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The effect of chirality on a plasma media

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ABSTRACT. The theoretical properties of a composite chiral-plasma medium are developed. By using the reaction theorem, for a magnetized chiroplasma, we obtain the proof of nonreciprocity based upon the constitutive relationships between electromagnetic vectors $\vec{E}$, $\vec{B}$, $\vec{H}$, $\vec{D}$. Using the Maxwell's equations and the proposed constitutive relations for a chiral-plasma medium, we derive the vectors $\vec{E}$ and $\vec{H}$ equations and from these equations, dispersion relations and $\vec{E}$-field polarization are obtained. For circularly polarized waves, a new mode conversion is founded due to the chiral effect. For the lower branch of the extraordinary wave mode there is no more bands of forbidden frequencies and the reflection point vanishes when the chiral parameter increases.

RESUMEN. Las propiedades teóricas de un medio compuesto del tipo quiroplasma son obtenidas. A través del teorema de reacción, para un quiroplasma magnetizado, se obtiene la prueba de reciprocidad en base a las relaciones constitutivas entre los vectores electromagnéticos $\vec{E}$, $\vec{B}$, $\vec{H}$, $\vec{D}$. Usando la ecuaciones de Maxwell y las relaciones constitutivas propuestas para un medio del tipo quiroplasma, se derivan las ecuaciones para $\vec{E}$ y $\vec{H}$, las relaciones de dispersión y la polarización del campo $\vec{E}$. Para ondas circularmente polarizadas, una nueva conversión de modos es obtenida debida al efecto quiral. Para la rama inferior del modo extraordinario no hay más bandas de frecuencias prohibidas y el punto de reflexión desaparece cuando el parámetro quiral aumenta.

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1. INTRODUCTION

Chirality is a purely geometric motion that refers to the lack of translational and rotational symmetry such that a chiral object cannot be superimposed on its mirror image. A chiral object has the property of handedness: It must be either left-handed or right-handed. An equal amount of right- and left-handed chiral objects would cause the sample to be racemic and absence of handedness all together would make an achiral sample. While ε and μ are sufficient to describe ordinary dielectric media permitting a single phase velocity (constitutive equations being: $\vec{D} = \varepsilon\vec{E}$ and $\vec{B} = \mu\vec{H}$), chiral media need an additional parameter called chirality parameter t to describe their handedness (positive for right-handed and negative for left-handed medium). As the chirality parameter charges sign on spatial inversion, t is a pseudoscalar. Based on the constitutive equations, it can be shown

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that linearly polarized waves cannot propagate through a chiral medium while right- and left-circularly polarized waves propagating with different phase velocities and attenuations are perfectly acceptable solutions of the wave equation in such media.

The electromagnetic chirality is also known as optical activity. The handedness of the uniformly distributed and randomly oriented chiral objects, which compose the chiral medium, is responsible for the observed optical activity. A chiral media when interacting with an electromagnetic wave can rotate the plane of polarization of the wave to the right or to the left depending on the handedness of the media. This fact is expected to play an important role in the potential application of chiral medias in the microwave and optical regimes. Plasma is a good media to simulate waves propagation by including the chiral effect. Chiral media [1–3] have been studied over the last decade for many applications. Chiral-media have been examined as coating for reducing radar cross section, for antennas and arrays, for antenna radomes in waveguides and for microstrip substrate. Here, we examine a chiral-plasma medium, which is non-reciprocal due to the external magnetic field. The chiral plasma composite can be modeled as a random distribution of chiral inclusions, considered as spheres or tetrahedrons, embedded in an infinitely nonchiral magnetized plasma. Such model may be important in calculation of an artificial plasma formation due to microwave pulse in the overlapping lower ionosphere and upper part of the neutral atmosphere, where we have the ozone layer. This plasma can have a property of chirality if we considerer aerosols and CFC's pollution.

To calculate the general dispersion relation, which gives the ω against $\vec{k}$ behavior, vector phasor Helmholtz based equations are derived. We determine the modal eigenvalue properties in the chiral-plasma medium, which is doubly anisotropic. For the case of waves which propagate parallel to the magnetic field, the medium is a cold magnetized chiral-plasma, which presents a new mode conversion. We compare our results with the typical results obtained for a cold plasma [4]. Also we obtain the chiral-faraday rotation which can be compared with the typical Faraday rotation for a pair of right- and left-handed circularly polarized waves. For waves which propagate perpendicular to the magnetic field, there is no mode conversion but a lower band of forbidden frequencies disappears.

