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Predictions for the meson masses in broken flavour SU(5) symmetry

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ABSTRACT. The 25 mesons formed out of the u, d, s, c, and b quarks and antiquarks are assumed to transform as $24 \oplus 1$ representation of the conventional flavour SU(5) group. Simple breaking of SU(5) leads to sum rules among the masses of the isospin multiplets which are obeyed to within a few percent. The masses of the yet undiscovered vector mesons $B_c^{*+}(\bar{b}c)$ and $B_s^{*0}(\bar{b}s)$ are predicted to be 6279 MeV and 5432 MeV. Predictions for the corresponding pseudoscalar mesons are also given.

RESUMEN. Se asume que los 25 mesones formados por quarks y antiquarks u, d, s, c, y b, se transforman como la representación $24 \oplus 1$ del grupo de sabor SU(5) convencional. Un sencillo rompimiento de SU(5) conduce a reglas de suma entre las masas de los multipletes de isoespín que son válidas hasta un pequeño orden porcentual. Las masas de los mesones vectoriales $B_c^{*+}(bc)$ y $B_s^{*0}(bs)$ aún no descubiertos se predicen en 6279 MeV y 5432 MeV. También se dan predicciones para los correspondientes mesones escalares.

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1. INTRODUCTION

In quantum chromodynamics (QCD), the interactions among the hadronic constituents (quarks, antiquarks and gluons) are completely flavour symmetric. The flavour symmetry is broken only by the quark mass terms. Mass-wise the five known quarks fall into two groups: (a) the light quarks, u, d and s; (b) the heavy quarks c and b. Different approximation methods and techniques have been developed and are used to understand the dynamics of the quarks in the two groupings. For example, chiral perturbation theory [1] is employed for the light quarks and more recently heavy quark effective theory [2] has been used for the c and b quarks. This observation has motivated the symmetry breaking pattern considered by us for the flavour SU(5) group used to classify hadronic states.

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In this brief note, we consider the low-lying $J^{PC} = 1^{--}$ vector mesons formed out of the u, d, s, c and b quarks and antiquarks. The 25 vector mesons (and their antiparticles) so obtained are classified according to conventional flavour SU(5) symmetry. They are assumed to transform as $\mathbf{24} \oplus \mathbf{1}$ representation of SU(5), with ideal mixing between the $\mathbf{1}$ and the $\mathbf{24}$ representations. The mesons in this 25-plet fall into 11 independent isospin multiplets. We give below the predictions for these 11 vector meson masses assuming a simple breaking of SU(5). The corresponding results for the pseudoscalar mesons, for which mixing is not ideal, are also given.

2. Transformation properties of the states

We use the notation suggested by the Particle Data Group [3] for the particle states. Explicitly, the eleven vector meson isospin multiplets are: $\rho(\rho^+ = u\bar{d}, \rho^0 = \frac{u\bar{u}-d\bar{d}}{\sqrt{2}}, \rho^- = d\bar{u})$, $\omega^0(\frac{u\bar{u}+d\bar{d}}{\sqrt{2}})$, $\phi^0(s\bar{s})$, $K^*(K^{*+} = u\bar{s}, K^{*0} = d\bar{s})$, $D^*(D^{*+} = c\bar{d}, D^{*0} = c\bar{u})$, $D_s^{*+}(c\bar{s})$, $\bar{B}^*(\bar{B}^{*0} = b\bar{d}$, $B^{*-} = b\bar{u})$, $\bar{B}_s^{*0}(b\bar{s})$, $B_c^-(b\bar{c})$, $\psi(1S, c\bar{c})$ and $\Upsilon(1S, b\bar{b})$. Their masses will be denoted by $M(V)$, where V will denote the particle label ignoring the electric charge specification as isospin symmetry will be preserved, *e.g.*, $M(\rho)$, $M(\omega)$, $M(B_s^*)$, $M(B^*)$, $M(B_c^*)$, etc.

Under usual flavour SU(3), ρ, ω, ϕ and K^*, $\bar{K}^*$ form a nonet $V(q)$ with ideal mixing, while $D(3) = (D^{*0}, D^{*-}, D_s^{*-})$ and $B(3) = (B^{*+}, B^{*0}, B_s^{*0})$ transform as a $\mathbf{3}$ with their antiparticles transforming as $\mathbf{3}^*$ representation of SU(3). The remaining four states ψ, Υ and $B_c^{*\pm}$ are SU(3) singlets as they contain only the c and b quarks and antiquarks. In the limit of $m_c, m_b \to \infty$ one expects a heavy quark flavour group $SU_Q(2)$ under which $\binom{c}{b}$ form a doublet with $I_Q = 1/2$. So that B_c^{*+} and B_c^{*-} are members of a $I_Q = 1$ multiplet with $I_{Q3} = +1$ and -1, while the $I_Q = 1$, $I_{Q3} = 0$ state mixes with the $I_Q = 0$ state to give the mass eigenstates ψ and Υ. These four states will be collectively denoted by V_Q.

