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Density fluctuations during the discharge of granular material from a two-dimensional bin

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ABSTRACT. We have found experimentally the existence of strong density fluctuations (density waves) during the discharge of monodisperse granular material from vertical flat bins. These density waves follow, approximately, a $1/f^\alpha$ fractal noise, where $\alpha = 1.32$, just above the exit of the bin and, a $1/f$ noise, just below the exit. The corresponding rescaled range analysis (Mandelbrot-Van Ness fractional Brownian motion) for the fractal noise gives a Hurst parameter $H = 0.16$ and a fractal dimension $D = 1.84$.

RESUMEN. Encontramos experimentalmente la existencia de grandes fluctuaciones en la densidad (ondas de densidad) durante la descarga de material granular monodisperso desde cajas verticales planas. Estas ondas de densidad siguen, aproximadamente, ruido fractal $1/f^\alpha$, donde $\alpha = 1.32$, justo arriba de la salida de la caja y un ruido $1/f$, justo abajo de la salida. El correspondiente análisis de intervalo reescalado (movimiento Browniano fraccional de Mandelbrot-Van Ness) para el ruido fractal da un parámetro de Hurst $H = 0.16$ y una dimensión fractal $D = 1.84$.

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1. INTRODUCTION

In the study of granular material, like sand, fluctuating phenomena have been recently noted under several conditions of composition and motion. A special class of these phenomena seems to contain the $1/f^\alpha$ noise which is a signal of random behavior as well as of lack of characteristic time scales [1–11]. Phenomena obeying noise with these characteristics have been detected during the surface flow in small sandpiles [1], in the sound propagation inside a granular material [2,3], in the flow through long narrow pipes [4,10], in the stresses on the side walls during the outflow in a three-dimensional (3D) conical hopper [11] and during the shear flow in narrow gaps between rough walls [12,13], among others.

Perhaps the main possible cause of these noisy behaviors can be the inherent internal disorder of the granular material which can respond with strong fluctuations to small induced changes. Experiments show that the flow under gravity of noncohesive granular material from flat hoppers do not flow uniformly but rather form density waves or time-dependent density patterns [14–16]; however, to our knowledge the corresponding experimental time scale analysis of the density waves has not been previously reported.

There is some evidence [16–18] that the existence of density waves can be the cause of some flow problems in hoppers, bins and silos and therefore, attempts to understand the origin and nature of density waves are relevant. Recently, Ristow *et al.* [19], using molecular dynamics simulations (MDS), have characterized the temporal fluctuations associated to the density waves in 2D hoppers as obeying a $1/f^{\alpha}$ noise, where $\alpha = 2.7$. In the present work we report experiments in nearly 2D vertical bins (flat bins) which let us establish the existence of $1/f^{\alpha}$ noise with strong different value of α compared with that obtained from simulations [19]. Like as in case of the dynamic stress measurements within conical hoppers [11], we have detected the possible fractional Brownian motion (fBm) [20–23] for the density patterns.

The present paper examines the existence of inhomogeneities or temporal number density variations during the gravity induced flow of monodisperse granular material in a flat bin. In order to reach this goal, in the next section we present the experimental setup and the measuring technique used to obtain the density fluctuations at two positions: Just above the exit (within the bin) and just below the exit (out of the bin), respectively. In Sect. 3, we search the noisy character of the time series through the power spectra and the fractal properties of the number density fluctuations. Finally, in Sect. 4, we present the final remarks and conclusions.

2. EXPERIMENTS

In order to observe and characterize the density fluctuations we did experiments in vertical glass-bins with the following dimensions: 40 cm length, 40 cm height and 0.4 cm width. The bins were filled by raining the grains down up to $h = 36$ cm. The granular material consist of monodisperse rough glass beads with a mean grain size $d_0 = 0.315 \pm 0.004$ cm in diameter, *one grain* density $\rho_p = 2.45$ gr/cm^3, and a friction coefficient $\mu = 0.57$ in the Coulomb sense. The size of the aperture, $d_a = 2$ cm was maintained fixed. It is to be noted that the ratio thickness of the bin to the particle diameter is close to 1.27, thus assuring a near 2D flow. The bottom walls forms a channel at the exit of size 4 cm length by 2 cm width.

