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Scalar sea contributions to spin 1/2 baryon structure and magnetic moments

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ABSTRACT. We treat the baryon as a composite system made out of a "core" of three quarks (as in the standard quark model) surrounded by a "sea" (of gluons and $q\bar{q}$-pairs) which is specified by its total quantum numbers like flavor, spin and color. Specifically, we assume the sea to be a flavor octet with spin 0 but no color. The general wavefunction for spin 1/2 baryons with such a sea component is given and an application to the magnetic moments is considered. Numerical analysis shows that the scalar sea can provide a good fit to the magnetic moment data *using experimental errors*.

RESUMEN. Se considera al barión como un sistema compuesto formado por un "núcleo" de tres quarks (como en el modelo de quarks estándar) rodeado por un "mar" (de gluones y pares $q\bar{q}$) especificado por sus números cuánticos totales como sabor, espín y color. Concretamente, se supone al mar como un octete de sabor con espín 0 y sin color. Se da la función de onda general para bariones de espín 1/2 con tal componente del mar y se considera una aplicación a los momentos magnéticos. El análisis numérico muestra que el mar escalar puede proporcionar un buen ajuste a los datos de los momentos magnéticos *usando errores experimentales*.

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1. INTRODUCTION

Attempts to understand the static properties of hadrons in the framework of the standard quark model (SQM) have had limited success. The belief that valence quarks were responsible for the spin of the proton has been shattered by recent experiments [1].

The naive valence quark picture of hadron structure is a simplification which does not properly take into account the fact that quarks interact through color forces mediated by vector gluons. The existence of the quark-gluon interaction, in QCD, implies that a hadron should be viewed as consisting of valence quarks surrounded by a "sea" which contains gluons and virtual quark-antiquark ($q\bar{q}$) pairs. Deep inelastic lepton-nucleon scattering has shown the existence of a sea component and its importance for nucleon structure

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functions. It is thus necessary to understand how this sea contributes to the baryon spin and other low energy properties.

Several authors [2–6] have studied the effect of the sea contributions on the hadron structure and the static properties of baryons. Some consider the sea as a single gluon or a $q\bar{q}$-pair. However the sea, in general, consists of any number of gluons and $q\bar{q}$-pairs. In this paper we "model" the general sea by its total quantum numbers (flavor, spin and color) which are such that the sea wave function when combined with the valence quark wave function gives the desired quantum numbers for the physical hadron. In particular, we explore the consequences of a "sea" with flavor and spin but no color [7] for the low energy properties of the spin 1/2 baryon octet $(p, n, \Lambda, \ldots)$.

For simplicity, we consider a flavor octet sea with spin 0. We find that the scalar (spin 0) sea described by two parameters gives a very good fit to the magnetic moment data using actual experimental errors which is our primary motivation.

In the next section, we discuss the wavefunctions for the physical baryons constructed from the SQM and from our model sea. Then, we obtain the magnetic moments from the modified wavefunction and give a general discussion of the results. The last section gives a summary and discussion.

2. Spin 1/2 octet baryon wavefunctions with sea

For the lowest-lying baryons, in the SQM, the three valence quarks are taken to be in a relative S-wave states so that the space wavefunction is symmetric and the color wavefunction is totally antisymmetric to give a color singlet baryon. Consequently, the flavor-spin wavefunction is totally symmetric. For the SU(3) flavor octet spin 1/2 baryons we denote this SQM or q^3 wavefunction by $\tilde{B}(\mathbf{8}, 1/2)$, the argument 1/2 refers to spin. The corresponding states are denoted by $\tilde{p}$, $\tilde{\Sigma}^+$, etc.

In our picture, the physical baryon octet $B(\mathbf{8}, 1/2)$ states are formed out of the q^3 "core" baryons described by $\tilde{B}(\mathbf{8}, 1/2)$ surrounded by a sea consisting of $q\bar{q}$-pairs and gluons. This sea is assumed to be color singlet but has flavor and spin properties such that its wavefunction Φ_s (subscript "s" for sea) when combined with the core baryons gives the physical baryons, that is

$$\tilde{B}(\mathbf{8}, 1/2) \otimes \Phi_s = B(\mathbf{8}, 1/2). \tag{1}$$

The possible SU(3) flavor and spin transformation properties the sea can have are $\mathbf{1}, \mathbf{8}, \mathbf{10}, \overline{\mathbf{10}}, \mathbf{27}$ for flavor and 0 for spin. The corresponding wavefunctions are denoted by $S(N)$ $(N = \mathbf{1}, \mathbf{8}, \ldots)$ and H_0. So, in general Φ_s is a combination of $S(N)H_0$. The SU(3) symmetric and spinless (color singlet) sea implicit in the SQM is discribed by the wavefunction $S(\mathbf{1})H_0$. Such a color singlet sea would require at least two gluons by themselves or a $q\bar{q}$-pair. The sea described by the wavefunctions $S(\mathbf{8})H_0$ would require a minimum of one $q\bar{q}$-pair. Gluons by themselves cannot give flavorful sea while, at least two $q\bar{q}$-pairs are required to give a sea with $N = \mathbf{10}, \overline{\mathbf{10}}, \mathbf{27}$. Guided by simplicity and the above remarks we limit our discussion to a sea described by the wavefunctions $S(\mathbf{1})H_0$ and $S(\mathbf{8})H_0$. The color singlet sea discribed by these wavefunctions can, in general, have any number of $q\bar{q}$-pairs and gluons consistent with its total flavor and spin quantum numbers. We also refer to a spin 0 sea as a scalar sea.