In our discussion of the mass formula below, the properties of the mesons with respect to the $SU(3) \times SU_Q(2)$ subgroup of SU(5) will play an underlying role. For our purpose we note that under this subgroup, $V(9)$ transforms as $(\mathbf{9}, I_Q = 0)$, $D(3)$ as $(\mathbf{3}, I_Q = 1/2, I_{Q3} = -1/2)$, $B(3)$ as $(\mathbf{3}, I_Q = 1/2, I_{Q3} = +1/2)$ and the states in set V_Q as $(\mathbf{1}, I_Q = 1 \oplus 0)$. Furthermore, note that the states in $D(3)$ and $B(3)$ form three $I_Q = 1/2$ doublets, *viz.*, $Q_1 \equiv (B^{*+}, \bar{D}^{*0})$, $Q_2 \equiv (B^{*0}, D^{*-})$ and $Q_3 \equiv (B_s^{*0}, D_s^{*-})$. Finally, we note that the vector 25-plet, $V(\mathbf{25})$, with ideal mixing can be written as a $5 \to 5$ matrix:

$$V(\mathbf{25}) = \begin{pmatrix} \rho^0/\sqrt{2} + \frac{\omega^0}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} & B^{*+} \\ \rho^- & -\rho^0/\sqrt{2} + \frac{\omega^0}{\sqrt{2}} & K^{*0} & D^{*-} & B^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} & B_s^{*0} \\ D^{*0} & D^{*+} & D_s^{*+} & \psi & B_c^{*+} \\ B^{*-} & \bar{B}^{*0} & \bar{B}_s^{*0} & B_c^{*-} & \Upsilon \end{pmatrix}$$

The matrix form is very convenient for calculational purposes. The pseudoscalar 25-plet $P(\mathbf{25})$ can be obtained from the above with obvious changes, the only point to be noted is that its SU(3) nonet does not have ideal mixing (see below Sect. 5).

3. Mass formula

To obtain the masses of the mesons we consider an isospin preserving mass operator of the general form

$$H = H_0 + H_5 + H_3 + H_1, \tag{1}$$

where H_0 is SU(5) symmetric and gives the common 25-plet mass M_0. The transformation property of the SU(5) breaking operators in Eq. (1) is as follows:

a) H_5 breaks down $SU(5) \to SU(3) \times SU_Q(2)$. This term is chosen because in the $\lim m_u \simeq m_d \simeq m_s =$ small and $m_c \simeq m_b \simeq$ large, QCD lagrangian would posses a flavour $SU(3) \times SU_Q(2)$ symmetry. We choose $H_5 = \alpha\lambda$ where λ is a traceless diagonal 5×5 matrix with diagonal elements $(2, 2, 2, -3, -3)$. This term will give a contribution to the masses proportional to the breaking parameter α.

b) H_3 transforms as the eighth component of an octet. It breaks flavour SU(3) while preserving $SU_Q(2)$ and isospin invariance. In general, it will give different effective reduced matrix elements α_3, α_3' and α_3'' for the SU(3) multiplets $V(9)$, $D(3)$ and $B(3)$. Since, H_3 commutes with $SU_Q(2)$ and $D(3) \to B(3)$ under $I_{Q3} \to -I_{Q3}$ one has $\alpha_3' = \alpha_3''$. We note that for the special choice $H_3 = \alpha_3\lambda_8$, where λ_8 is a diagonal generator of SU(5) with diagonal elements $(1, 1, -2, 0, 0)$, only one parameter α_3 $(= \alpha_3' = \alpha_3'')$ will provide a measure of SU(3) breaking.

c) H_1 is chosen to transform as a SU(3) singlet which has $I_Q = 1$, $I_{Q3} = 0$. It breaks $SU_Q(2)$ symmetry but commutes with flavour SU(3). Such a term is necessary for a realistic mass formula since $m_b \simeq 3m_c$ implies that $SU_Q(2)$ cannot be a viable flavour symmetry. In general, H_1 would give different effective reduced matrix elements α_1, α_1', α_1'' and β_1 for the $SU_Q(2)$ multiplets Q_1, Q_2, Q_3 (defined earlier) and V_Q. However, $\alpha_1 = \alpha_1' = \alpha_1''$ because H_1 preserves SU(3) and the doublets Q, can go into each other under SU(3) transformations. We note that instead of two parameters α_1 and β_1, the special choice $H_1 = \alpha_1\lambda_Q$ will give only one $SU_Q(2)$ breaking parameters α_1 $(= \beta_1)$, where λ_Q is the 5×5 traceless diagonal matrix with diagonal elements $(0, 0, 0, 1, -1)$.