Our measuring technique uses the light produced by an halogen lamp passing through the granular material at time t, which is captured by a silicon solar cell, type Archer 276-124A, situated in front of the bin (Fig. 1). The photocell, positioned 0.6 cm away from the bin wall, has a cross-sectional area 2 cm $\times$ 4 cm and, therefore, the actual measurement cross-sectional area was almost the same than that of the photocell. The light variations produced continuously by the granular flow were monitored in real time by the photocell and the voltage variations were transmitted into a PC by using an A/D converter coupled to a data acquisition card PCL-818. With this method we resolve even a fraction of a single grain.

We have searched the effect of the exit locations on the number density. This was motived by other authors [24–26] who detected a noticeable increase in the mass flow rate as the exit is moved from the center to the border. Thus, we have done measurements of the time-dependent number density using two different configurations: (I) The exit located

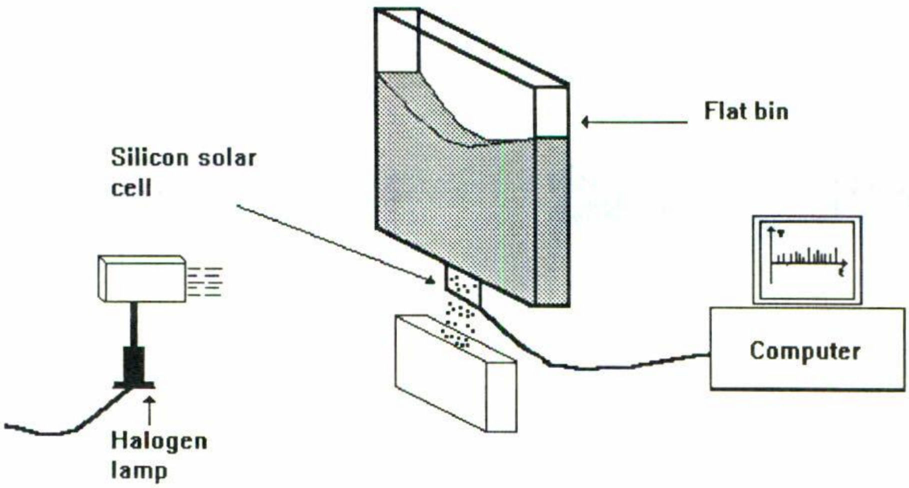

FIGURE 1. Schematic lateral view of a flat bin. The bottom exit was placed in two different positions: (I) at the center and, (II) at the border. In both cases we have measured the number density in the vicinity just above the exit of the bin and just below the exit.

at the center, and (II) the exit located at the border. Also, the photocell was placed in two different relative locations: (1) Just above the exit (to detect the number density variations within the bin), and (2) just below the exit (to detect the number density variations out of the bin). In cases (I), the voltage measurements were made approximately each 2×10^{-3} s and each 1×10^{-3} s in case (II). The measurements showed considerable fluctuations in the voltage which corresponds, after calibrations, to considerable fluctuations in the number density. A full account of our work, including a detailed discussion of the instrumental noise in experiments, will be published elsewhere [27].

Depending on the photocell position we found two different calibrations relating the measured voltage V with the number of grains in the cross-sectional area. The nonlinear calibration formulas are

$$V(t) - V_0 = k\left[n^2(t) - n_0^2\right], \quad \text{for cases (1);} \tag{1}$$

and

$$V(t) - V_0 = kn^2(t), \quad \text{for cases (2);} \tag{2}$$

where $k = -1.6 \times 10^{-5}$ V and V_0 is the initial voltage without flow, n_0 is the initial number of grains in a cross-sectional area with same dimensions than that of the photocell, and $n(t)$ is the number of grains at time t. Clearly, in Eq. (2) there is not an initial number of grains because the solar cell is placed just below the exit, while in Eq. (1) there is an initial number of grains because the solar cell detect light passing a zone within the bin. By using Eqs. (1) or (2) we performed a calculation of the number of grains in a 2 cm $\times$ 4 cm cross-sectional area, as a time function, within the bin (Figs. 2(a)-(c)) and for comparison, out of the bin (Fig. 2(d)). We have used $n_0 = 80$ in Figs. 2(a)-(c), which was obtained from experiments, and $n = 0$ in Fig. 2(d). The small fluctuations around $n = 0$