The total flavor-spin wavefunction of a spin up (↑) physical baryon which consists of 3 valence quarks and a sea component (as discussed above) can be written schematically as

$$B(1/2\uparrow) = \tilde{B}(\mathbf{8}, 1/2\uparrow)H_0 S(\mathbf{1}) + \sum_N a(N)\left[\tilde{B}(\mathbf{8}, 1/2\uparrow)H_0 \otimes S(\mathbf{8})\right]_N . \quad (2)$$

The normalization not indicated here is discussed later. The first term is the usual q^3-wavefunction of the SQM (with a trivial sea), in this term the sea is a flavor singlet. The second term in Eq. (2) contains a scalar sea which transforms as a flavor octet. The various SU(3) flavor representations obtained from $\tilde{B}(\mathbf{8}) \otimes S(\mathbf{8})$ are labelled by $N = \mathbf{1}, \mathbf{8_F}, \mathbf{8_D}, \mathbf{10}, \mathbf{\overline{10}}, \mathbf{27}$. As it stands, Eq. (2) represents a spin 1/2↑ baryon which is not *a pure flavor octet* but has an admixture of other SU(3) representations weighted by the unspecified constants $a(N)$. It will be a flavor octet if $a(N) = 0$ for $N = \mathbf{1}, \mathbf{10}, \mathbf{\overline{10}}, \mathbf{27}$. The color wavefunctions have not been indicated as the three valence quarks in the core $\tilde{B}$ and the sea (by assumption) are in a color singlet state.

For our applications we adopt the phenomenological wavefunction given in Eq. (2), where the physical spin 1/2 baryons have admixtures of flavor SU(3) determined by the coefficients $a(N)$, $N = \mathbf{1}, \mathbf{10}, \mathbf{\overline{10}}, \mathbf{27}$. As we shall see, such a wavefunction which respects the isospin and hypercharge properties of the usual spin 1/2 baryon states is general enough to provide an excellent fit to the magnetic moments data. Surprisingly, only 2 or 3 of the six parameters in Eq. (2) are needed for this purpose. For the moment we discuss the general wavefunction in Eq. (2) as it is. Incidentally, such a wavefunction could arise in general since we know flavor is broken by mass terms in the QCD Lagrangian.

The sea isospin multiplets contained in the octet $S(\mathbf{8})$ are denoted as

$$(S_{\pi^+}, S_{\pi^0}, S_{\pi^-}), \qquad (S_{K^+}, S_{K^0}), \qquad (S_{\bar{K}^0}, S_{K^-}), \quad \text{and} \quad S_\eta . \quad (3)$$

The suffix on the components label the isospin and hypercharge quantum numbers. For example, S_{π^+} has $I = 1$, $I_3 = 1$ and $Y = 0$; S_{K^-} has $I = 1/2$, $I_3 = -1/2$, and $Y = -1$; etc. These flavor quantum numbers when combined with those of the three valence quarks states $\tilde{B}$ will give the observed I, I_3, and Y for the physical states B. The flavor combinations in the second term in Eq. (2) imply that the physical states $B(Y, I, I_3)$ are expressed as a sum of products of $\tilde{B}(Y, I, I_3)$ and the sea components $S(Y, I, I_3)$, weighted by coefficients which are linear combinations of the coefficients $a(N)$. Schematically, the flavor content of the second term in Eq. (2) is of the form (suppressing I_3)

$$B(Y, I) = \sum_i \alpha_i(Y_1, Y_2, I_1, I_2)\left[\tilde{B}(Y_1, I_1)\, S(Y_2, I_2)\right]_i , \quad (4)$$

where the sum is over all Y_i, I_i, $(i = 1, 2)$; such that: $Y = Y_1 + Y_2$ and $\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$. The flavor content of $B(Y, I, I_3)$ in terms of $\tilde{B}(Y, I, I_3)$ and sea components are given in Table I. The corresponding coefficients $\bar{\beta}_i$, β_i, etc. expressed in terms of the coefficients $a(N)$ are recorded in Table II. In Table I we have denoted $\tilde{B}(Y, I, I_3)$ and $S(Y, I, I_3)$ by appropiate

TABLE I. Contribution to the physical baryon state $B(Y,I,I_3)$ formed out of $\tilde{B}(Y,I,I_3)$ and flavor octet states $S(Y,I,I_3)$ (see second term in Eq. (2)). The core baryon states $\tilde{B}$ denoted by $\tilde{p}$, $\tilde{n}$, etc. are the normal 3 valence quark states of SQM. The sea octet states are denoted by $S_{\pi^+} = S(0,1,1)$, etc. as in Eq. (3). Further, $(\tilde{N}S_\pi)_{I,I_3}$, $(\tilde{\Sigma}S_{\bar{K}})_{I,I_3}$, $(\tilde{\Sigma}S_\pi)_{I,I_3}$, ... stand for total I, I_3 *normalized* combinations of $\tilde{N}$ and S_π, etc. See Table II for the coefficients $\bar{\beta}_i$, β_i, γ_i, and δ_i.