Having specified the choice of the SU(5) breaking terms in Eq. (1), it is straightforward to express the masses of the 11 meson isomultiplets in terms of the six parameters M_0, α, α_3, α_3', α_1 and β_1. We also remark on the result for the special choice for H_3 and H_1 (which gives $\alpha_3 = \alpha_3'$ and $\alpha_1 = \beta_1$).

4. Vector mesons

For the vector meson masses, the general mass operator in Eq. (1) gives

$$M(\rho) = M(\omega) = M_0 + 4\alpha + 2\alpha_3, \tag{2}$$

$$M(\phi) = M_0 + 4\alpha - 4\alpha_3, \tag{3}$$

$$M(K^*) = M_0 + 4\alpha - \alpha_3, \tag{4}$$

$$M(D^*) = M_0 - \alpha + \alpha_3' + \alpha_1, \tag{5}$$
$$M(D_s^*) = M_0 - \alpha - 2\alpha_3' + \alpha_1, \tag{6}$$

$$M(B^*) = M_0 - \alpha + \alpha_3' - \alpha_1, \tag{7}$$
$$M(B_s^*) = M_0 - \alpha - 2\alpha_3' - \alpha_1, \tag{8}$$

$$M(B_c^*) = M_0 - 6\alpha, \tag{9}$$
$$M(\psi) = M_0 - 6\alpha + 2\beta_1, \tag{10}$$
$$M(\Upsilon) = M_0 - 6\alpha - 2\beta_1. \tag{11}$$

The mass states have been grouped according to their transformation property under $SU(3) \times SU_Q(2)$. Since, 11 masses are expressed linearly in terms of 6 parameters we expect 5 sum rules.

Equations (2)–(4) give the two well known relations:

$$M(\rho) = M(\omega), \tag{12}$$
$$4M(K^*) = M(\rho) + M(\omega) + 2M(\phi), \tag{13}$$

of which Eq. (12) is acceptable, while Eq. (13) is well satisfied to within 1% accuracy.

Equations (5)–(6) and (7)–(8) yield the sum rule

$$M(D^*) + M(B_s^*) = M(D_s^*) + M(B^*). \tag{14}$$

Using the experimentally known masses [3] this predicts the unknown mass

$$M(B_s^*) = 5432 \text{ MeV}. \tag{15}$$

Note $M(B^*) = 5330 \pm 5$ MeV as given in the full listings in Ref. [3] was used to obtain Eq. (15). No errors have been given in Eq. (15) and in numerical results given below since the sum rules or mass relations used themselves are not expected to be satisfied (at best) to better than a few percent accuracy. So it is not meaningful to quote errors of a few MeV in predictions like Eq. (15). The masses of the states in the multiplet V_Q are given by Eqs. (9)–(11) and they satisfy a relation which predicts the as yet unknown mass $M(B_c^*)$ of $B_c^{*\pm}$, giving

$$M(B_c^*) = \tfrac{1}{2}[M(\psi) + M(\Upsilon)] = 6279 \text{ MeV}. \tag{16}$$

In addition to the above 4 sum rules, the fifth provides a relation between the mean masses

$$M(9) = \frac{3M(\rho) + M(\omega) + M(\phi) + 4M(K^*)}{9} = M_0 + 4\alpha, \tag{17}$$
$$M(3) = \frac{2M(D^*) + 2M(B^*) + M(D_s^*) + M(B_s^*)}{6} = M_0 - \alpha, \tag{18}$$
$$M(1) = \frac{M(\psi) + M(\Upsilon) + 2M(B_c^*)}{4} = M_0 - 6\alpha, \tag{19}$$

of the multiplets $V(9)$, $D(3)$ and $B(3)$ and V_Q. The resultant inter-multiplet sum rule

$$2M(3) = M(1) + M(9), \tag{20}$$

can be simplified using Eqs. (12)–(13) and (16) to read simply

$$2M(D^*) + 2M(B^*) + M(D_s^*) + M(B^a st_s) = 3M(B_c^*) + M(\rho) + M(\omega) + M(\phi). \tag{21}$$

This sum rule, like the well-known sum rules in Eqs. (12)–(13), provides a test of our choice for the mass operator since the two sides can be calculated. Putting in the mass values, Eq. (20) or (21) gives 22.20 GeV $\simeq$ 21.39 GeV. In other words, the sum rule is satisfied within 4%. This is rather gratifying considering our simple mass formula.

