SA modes have their stop points at $\omega_{\mathrm{L}x} = 2.361 \times 10^{14}$ rad/s, where $\epsilon_x = 0$ and $\alpha_3 = 0$. At higher frequencies α_3 is pure

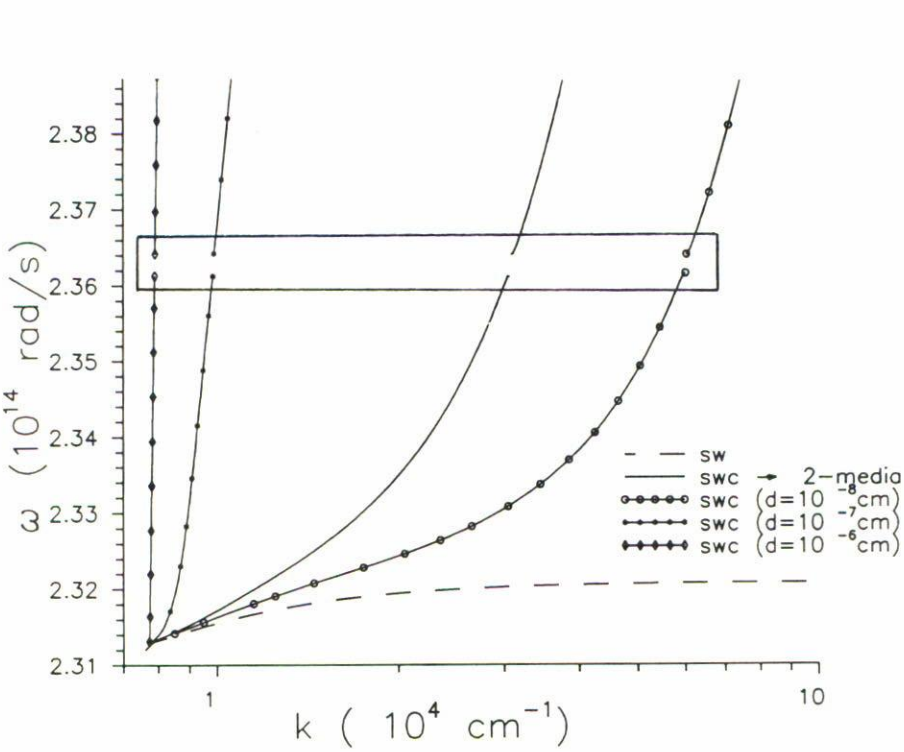

FIGURE 2. Dispersion curves in vacuum-silver-α-quartz system. Notice the logarithmic scale in k. The gap region between the SA and SB is indicated by a rectangle.

imaginary and the SW becomes a bulk polariton. At $\omega_{\mathrm{L}x} = 2.364 \times 10^{14}$ rad/s, where $\epsilon_{z3} = 0$ and $\alpha_3 = \infty$, a new (SB) mode with real α_3 starts.

Note that the dispersion curve obtained from (2) is far from the one obtained from Eq. (1) with $d = 10^{-8}$ cm, even though d is so small that the condition $d \ll 1/\alpha_i$ is satisfied [1]. This means that to study SWC in real systems it is necessary to consider a three media model instead of Nakayama's. However the last one is useful to describe qualitatively the general features of the dispersion relations.

4. VACCUM-METAL-α-QUARTZ SYSTEMS

Consider now instead of a silver sheet, a series of different metals with different widths, taken in such a way that the surface conductivity $\sigma^{\mathrm{s}} = \sigma d$ and $\omega_{ps} = \omega_p d^{1/2}$ remains constant. This means that if we increase d, then the conductivity of medium 2 decreases.

Figure 3 ilustrates the general features of the modes obtained form Eq. (1) by taking $\omega_{ps} = 10^{11}$ rad/s cm$^{1/2}$ in the frequency region above $\omega_{\mathrm{T}x} = 2.313 \times 10^{14}$ rad/s. The symbol sw refers to the same curve as in Fig. 2. The symbol swc refers to Eq. (2) taking $\sigma^{\mathrm{s}} = i\omega_{ps}^2/\omega$. The remaining curves are obtained from (1) for $d = 10^{-8}$ cm, $d = 9\times10^{-8}$ cm, $d = 10^{-7}$ cm, $d = 1.5 \times 10^{-7}$ cm, $d = 2 \times 10^{-7}$ cm and $d = 10^{-6}$ cm.