$B(Y,I,I_3)$	$\tilde{B}(Y,I,I_3)$ and $S(Y,I,I_3)$
p	$\bar{\beta}_1\tilde{p}S_\eta + \bar{\beta}_2\tilde{\Lambda}S_{K^+} + \bar{\beta}_3(\tilde{N}S_\pi)_{1/2,1/2} + \bar{\beta}_4(\tilde{\Sigma}S_K)_{1/2,1/2}$
n	$\bar{\beta}_1\tilde{n}S_\eta + \bar{\beta}_2\tilde{\Lambda}S_{K^0} + \bar{\beta}_3(\tilde{N}S_\pi)_{1/2,-1/2} + \bar{\beta}_4(\tilde{\Sigma}S_K)_{1/2,-1/2}$
Ξ^0	$\beta_1\tilde{\Xi}^0S_\eta + \beta_2\tilde{\Lambda}S_{\bar{K}^0} + \beta_3(\tilde{\Xi}S_\pi)_{1/2,1/2} + \beta_4(\tilde{\Sigma}S_{\bar{K}})_{1/2,1/2}$
Ξ^-	$\beta_1\tilde{\Xi}^-S_\eta + \beta_2\tilde{\Lambda}S_{\bar{K}^-} + \beta_3(\tilde{\Xi}S_\pi)_{1/2,-1/2} + \beta_4(\tilde{\Sigma}S_{\bar{K}})_{1/2,-1/2}$
Σ^+	$\gamma_1\tilde{p}S_{\bar{K}^0} + \gamma_2\tilde{\Xi}^0S_{K^+} + \gamma_3\tilde{\Lambda}S_{\pi^+} + \gamma_4\tilde{\Sigma}^+S_\eta + \gamma_5(\tilde{\Sigma}S_\pi)_{1,1}$
Σ^-	$\gamma_1\tilde{n}S_{K^-} + \gamma_2\tilde{\Xi}^-S_{K^0} + \gamma_3\tilde{\Lambda}S_{\pi^-} + \gamma_4\tilde{\Sigma}^-S_\eta + \gamma_5(\tilde{\Sigma}S_\pi)_{1,-1}$
Σ^0	$\gamma_1(\tilde{N}S_{\bar{K}})_{1,0} + \gamma_2(\tilde{\Xi}S_K)_{1,0} + \gamma_3\tilde{\Lambda}S_{\pi^0} + \gamma_4\tilde{\Sigma}^0S_\eta + \gamma_5(\tilde{\Sigma}S_\pi)_{1,0}$
Λ	$\delta_1(\tilde{N}S_{\bar{K}})_{0,0} + \delta_2(\tilde{\Xi}S_K)_{0,0} + \delta_3\tilde{\Lambda}S_\eta + \delta_4(\tilde{\Sigma}S_\pi)_{0,0}$

symbols, e.g., $\tilde{B}(1,1,1/2)$ by $\tilde{p}$, $S(0,1,1)$ by S_{π^+}, etc. In Tables I and II for the reduction of $\tilde{B}(\mathbf{8}) \otimes S(\mathbf{8})$ into various SU(3) representations we have followed the convention used by Carruthers [8].

The normalization of the physical baryons wavefunction in Eq. (2) can be obtained by using $\langle\tilde{B}(Y,I,I_3)|\tilde{B}(Y',I',I_3')\rangle = \langle S(Y,I,I_3)|S(Y',I',I_3')\rangle = \delta_{YY'}\delta_{II'}\delta_{I_3I_3'}$. However, it should be noted that the normalization are different, in general, for each $B(Y,I)$ state. This is because not all $a(N)$ contribute to a given (Y,I)-multiplet as is clear from Tables I and II. For example, $a(\mathbf{1})$ contributes only to Λ while $a(\mathbf{10})$ does not contribute to the nucleon states. Denoting by N_1, N_2, N_3, and N_4, the normalization constants for the (p,n), (Ξ^0,Ξ^-), $(\Sigma^\pm,\Sigma^0)$, and Λ isospin multiplets, one has

$$N_1^2 = N_0^2 + a^2(\mathbf{\overline{10}}), \qquad N_2^2 = N_0^2 + a^2(\mathbf{10}), \tag{5}$$

$$N_3^2 = N_0^2 + \sum_{N=\mathbf{10},\mathbf{\overline{10}}} a^2(N), \qquad N_4^2 = N_0^2 + a^2(\mathbf{1}), \tag{6}$$

where

$$N_0^2 = 1 + \sum_{N=\mathbf{8_D},\mathbf{8_F},\mathbf{27}} a^2(N). \tag{7}$$

For example, using Tables I and II, and Eqs. (5)–(7), the physical spin-up proton state as given by Eq. (2) is

$$\begin{aligned} N_1|p\uparrow\rangle &= |\tilde{p}\uparrow\rangle S(\mathbf{1}) + \bar{\beta}_1|\tilde{p}\uparrow\rangle S_\eta + \bar{\beta}_2|\tilde{\Lambda}\uparrow\rangle S_{K^+} \\ &\quad + \bar{\beta}_3\Big|(\tilde{N}\uparrow S_\pi)_{1/2,1/2}\rangle + \bar{\beta}_4|(\tilde{\Sigma}\uparrow S_K)_{1/2,1/2}\Big\rangle. \end{aligned} \tag{8}$$