One can make more predictions for the masses if we make the special choice *viz.* $H_3 = \alpha_3\lambda_8$ and $H_1 = \alpha_1\lambda_Q$ mentioned earlier. This choice means putting $\alpha_3' = \alpha_3$ and $\beta_1 = \alpha_1$ in Eqs. (5)-(11). One obtains two more inter-multiplet relations by solving for α_3 and α_1 in terms of different sets of masses. Thus,

$$2M(\phi) - M(\rho) - M(\omega) = 4M(D_s^*) - 4M(D^*), \tag{22}$$

and

$$2M(B^*) - 2M(D^*) = M(\Upsilon) - M(\psi). \tag{23}$$

These relations involve known masses and provide a test of the special choice for H_3 and H_1. Putting in the masses, Eq. (22) and Eq. (23) give 490 MeV $\simeq$ 400 MeV and 6643 MeV $\simeq$ 6363 MeV. That is, Eq. (23) (due to $\beta_1 = \alpha_1$) is satisfied within 5% accuracy while Eq. (22) (corresponding to $\alpha_3 = \alpha_3'$) is violated by about 20%. This tells us that the description of $SU_Q(2)$ breaking by one parameter is acceptable while SU(3) breaking should be described by the general H_3 which requires two parameters. The overall conclusion is that vector meson masses are reasonably well described by the general mass operator given by Eq. (1).

5. Pseudoscalar mesons

The expression for the masses for this case can be easily obtained from Eqs. (5)–(11) by replacing the vector meson state by the corresponding pseudoscalar (ps) meson state, *e.g.*, $D^* \to D$, $B_s^* \to B_s$, $\psi \to \eta_c$ etc. The major change is in Eqs. (2)–(4) since the ps mesons π, K, $\eta(548)$ and $\eta'(958)$ do not form an ideally mixed nonet. Consequently, only the well-known mass relation

$$4M(K) = M(\pi) + 3M(\eta)\cos^2\theta_P + 3M(\eta')\sin^2\theta_P \tag{24}$$

is obtained instead of the two Eqs. (12)–(13) for vector mesons. Here θ_P is singlet-octet mixing angle. For the ps-mesons we will use below linear mass formulae to make numerical predictions. In this case one obtains [3] $\theta_P = -23°$.

The sum rule corresponding to Eq. (14) predicts

$$M(B_s) = 5380.5 \text{ MeV}. \tag{25}$$

This is in remarkably good agreement with the experimental value [4] of 5375 ± 5 MeV.

Consequently, the sum rule corresponding to Eqs. (20) and (21) enables one to predict

$$M(B_c) = 2M_P(3) - M_P(8) = 6837.7 \text{ MeV}, \tag{26}$$

where

$$M_P(3) = \tfrac{1}{6}(2M(D) + 2M(B) + M(D_s) + M(B_s)) \tag{27}$$

and

$$M_P(8) = \tfrac{1}{8}[3M(\pi) + 4M(K) + M(\eta)\cos^2\theta_P + M(\eta')\sin^2\theta_P]. \tag{28}$$

Knowledge of $M(B_c)$ and the ps mass sum rule corresponding to Eq. (16) permits one to predict

$$M(\eta_b) = 10,695.8 \text{ MeV}. \tag{29}$$

Predictions in Eqs. (26)–(28) and Eq. (29) are disturbing as they give $M(B_c) - M(B_c^*) \simeq$ 560 MeV and $M(\eta_b) - M(\Upsilon) \simeq 1235$ MeV! Use of a quadratic mass formula throughout does not help as it predicts $M(B_s) = 5.31$ GeV, $M(B_c) = 5.61$ GeV and $M(\eta_b) =$ 7.35 GeV!

In either case, one does not obtain credible predictions for the heavier pseudoscalar mesons even through the prediction [Eq. (25)] for the medium mass sector in good. The reason for the poor predictions we feel, is due to the use of the inter-multiplet sum rule [Eq. (26)] which relates the masses of the meson in the light, medium and heavy mass sectors. The light masses sector contains the SU(3) octet and nonet mesons. The medium mass sector consist of the SU(3) triplet states like D, B^* etc. The heavy mass sector contains the $SU_Q(2)$ states. This problem was not apparent in the vector mason case since we did not use the inter-multiplet sum rule [Eq. (21)] to predict a mass but merely checked it. However, if we use Eq. (21) [with Eq. (15)] to predict $M(B_c^*)$ one finds a value of 6545 MeV which is about 265 MeV higher than that of Eq. (16). Unreliability of the inter-multiplet sum rule implies that the SU(5) breaking is more complicated than considered here. However, for both vector and pseudo scalar mesons, we expect that the sum rules in each sector will be reliable to about 1% accuracy. It would be very heartering if the experimental discovery of the B_s^{*0} and B_c^{*-} lend support to our results.

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