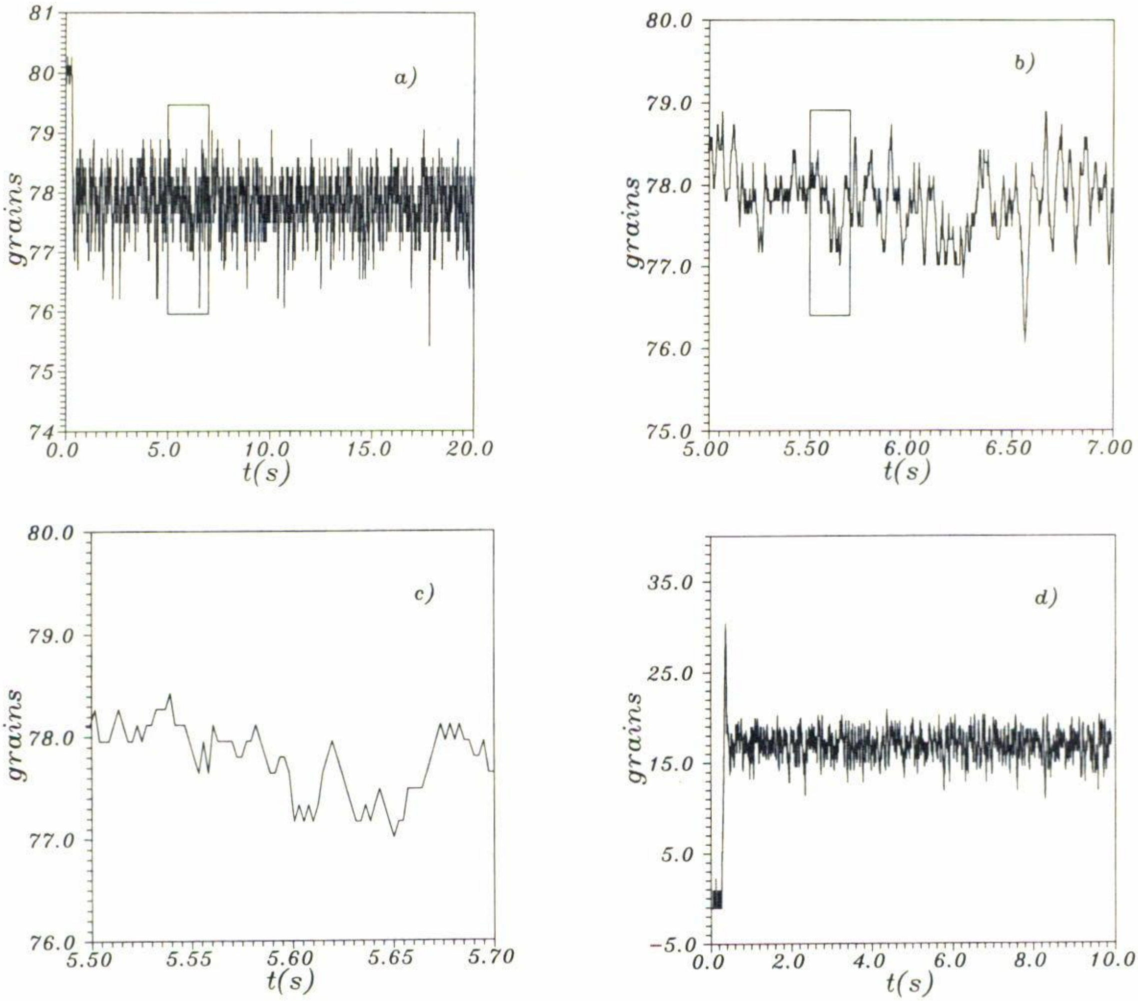

FIGURE 2. Typical noisy time series of the number of grains in a 2 cm × 4 cm cross-sectional area: Figs. (a)-(c) are three time intervals for cases (1). Fig. (d) is typical for cases (2). Initially, in cases (1), there is a certain quantity, $n_0 \simeq 80$, of grains while, in cases (2), $n_0 \simeq 0$; in all cases the small fluctuations around n_0 are due to the measuring technique and have no physical significance. The time series in (a) has a larger duration than that in (d) because the first series is typical of measurements with exit at the center while the second series is typical of measurements with exit at the border, *i.e.*, this last one corresponds to the larger mass flow rate.

are due to the noise produced by the measuring technique and the smaller triangles visible in curve of Fig. 2(c) are due to the A/D converter and have no physical significance.