TABLE II. The coefficients $\bar{\beta}_i$, β_i, γ_i, and δ_i in Table I expressed in terms of the coefficients $a(N)$, $N = \mathbf{1}, \mathbf{8_F}, \mathbf{8_D}, \mathbf{10}, \mathbf{\bar{10}}, \mathbf{27}$, in the 2nd term (from scalar sea) in Eq. (2).

$$\beta_1 = \frac{1}{\sqrt{20}}(3a(\mathbf{27}) - a(\mathbf{8_D})) - \frac{1}{2}(a(\mathbf{8_F}) - a(\mathbf{10}))$$

$$\beta_2 = \frac{1}{\sqrt{20}}(3a(\mathbf{27}) - a(\mathbf{8_D})) + \frac{1}{2}(a(\mathbf{8_F}) - a(\mathbf{10}))$$

$$\beta_3 = -\frac{1}{\sqrt{20}}(a(\mathbf{27}) + 3a(\mathbf{8_D})) + \frac{1}{2}(a(\mathbf{8_F}) + a(\mathbf{10}))$$

$$\beta_4 = \frac{1}{\sqrt{20}}(a(\mathbf{27}) + 3a(\mathbf{8_D})) + \frac{1}{2}(a(\mathbf{8_F}) + a(\mathbf{10}))$$

$$\bar{\beta}_1 = \frac{1}{\sqrt{20}}(3a(\mathbf{27}) - a(\mathbf{8_D})) + \frac{1}{2}(a(\mathbf{8_F}) + a(\mathbf{\bar{10}}))$$

$$\bar{\beta}_2 = \frac{1}{\sqrt{20}}(3a(\mathbf{27}) - a(\mathbf{8_D})) - \frac{1}{2}(a(\mathbf{8_F}) + a(\mathbf{\bar{10}}))$$

$$\bar{\beta}_3 = \frac{1}{\sqrt{20}}(a(\mathbf{27}) + 3a(\mathbf{8_D})) + \frac{1}{2}(a(\mathbf{8_F}) - a(\mathbf{\bar{10}}))$$

$$\bar{\beta}_4 = -\frac{1}{\sqrt{20}}(a(\mathbf{27}) + 3a(\mathbf{8_D})) + \frac{1}{2}(a(\mathbf{8_F}) - a(\mathbf{\bar{10}}))$$

$$\gamma_1 = \frac{1}{\sqrt{10}}(\sqrt{2}a(\mathbf{27}) - \sqrt{3}a(\mathbf{8_D})) + \frac{1}{\sqrt{6}}(a(\mathbf{8_F}) - a(\mathbf{10}) + a(\mathbf{\bar{10}}))$$

$$\gamma_2 = \frac{1}{\sqrt{10}}(\sqrt{2}a(\mathbf{27}) - \sqrt{3}a(\mathbf{8_D})) - \frac{1}{\sqrt{6}}(a(\mathbf{8_F}) - a(\mathbf{10}) + a(\mathbf{\bar{10}}))$$

$$\gamma_3 = \frac{1}{\sqrt{10}}(\sqrt{3}a(\mathbf{27}) + \sqrt{2}a(\mathbf{8_D})) - \frac{1}{2}(a(\mathbf{10}) + a(\mathbf{\bar{10}}))$$

$$\gamma_4 = \frac{1}{\sqrt{10}}(\sqrt{3}a(\mathbf{27}) + \sqrt{2}a(\mathbf{8_D})) + \frac{1}{2}(a(\mathbf{10}) + a(\mathbf{\bar{10}}))$$

$$\gamma_5 = \frac{1}{\sqrt{6}}(2a(\mathbf{8_F}) + a(\mathbf{10}) - a(\mathbf{\bar{10}}))$$

$$\delta_1 = \frac{1}{\sqrt{20}}(\sqrt{3}a(\mathbf{27}) + \sqrt{2}a(\mathbf{8_D})) + \frac{1}{2}(\sqrt{2}a(\mathbf{8_F}) + a(\mathbf{1}))$$

$$\delta_2 = -\frac{1}{\sqrt{20}}(\sqrt{3}a(\mathbf{27}) + \sqrt{2}a(\mathbf{8_D})) + \frac{1}{2}(\sqrt{2}a(\mathbf{8_F}) - a(\mathbf{1}))$$

$$\delta_3 = \frac{3\sqrt{3}}{\sqrt{40}}a(\mathbf{27}) - \frac{1}{\sqrt{5}}a(\mathbf{8_D}) - \frac{\sqrt{2}}{4}a(\mathbf{1})$$

$$\delta_4 = -\frac{1}{\sqrt{40}}a(\mathbf{27}) - \sqrt{\frac{3}{5}}a(\mathbf{8_D}) + \frac{\sqrt{6}}{4}a(\mathbf{1})$$