2. THEORETICAL FOUNDATIONS

We propose the following constitutive relations for chiral-plasma media:

$$\vec{D} = \epsilon_0 \overleftrightarrow{\epsilon} \cdot \vec{E} + t_1 \vec{H} \tag{1}$$

$$\vec{B} = \mu \vec{H} + t_2 \vec{E}. \tag{2}$$

Plasma media constitutive relations are [4]

$$\vec{D}_{\mathrm{p}} = \epsilon_0 \overleftrightarrow{\epsilon}_{\mathrm{p}} \cdot \vec{E}_{\mathrm{p}}, \tag{3}$$

$$\vec{B}_{\mathrm{p}} = \mu_0 \vec{H}_{\mathrm{p}}, \tag{4}$$

where

$$\overleftrightarrow{\epsilon}_{\mathrm{p}} = \begin{vmatrix} \epsilon_1 & i\epsilon_2 & 0 \\ -i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{vmatrix}. \tag{5}$$

$\overleftrightarrow{\epsilon}$ and $t_{1,2}$ represent the permitivity tensor and chirality parameters of the composite medium, respectively. The lossless character of the magnetized cold plasma medium is implied by the hermitian nature of the tensor $(\overleftrightarrow{\epsilon}^{*}_{\mathrm{p}})^T = \overleftrightarrow{\epsilon}_{\mathrm{p}}$. The superscripts * and ^{T} denote the complex conjugate and the transpose, respectively.

In the search for new media, which displays nonreciprocal properties, it is essential to establish the nature of the chirality parameter t_1 and t_2. The anisotropic reaction theorem [5] is

$$\int \vec{E}_b \cdot \vec{J}_a \, dv = \int \tilde{E}_a \cdot \tilde{J}_b \, dv. \tag{6}$$

Here, we see that source currents $\vec{J}_a$ and $\vec{J}_b$ produce fields $\vec{E}_a$ and $\vec{E}_b$, respectively, and the *tilde* over the fields indicates a new media altered from the original media. Thus, we obtain 6×6 constitutive tensors

$$\tilde{A} = \begin{vmatrix} \hat{\epsilon}^T & -\hat{t}\,'^T \\ -\hat{t}^T & \hat{\mu}^T \end{vmatrix} \tag{7}$$

and

$$\hat{A} = \begin{vmatrix} \hat{\epsilon} & -\hat{t} \\ -\hat{t}\,' & \hat{\mu} \end{vmatrix}, \tag{8}$$

with $\hat{t}$ and $\hat{t}\,'$ being the optical activity 3×3 tensors.

Reciprocity occurs only if

$$\int \vec{E}_b \cdot \vec{J}_a \, dv = \int \vec{E}_a \cdot \vec{J}_b \, dv,$$

that is, by Eq. (6) it requires

$$\tilde{A} = \hat{A}. \tag{9}$$

For chiral media we must obtain

$$\hat{\epsilon} = 0, \qquad \hat{t} = t_1 I, \qquad \hat{t}\,' - t_2 I, \qquad \hat{\mu} = 0. \tag{10}$$

To obtain reciprocity, Eq. (9) imposes

$$-t_2 I^T = t_1 I, \qquad -t_1 I^T = t_2 I, \tag{11}$$

that is,

$$t_1 = -t_2. \tag{12}$$

For plasma media Eqs. (4) hold leading to

$$\hat{\epsilon} = \overleftrightarrow{\epsilon}, \qquad \hat{t} = \hat{t}' = 0, \qquad \hat{\mu} = \mu_0 = 1. \tag{13}$$

Then for the proposed constitutive relations [Eqs. (1) and (2)] we have

$$\vec{D} = \epsilon_0 \overleftrightarrow{\epsilon} \cdot \vec{E} + t_1 \vec{H}, \tag{14}$$

$$\vec{B} = \mu_0 \vec{H} + t_2 \vec{E}. \tag{15}$$

3. VECTOR HELMHOLTZ EQUATIONS

The E–field vector Helmholtz equation is derived by inserting the constitutive relation Eqs. (14) and (15) into Maxwell's equations

$$\nabla \times \vec{E} = -i\omega \vec{B}, \tag{16}$$

$$\nabla \times \vec{H} = i\omega \vec{D} + \vec{J}, \tag{17}$$

so

$$\nabla \times \vec{E} = -i\omega\mu_0 \vec{H} - i\omega t_2 \vec{E}, \tag{18}$$

$$\nabla \times \vec{H} = i\omega\epsilon_0 \overleftrightarrow{\epsilon} \cdot \vec{E} + i\omega t_1 \vec{H}. \tag{19}$$

Solving for $\vec{H}$, Eq. (18) gives

$$\vec{H} = \frac{1}{\mu_0}\left(\frac{i}{\omega}\nabla \times \vec{E} - t_2 \vec{E}\right), \tag{20}$$

and putting this into Eq. (19) we obtain

$$\nabla \times \vec{H} = \frac{1}{\mu_0}\frac{i}{\omega}(\nabla \times \nabla \times \vec{E}) - \frac{t_2}{\mu_0}\nabla \times \vec{E}. \tag{21}$$

Then the $\vec{E}$-field vector equation is

$$\nabla \times \nabla \times \vec{E} + i\omega(t_2 - t_1)\nabla \times \vec{E} - \omega^2\mu_0\epsilon_0\left(\overleftrightarrow{\epsilon} - \frac{t_1 t_2}{\mu_0\epsilon_0}\right) \cdot \vec{E} = 0. \tag{22}$$

Here, the plasma current is included in the permitivity tensor $\overleftrightarrow{\epsilon}$.