3. SPECTRAL AND FRACTAL PROPERTIES

We have performed a fast Fourier transform (FFT) analysis for the time series which let us obtain the power spectra (the spectral logarithmic slope α) and the possible characteristic frequencies associated to the time series of the number of grains for both cases (I) and (II) and for both cases (1) and (2).

In experiments ten trails have been studied in each one of the four cases. We have found different power spectra between cases (1) and (2) but not between cases (I) and (II), *i.e.*, the exit position was not important in relation with the unique well defined value of the α exponents. In cases (1) the power spectra of the number of grains, $n(t)$, goes like $1/f^\alpha$ with $\alpha = 1.32$ over a considerable range of frequency f (Fig. 3(a)) which is in disagreement with the predicted spectrum by Ristow *et al.* [19] for two-dimensional hoppers; here [19], there is an overestimation of the exponent α compared with that obtained in our experiments. In cases (2) the power spectra of the number of grains shows, at least approximately, an exponent $\alpha = 1.0$, which formally corresponds to the ubiquitous $1/f$ noise (Fig. 3(b)). All the α exponents were obtained using a least squares fit; for each case treated here the values of α has less than 3% in error.

The Mandelbrot-Van Ness fractional Brownian motion (MV fBm) [20] has been widely used in the modeling of various physical phenomena which have empirical spectra of power-law type, $1/f^\alpha$, with $1 < \alpha < 3$ [20–23]; the random process characterized by Brownian motion yields a power law spectrum with exactly $\alpha = 2$.

Our data for the number density within the bin (cases (1)), with α near but typically larger than 1, suggest the possibility of MV fBm. Such a possibility for the series can be obtained using the R/S analysis, which we will give a brief outline here [28]. In this work we obtained records in time of the number density variations. By choosing a time period (referred as a lag) shorter than the total record, we first calculate the increments I, the change in the signal between two adjacent times, for this lag and then the time average A and the standard deviation S of the increments. By constructing a set of new increments $(I - A)$ and sequentially adding them, we reconstruct an image of the original curve in this lag along the horizontal axis. The range R is defined as the maximum value minus the minimum value of the curve, and by dividing R with S we obtain a dimensionless number R/S for the lag in question. If the lag was chosen say 1/100 of the total period, we have one hundred independent estimates of the R/S for this lag, and the final value is the average of all of them.

For many natural phenomena it has been found that the number R/S has the form

$$R/S \sim (\text{lag})^H , \tag{3}$$

where H is now referred to as the scaling exponent or Hurst parameter. A MV fBm with $0 < H < 1/2$ is said to be antipersistent in the sense that positive increments for some t imply (statistically) negative increments for $t+\text{lag}$, and vice versa. The range $1/2 < H < 1$ is called the persistent range: Positive increments for some t imply (statistically) positive increments for $t + \text{lag}$. The special value $H = 1/2$ yields the familiar Brownian motion, for which successive increments are independent [21]. Also, it is easy to prove that the α coefficient of the spectra and the Hurst parameter can be related by [20–22]