Other baryon wavefunctions will have a similar structure. Also, $(\tilde{N} \uparrow S_\pi)_{1/2,1/2}$ ($(\tilde{\Sigma} \uparrow S_K)_{1/2,1/2}$) stand for the $I = I_3 = 1/2$ combination of the $I = 1/2$ $\tilde{N}$ (S_K) and $I = 1$ S_π ($\tilde{\Sigma}$) multiplets. For any operator $\hat{O}$ which depends only on quarks, the matrix elements are easily obtained using the ortogonality of the sea components. Clearly $\langle p \uparrow | \hat{O} | p \uparrow \rangle$ will be a linear combination of the matrix elements $\langle \tilde{B} \uparrow | \hat{O} | \tilde{B}' \uparrow \rangle$ (known from SQM) with coefficients which depend on the coefficients in the wavefunction. Note, for the physical baryons to have $J^P = \frac{1}{2}^+$ the scalar sea has $J^P = 0^+$ since $\tilde{B}$ have $J^P = \frac{1}{2}^+$.

For applications, we need the quantities $(\Delta q)^B$, $q = u, d, s$; for each spin-up baryon B. These are defined as

$$(\Delta q)^B = n^B(q \uparrow) - n^B(q \downarrow) + n^B(\bar{q} \uparrow) - n^B(\bar{q} \downarrow), \tag{9}$$

where $n^B(q\uparrow)$ ($n^B(q\downarrow)$) are the number of spin-up (spin-down) quarks of flavor q in the spin-up baryon B. Also, $n^B(\bar{q}\uparrow)$ and $n^B(\bar{q}\downarrow)$ have a similar meaning for antiquarks. However, these are zero as there are no explicit antiquarks in the wavefunctions given by Eq. (2). The expressions for $(\Delta q)^B$ are given in Table III in terms of the coefficients $\bar{\beta}_i$, etc. The expressions for $(\Delta q)^B$ reduce to the SQM values if there is no sea contribution, that is, $a(N)=0$, $N=\mathbf{1},\mathbf{8_F},\mathbf{8_D},\mathbf{10},\overline{\mathbf{10}},\mathbf{27}$. Moreover, the total spin S_Z of a baryon is given by $S_Z^B=(1/2)\sum_q(\Delta q)^B$. For $S_Z^B=1/2$, we expect $\sum_q(\Delta q)^B=1$ for a purely scalar sea. This is indeed true for each baryon as can be seen from Table III. There are three $(\Delta q)^B$ ($q=u,d,s$) for each (Y,I)-multiplet. These twelve quantities and $(\Delta q)^{\Sigma^0\Lambda}$ are given in terms of the six parameters of Eq. (2) as our spin 1/2 baryons do not belong to a definite representation of SU(3). To obtain a flavor octet physical baryon one restricts N to $\mathbf{8_F}$ and $\mathbf{8_D}$ in Eq. (2), that is, put $a(N)=0$ for $N=\mathbf{27},\mathbf{10},\overline{\mathbf{10}},\mathbf{1}$, so that the twelve $(\Delta q)^B$ are given in terms of two parameters $a(N)$ with $N=\mathbf{8_F},\mathbf{8_D}$. It is clear that our wavefunction, Eq. (2), provides an explicit model for spin 1/2 baryons to be compared to the phenomenological model considered by some authors [9] recently to fit the baryon magnetic moments. These authors take the three quantities $(\Delta q)^p$ ($q=u,d,s$) as parameters to be determined from data but use flavor $SU(3)$ to express all the other $(\Delta q)^B$ in terms of the $(\Delta q)^p$. In our case the various $(\Delta q)^B$ are not simply related by flavor SU(3) because of the non-trivial flavor properties of the sea and thus provides an explicit and very different model for the baryons.

3. Application to magnetic moments

We assume the baryon magnetic moment operator $\hat{\mu}$ to be expressed solely in terms of quarks as is usual in the quark model. So that $\hat{\mu}=\sum_q(e_q/2m_q)\sigma_Z^q$ ($q=u,d,s$). It is clear from Eq. (2) that $\mu_B=\langle B|\hat{\mu}|B\rangle$ will be a linear combination of $\mu_{\tilde{B}}$ and $\mu_{\tilde{\Sigma}^0\tilde{\Lambda}}$ weighted by the coefficients which depend on $a(N)$'s. The magnetic moments $\mu_{\tilde{B}}$ and the transition moment $\mu_{\tilde{\Sigma}^0\tilde{\Lambda}}$ (for the core baryons) are given in terms of the quark magnetic moments μ_q as per SQM. For example, $\mu_{\tilde{p}}=(4\mu_u-\mu_d)/3$, $\mu_{\tilde{\Lambda}}=\mu_s$, $\mu_{\tilde{\Sigma}^0\tilde{\Lambda}}=(\mu_u-\mu_d)/\sqrt{3}$, etc. Consequently, all the magnetic moments and the $\Sigma^0\to\Lambda$ transition magnetic moment in our model can be written simply as

$$\mu_B=\sum_q(\Delta q)^B\mu_q,\quad (q=u,d,s); \tag{10}$$

$$\mu_{\Sigma^0\Lambda}=\sum_q(\Delta q)^{\Sigma^0\Lambda}\mu_q,\quad (q=u,d); \tag{11}$$

where the $(\Delta q)^B$ and $(\Delta q)^{\Sigma^0\Lambda}$ are given in Table III and $B=p,n,\Lambda,\ldots$.