In a semi-infinite metal, plasmon-like polaritons propagate only for $\omega > \omega_p$ (bulk modes) or $\omega < \omega_p/\sqrt{2}$ (surface modes). For a metal sheet of width d, and electromagnetic exci-

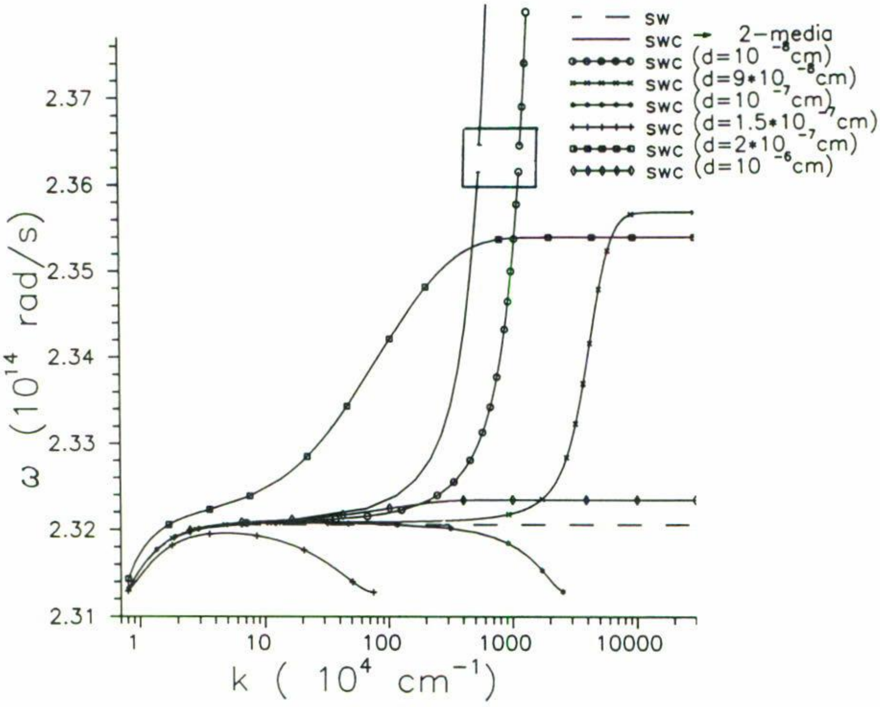

FIGURE 3. Dispersion curves in vacuum-metal-α-quartz system. Notice the logarithmic scale in k. The gap region between the SA and SB is indicated by a rectangle.

tation with frequency ω in the "forbidden" interval $\omega_p/\sqrt{2} < \omega < \omega_p$ can "tunnel" from vacuum to quartz only if $kd < 1$.

For $d = 10^{-8}$ cm, $\omega_p/\sqrt{2} = 7.071 \times 10^{14}$ s^{-1} and surface waves propagate below this frequency with stop points at $\omega_{\mathrm{L}z} = 2.364 \times 10^{14}$ rad/s (SB) and $\omega_{\mathrm{L}x} = 2.361 \times 10^{14}$ rad/s (SA). As d increases to 9×10^{-8} cm, $\omega_p/\sqrt{2}$ moves below $\omega_{\mathrm{L}x}$, no stop points are found in the interval $\omega_p/\sqrt{2} > \omega > \omega_{\mathrm{T}x}$ and the mode is real. For $d = 10^{-7}$ cm and 1.5×10^{-7} cm, $\omega_p/\sqrt{2} < \omega_{\mathrm{T}x}$ so that in the forbidden region $\omega_p/\sqrt{2} > \omega > \omega_{\mathrm{T}x}$ one has SA modes with $kd < 1$ and anomalous dispersion. When $d = 2 \times 10^{-7}$ cm and $d = 10^{-6}$ cm, $\omega_p < \omega_{\mathrm{T}x}$ and real modes approaching the dispersion law of surface polaritons in the vacuum–α-quartz system propagate for $\omega > \omega_{\mathrm{T}x}$.

The same picture is obtained for modes at frequencies below $\omega_{\mathrm{T}x}$. In the limit $d \to \infty$ the dispersion curves from Eq. (1) approach to those of the vacuum–α-quartz interface. Then both real and SA modes of the system vacuum–metal–α-quartz turn to real and SA modes of the two media system.

5. CONCLUSIONS

The dispersion relation for surface polaritons propagating in the three-media system anisotropic dielectric–isotropic conductor–vacuum has been obtained and the existence of the corresponding SA and SB modes has been shown.

As pointed out by Borstel and Falge [3], SA modes can only exist in systems involving some anisotropic media. On the other hand, Nakayama's results [1] show that coupling to charge carriers does not give raise to Stop modes. From the present work it becomes clear that the existence of SB modes requires both anisotropy and a medium with negative dielectric function (as a metal for $\omega < \omega_p$). More precisely, SB modes exist only when $\omega_p/\sqrt{2} > \omega_{Lz}$ and disappear as the sandwiched medium becomes less conductive. Metal superlattices, where SB modes were found by Apell and Hundery [2], are particularly interesting systems for our theory since they have conduction layers and are anisotropic.

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