$$\alpha = 2H + 1. \tag{4}$$

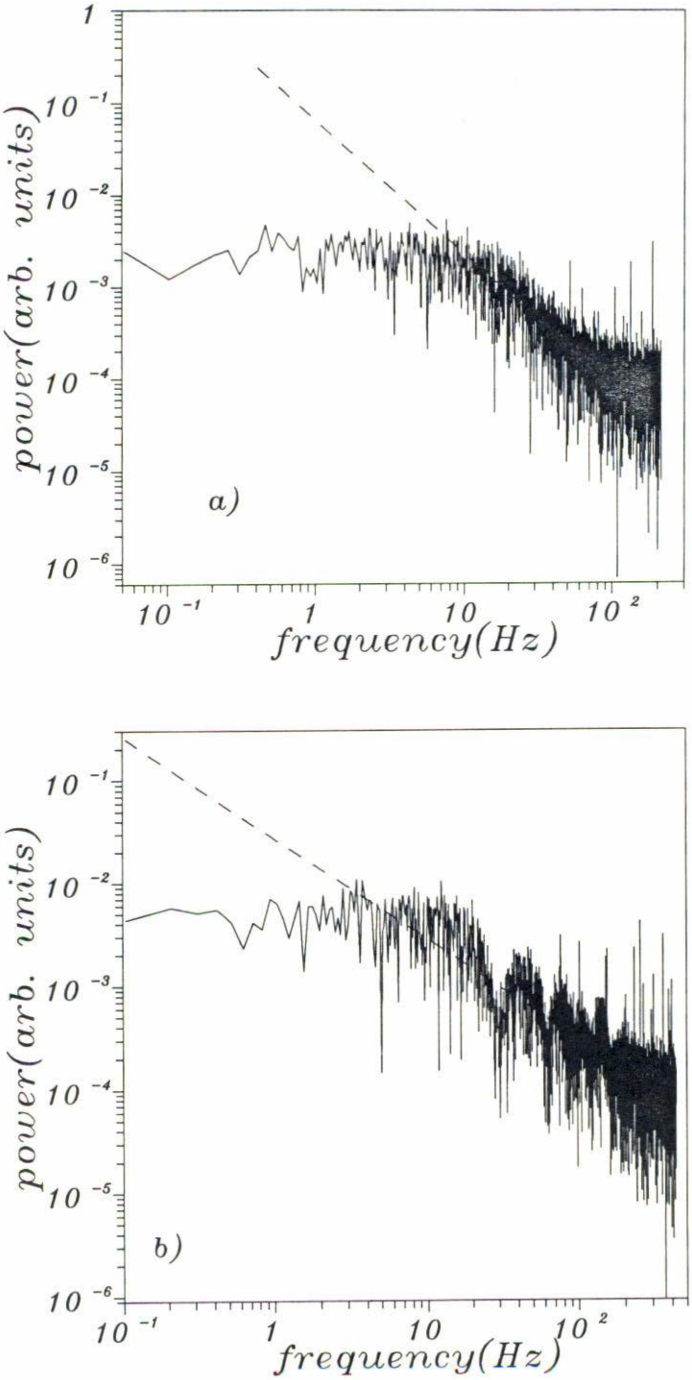

FIGURE 3. Log-log plot of the power spectra corresponding to the full frequency range available from a typical experiment for the two cases here studied: (a) within the bin and (b) at the exit of the bottom opening. In cases (1) $\alpha = 1.32 \pm 3\%$, and in cases (2) $\alpha = 1.0 \pm 3\%$. The corresponding exponents were obtained by using a least squares fit for the data.