A class of models [9] have been recently considered in which the magnetic moments were expressed in terms of μ_q and $(\Delta q)^p$ ($q=u,d,s$) without giving an explicit wavefunction. Interestingly, Eqs. (10)–(11) have the same general structure except that here the twelve $(\Delta q)^B$ and $(\Delta q)^{\Sigma^0\Lambda}$ are not related but depend on six parameters, namely, the six $a(N)$'s

TABLE III. $(\Delta q)^B$ defined in Eq. (8) for physical baryon B given by general wavefunction in Eq. (2). The normalizations N_1, N_2, N_3, and N_4 are given in Eqs. (6). The $(\Delta q)^{\Sigma^0\Lambda}$ for the $\Sigma^0 \to \Lambda$ transition magnetic moment is also given.

$$(\Delta u)^p = \frac{1}{3N_1^2}(4 + 4\bar\beta_1^2 + \tfrac{2}{3}\bar\beta_3^2 + \tfrac{10}{3}\bar\beta_4^2 - 2\bar\beta_2\bar\beta_4)$$

$$(\Delta d)^p = \frac{1}{3N_1^2}(-1 - \bar\beta_1^2 + \tfrac{7}{3}\bar\beta_3^2 + \tfrac{2}{3}\bar\beta_4^2 + 2\bar\beta_2\bar\beta_4) \qquad (\Delta s)^p = \frac{1}{3N_1^2}(3\bar\beta_2^2 - \bar\beta_4^2)$$

$$(\Delta u)^n = (\Delta d)^p \qquad (\Delta d)^n = (\Delta u)^p \qquad (\Delta s)^n = (\Delta s)^p$$

$$(\Delta u)^{\Xi^0} = \frac{1}{3N_2^2}(-1 - \beta_1^2 - \tfrac{1}{3}\beta_3^2 + \tfrac{10}{3}\beta_4^2 - 2\beta_2\beta_4)$$

$$(\Delta d)^{\Xi^0} = \frac{1}{3N_2^2}(-\tfrac{2}{3}\beta_3^2 + \tfrac{2}{3}\beta_4^2 + 2\beta_2\beta_4) \qquad (\Delta s)^{\Xi^0} = \frac{1}{3N_2^2}(4 + 4\beta_1^2 + 3\beta_2^2 + 4\beta_3^2 - \beta_4^2)$$

$$(\Delta u)^{\Xi^-} = (\Delta d)^{\Xi^0} \qquad (\Delta d)^{\Xi^-} = (\Delta u)^{\Xi^0} \qquad (\Delta s)^{\Xi^-} = (\Delta s)^{\Xi^0}$$

$$(\Delta u)^{\Sigma^+} = \frac{1}{3N_3^2}(4 + 4\gamma_1^2 - \gamma_2^2 + 4\gamma_4^2 + 3\gamma_5^2 - \sqrt{6}\gamma_3\gamma_5)$$

$$(\Delta d)^{\Sigma^+} = \frac{1}{3N_3^2}(-\gamma_1^2 + \gamma_5^2 + \sqrt{6}\gamma_3\gamma_5) \qquad (\Delta s)^{\Sigma^+} = \frac{1}{3N_3^2}(-1 + 4\gamma_2^2 + 3\gamma_3^2 - \gamma_4^2 - \gamma_5^2)$$

$$(\Delta u)^{\Sigma^-} = (\Delta d)^{\Sigma^+} \qquad (\Delta d)^{\Sigma^-} = (\Delta u)^{\Sigma^+} \qquad (\Delta s)^{\Sigma^-} = (\Delta s)^{\Sigma^+}$$

$$(\Delta u)^{\Sigma^0} = \tfrac{1}{2}[(\Delta u)^{\Sigma^+} + (\Delta u)^{\Sigma^-}] \qquad (\Delta d)^{\Sigma^0} = (\Delta u)^{\Sigma^0} \qquad (\Delta s)^{\Sigma^0} = (\Delta s)^{\Sigma^+}$$

$$(\Delta u)^{\Lambda} = \frac{1}{3N_4^2}(\tfrac{3}{2}\delta_1^2 - \tfrac{1}{2}\delta_2^2 + 2\delta_4^2)$$

$$(\Delta d)^{\Lambda} = (\Delta u)^{\Lambda} \qquad (\Delta s)^{\Lambda} = \frac{1}{3N_4^2}(3 + 4\delta_2^2 + 3\delta_3^2 - \delta_4^2)$$

$$(\Delta u)^{\Sigma^0\Lambda} = \frac{1}{N_3N_4}(\tfrac{1}{\sqrt{3}} + \tfrac{1}{\sqrt{3}}\gamma_4\delta_3 - \tfrac{1}{3}\gamma_3\delta_4 + \tfrac{5}{6}\gamma_1\delta_1 - \tfrac{1}{6}\gamma_2\delta_2 + \tfrac{4}{3\sqrt{6}}\gamma_5\delta_4)$$

$$(\Delta d)^{\Sigma^0\Lambda} = -(\Delta u)^{\Sigma^0\Lambda} \qquad (\Delta s)^{\Sigma^0\Lambda} = 0$$

for the scalar sea. Despite, our general wavefunction, the isospin sum rule [11]

$$\mu_{\Sigma^0} = \tfrac{1}{2}(\mu_{\Sigma^+} + \mu_{\Sigma^-}) \tag{12}$$

holds. This is because $\hat\mu$ transforms as $(I=0)\oplus(I=1)$ in isospin space and the wavefunction of Eq. (2) respects isospin thus giving $(\Delta q)^{\Sigma^0} = ((\Delta q)^{\Sigma^+} + (\Delta q)^{\Sigma^-})/2$ (see Table III).

Further, if we require the physical baryon states given by Eq. (2) to transform as an SU(3) octet (i.e., put $a(N) = 0$, $N = \mathbf{1}, \mathbf{10}, \overline{\mathbf{10}}, \mathbf{27}$) then the seven μ_B's and $\mu_{\Sigma^0\Lambda}$ depend non-linearly on two parameters ($a(\mathbf{8_F})$ and $a(\mathbf{8_D})$) of the wavefunction and the three μ_q's. Even so, three sum rules, namely

$$\mu_p - \mu_n = (\mu_{\Sigma^+} - \mu_{\Sigma^-}) - (\mu_{\Xi^0} - \mu_{\Xi^-}) \tag{13}$$

$$((4.70589019 \pm 5\times10^{-7})\ \mu_N) \qquad ((4.217 \pm 0.031)\ \mu_N) \tag{14}$$

$$-6\mu_\Lambda = \mu_{\Sigma^+} + \mu_{\Sigma^-} - 2(\mu_p + \mu_n + \mu_{\Xi^0} + \mu_{\Xi^-}) \tag{15}$$

$$((3.678 \pm 0.024)\ \mu_N) \qquad ((3.340 \pm 0.039)\ \mu_N) \tag{16}$$

$$2\sqrt{3}\mu_{\Sigma^0\Lambda} = 2(\mu_p - \mu_n) - (\mu_{\Sigma^+} - \mu_{\Sigma^-}) \tag{17}$$

$$((5.577 \pm 0.277)\ \mu_N) \qquad ((5.794 \pm 0.027)\ \mu_N) \tag{18}$$

emerge. These have been noted earlier in the context of other models [6,9]. The values of the two sides taken from data [10] are shown in parentheses. The reason these sum rules hold despite the number of parameters is because they are a consequence of flavor SU(3) since baryons form a SU(3) octet and $\hat{\mu}$ transforms as $\mathbf{1} \oplus \mathbf{8}$. However, as can be seen, the first two SU(3) sum rules are not well satisfied experimentally. To avoid them one could modify the SU(3) transformation properties of $\hat{\mu}$ or the baryons. A group-theoretic analysis with the most general $\hat{\mu}$ which would contribute to the magnetic moments of an octet was done by Dothan [12] over a decade ago. Such a $\hat{\mu}$ could arise from SU(3) breaking effects. However, several authors [13] have considered models in which they modify the baryon wavefunction. In our approach, we keep $\hat{\mu}$ as given by the quark model but modify the baryon wavefunction by taking a sea with flavor and spin into account.

As noted above the poorly satisfied $SU(3)$ sum rules Eqs. (13)–(14) and (15)–(16) will hold as long the physical baryon is restricted to be an octet. This means we must include SU(3) breaking effects in the baryon wavefunction by considering non-zero $a(N)$ with $N = \mathbf{1}, \mathbf{10}, \overline{\mathbf{10}}, \mathbf{27}$.

In making our fits we have used *experimental errors* as given by Particle Data Group [10]. This is in contrast to many authors who use "theoretical errors" of the order of a few percent or more to fit the data. In actual fact the experimental errors are very much smaller. Furthermore, we keep in mind that the constituent quark masses are m_u, $m_d \approx 300\,\text{MeV}$, and $m_s \approx 500\,\text{Mev}$, so we expect $\mu_u \cong -2\mu_d > 0$ and $\mu_s \cong 0.6\mu_d$. Current quark masses would give a very different numerical range for the ratios μ_u/μ_d and μ_s/μ_d. Also, if the core baryon contribution is dominant then the parameters determining the sea should be small compared to unity. Furthermore, for a dominantly SU(3) octet physical baryon $a(\mathbf{8_F})$ and $a(\mathbf{8_D})$ should be larger than the other parameters in the wave function.

SQM has three parameters μ_q ($q = u$, d, s) the quark magnetic moments in nuclear magnetons μ_N. A fit using experimental errors gives $\chi^2/\text{DOF} = 1818/5$ with $\mu_u = 1.852$, $\mu_d = -0.972$, $\mu_s = -0.701$.