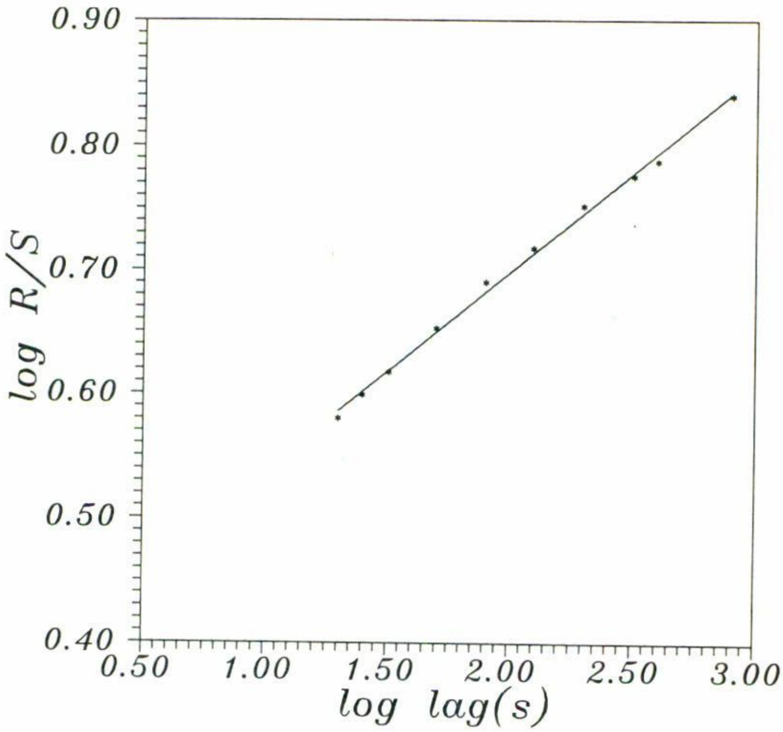

FIGURE 4. The R/S analysis for the time series of the Fig. 2(a) for both exit located at the center and exit located at the border. The line which adjust the points has a slope $H = 0.16 \pm 3\%$ for both cases.

The fractal dimension D, a qualitative measure of its filling properties [21], can be expressed in terms of H as

$$D = 2 - H. \tag{5}$$

Therefore, we can obtain the Hurst parameter for the data obtained in cases (1) by using the R/S analysis or by using relation (4) with $\alpha = 1.32$; both approaches give an unique, well defined, scaling exponent $H = 0.16$ and, therefore, we obtain a fractal dimension $D = 1.84$. In Fig. (4) we show the R/S analysis, for cases (1) and is insensitive to the exit position. This result indicates that the time series in cases (1) have fractal noise and are random walks negatively correlated.

4. REMARKS AND CONCLUSIONS

Using a simple experimental setup we have shown the existence of noise associated to the number density in monodisperse material. We therefore conclude that the rotational hindrance is not the supply of fluctuations in the density as was suggested by other authors [15]. In addition, the simulations of Ristow *et al.* [19] which gives $1/f^{\alpha}$ noise, where $\alpha = 2.7$, can also show the existence of this type of noise for monodisperse granular material, however, it is clear that the MDS does not reproduce our results for cases (1) (approximately $\alpha = 1.32 \simeq 4/3$) overestimating strongly the value of the α exponent; neither can to exhibit the presence of $1/f$ noise just below the exit of the 2D bin.

We will remark that our power-law fits in the power spectra were over more than one decade in the frequency range. Recent experiments [10] and simulations in pipes [8] to detect density waves, gave values of $\alpha \simeq 1.5$ and $\alpha \simeq 4/3$, respectively. Incidentally, the data presented in Ref. [10] covers a frequency range of one decade, just as ours. This shows the dependence of the laboratory scale on the results. The closed related values of α obtained in these works, merits a deep theoretical study because measurements in pipes are 3D while our experiments are 2D. Air effects on this quantity also are interesting, depending on the discharge rate.

In relation with Figs. 3(a) and 3(b) we shown the existence of cutoffs in the power-law of the power spectra, which has not been observed using MDS. The existence of low and high frequency cutoffs in the power spectra ensure, actually, that the total power does not diverge to infinity [29].

In order to give rise a more realistic value of the α exponent for the power spectrum within the bin, we suggest to introduce the shear friction force in MDS proportional to the square of the relative velocity of the grains. We have recently showed that this form gives a good approximation to the dissipative stresses in a rapid granular flow within a bin [30].

In summary, we have evidenced the noisy behavior of the density waves in a 2D bin. From a physical point of view the existence of $1/f^{\alpha}$ noise let us confirm the lack of characteristic time scales during the gravity induced flow from flat bins and the existence of long time correlations exponentially decaying. We should again emphasize that the position of the exit was not important in relation with the unique, well defined value of these exponents, which seems to indicate that the asymmetry in the value of the mass flow rate noted by other authors [24–26] is not related with the local density variations. As a final comment we suggest that a possible interpretation of our results, mainly that related with the origin of the $1/f$ noise, appears to be adequate in the frame of the theoretical model of vehicles in traffic jam [31]. In this model, the outflow from a large jam (in our experiments the pass of grains through the exit channel) self-organizes to the maximum throughput (after the exit channel the grains should accelerate freely under gravity) which is robust and critical, *i.e.*, obeys a $1/f$ noise. This last result is very important for the case of flow in a narrow pipe because in experiments one usually has a bin above the pipe to ensure constant refilling; obviously, the $1/f$ density fluctuations noted in experiments [4], and simulations [9], can be originated in the outflow from the bin.

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