The situation improves a little for a pure octet physical baryon with scalar sea described by $a(\mathbf{8_F})$ and $a(\mathbf{8_D})$. These two sea parameters enter Eqs. (10)–(11) only through the three combinations given by $(\Delta q)^p$. Hence, the 3 sum rules in Eqs. (13)-(18). For *experimental errors* with μ_q also as parameters one obtains $\chi^2/\text{DOF} = 652/3$. Most of the contribution to χ^2 comes from a poor fit to μ_{Σ^+}, μ_{Σ^-}, and μ_{Ξ^0}. This is a clear indication that admixture of other SU(3) representations in our wavefunction need to be considered.

To get a feeling for how the sea contributes we did extensive and systematic numerical analysis. In all the fits, in addition to the sea parameters, μ_q were treated as parameters.

In general, there are six parameters $a(N)$'s ($N = \mathbf{1}$, $\mathbf{8_F}$, $\mathbf{8_D}$, $\mathbf{10}$, $\overline{\mathbf{10}}$, $\mathbf{27}$) in the wavefunction, and the three μ_q's. These nine parameters provide a perfect fit with $\chi^2 = 1.5 \times 10^{-4}$. This clearly means that the scalar sea contribution modifies the values of $(\Delta q)^B$ in the right direction for a fit to the baryon magnetic moments. However, a seven parameter fit with $a(\mathbf{1}) = 0.0625$, $a(\mathbf{8_D}) = -0.1558$, $a(\mathbf{8_F}) = 0.1896$, $a(\mathbf{10}) = 0.4297$, and $\mu_u = 1.85893$, $\mu_d =$

TABLE IV. Predictions for the baryon magnetic moments (in N.M.), along with the eight experimental measurements currently available.

Baryon	Data	SQM	Scalar sea
p	$2.79284739 \pm 6 \times 10^{-8}$	2.7928	2.7928
n	$-1.9130428 \pm 5 \times 10^{-7}$	−1.9130	−1.9130
Λ	-0.613 ± 0.004	−0.701	−0.616
Σ^+	2.458 ± 0.010	2.703	2.456
Σ^0	——	0.8203	0.6238
Σ^-	-1.160 ± 0.025	−1.062	−1.208
Ξ^0	-1.250 ± 0.014	−1.552	−1.248
Ξ^-	-0.6507 ± 0.0025	−0.6111	−0.6500
$\lvert\Sigma^0 \to \Lambda\rvert$	1.61 ± 0.08	1.63	1.52
χ^2/DOF	——	1818/5	5.60/2

-0.99884, and $\mu_s = -0.65302$, provides an excellent fit with $\chi^2/\text{DOF} = 0.838$. The μ_q (in units of μ_N) imply for the quark masses the values $m_u = 336.49\,\text{MeV}$, $m_d = 313.12\,\text{MeV}$, and $m_s = 478.94\,\text{MeV}$, which are in accord with the constituent quark model. A noteworthy six parameter fit with $\chi^2/\text{DOF} = 5.60/2$ is given by $a(\mathbf{8_D}) = -0.2262$, $a(\mathbf{8_F}) = 0.2776$, and $a(\mathbf{10}) = 0.4216$, with $\mu_u = 1.86688$, $\mu_d = -1.02558$, and $\mu_s = -0.64663$. The predictions of this six parameter fit are displayed in the "Scalar sea" column of Table IV. This fit gives $(\Delta u)^p = 1.322$, $(\Delta d)^p = -0.304$, and $(\Delta s)^p = -0.018$, which are near SQM values with a small strangeness content in the nucleon.

4. SUMMARY

In summary, we have considered the physical spin 1/2 low-lying baryons to be formed out "core" baryons (described by the q^3-wavefunction of SQM) and a flavor octet but color singlet "sea".

This sea (which may contain arbitrary number of gluons and $q\bar{q}$-pairs) is specified only by its total flavor and spin quantum numbers. The most general wavefunction for the physical baryons for an octet sea with spin 0 was considered which respected isospin and hypercharge (or strangeness). Owing to the flavor properties of the sea the nucleons can have a non-zero strange quark content (giving $(\Delta s)^p = (\Delta s)^n \neq 0$) through the strange core baryons. In this model the eight baryons no longer form an exact SU(3) octet. The admixture of other flavor SU(3) representations in the wavefunction is understood to represent broken SU(3) effects. The parameters in the wavefunction describing the sea were determined by application to the available data on baryon magnetic moments. To these 8 pieces of data we found good fits with six parameters *using available experimental errors* [10]. Three of these parameters determined the sea contribution while the other three were μ_q's (or m_q's, $q = u, d, s$) the quark magnetic moments (masses). The modified baryon wavefunction including such a sea suggested by the fits to the data is simply

$$B(1/2\uparrow) = \tilde{B}(\mathbf{8}, 1/2\uparrow)H_0 S(\mathbf{1}) + \sum_{N=\mathbf{8_D},\mathbf{8_F},\mathbf{10}} a(N)\left[\tilde{B}(\mathbf{8}, 1/2\uparrow)H_0 \otimes S(\mathbf{8})\right]_N . \qquad (19)$$

Our results suggest that the physical spin 1/2 "octet" baryons contain an admixture of primarily the **10** representation. Why SU(3) breaking (which we have invoked through a flavor octet sea) induce this representation is a question for the future when one is able to calculate the parameters in the wavefunction of Eq. (2) reliably from quantum chromodynamics.

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