Similarly the $\vec{H}$-field vector equation is

$$\nabla \times \overleftrightarrow{\epsilon}^{-1} \nabla \times \vec{H} + i\omega(t_2 \overleftrightarrow{\epsilon}^{-1} \nabla \times \vec{H} - t_1 \nabla \times \overleftrightarrow{\epsilon}^{-1} \times \vec{H}) - \omega^2 \mu_0 \left(\mathbf{I} - \frac{t_1 t_2}{\mu_0} \overleftrightarrow{\epsilon}^{-1}\right) \vec{H} = 0. \quad (23)$$

The inverse permitivity tensor is

$$\overleftrightarrow{\epsilon}^{-1} = \begin{vmatrix} \epsilon_1 & -i\epsilon_2 & 0 \\ i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \frac{\epsilon_1^2 - \epsilon_2^2}{\epsilon_3} \end{vmatrix}. \quad (24)$$

4. Dispersion relations

The dispersion relation for the propagation vector $\vec{k}$ against ω can be obtained from $\vec{E}$- or $\vec{H}$-vector equation. We start with the $\vec{E}$-field relation which is simpler than the $\vec{H}$-equation.

Defining $\vec{E}$ as

$$\vec{E} = \vec{E}_0 \, e^{-i\vec{k}\cdot\vec{r}}, \quad (25)$$

we obtain

$$-\vec{k} \times \vec{k} \times \vec{E}_0 + \omega(t_2 - t_1)\vec{k} \times \vec{E}_0 - \omega^2 \mu_0 \epsilon_0 \left(\overleftrightarrow{\epsilon} - \frac{t_1 t_2}{\mu_0 \epsilon_0}\right) \vec{E}_0 = 0. \quad (26)$$

Putting $\vec{E}_0$ into rectangular coordinates,

$$\vec{E}_0 = E_x \hat{x} + E_y \hat{y} + E_z \hat{z}, \quad (27)$$

we obtain three components system of equations which determine the eigenvector, and the determinant of the coefficient component matrix M_k will determine the eigenvalues, thereby yielding the ω against $\vec{k}$ dispersion diagram in phase-space. Writing $\mathrm{Det}(M_k) = 0$, with $k_x = 0$ and with symmetry about the z-axis we obtain

$$\begin{vmatrix} 1 - \frac{\epsilon_1}{n^2}\left(1 - \frac{t_1 t_2}{\epsilon_0 \mu_0 \epsilon_1}\right) & -\frac{i\epsilon_2}{n^2} - \frac{\cos\theta(t_2 - t_1)}{n\sqrt{\mu_0\epsilon_0}} & \frac{\sin\theta(t_2 - t_1)}{n\sqrt{\mu_0\epsilon_0}} \\ \frac{i\epsilon_2}{n^2} + \frac{\cos\theta(t_2 - t_1)}{n\sqrt{\mu_0\epsilon_0}} & \cos^2\theta - \frac{\epsilon_1}{n^2}\left(1 - \frac{t_1 t_2}{\epsilon_0\mu_0\epsilon_1}\right) & -\sin\theta\cos\theta \\ -\frac{\sin\theta(t_2 - t_1)}{n\sqrt{\mu_0\epsilon_0}} & -\sin\theta\cos\theta & \sin^2\theta - \frac{\epsilon_3}{n^2}\left(1 - \frac{t_1 t_2}{\epsilon_3\mu_0\epsilon_0}\right) \end{vmatrix} = 0. \quad (28)$$

Here, the refractive index n is defined as

$$n = \frac{ck}{\omega}, \quad \text{where} \quad c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.$$

If $\mu_0 = 1$, $\epsilon_0 = 1$ and $t_1 = t_2 = 0$ we obtain the same results given by Krall and Trivelpiece for a magneto-plasma [4].

For a loss less chiroplasma, *i.e.*, $t_1 = it\sqrt{\mu_0\epsilon_0}$ and $t_2 = -it\sqrt{\mu_0\epsilon_0}$, the non-trivial solution of this system comes from setting the determinant of the coefficients equal to zero, giving

$$f(\theta) = F(n^2, \omega, \epsilon_1, \epsilon_2, t, \epsilon_3, k). \tag{29}$$

Equation (29) is then the general dispersion relation for waves propagating in a cold collision less homogeneous chiroplasma in a uniform magnetic field. For a given plasma frequency ω_{p}, a given cyclotron frequency ω_{c}, a given wave frequency ω and a given direction of propagation θ, Eq. (29) can be solved for the index of refraction n, having as parameter the chirality t.

In terms of k, the dispersion relation is

$$a_1 k^4 + a_2 k^3 + a_3 k^2 + a_4 k + a_5 = 0, \tag{30}$$

where

$$a_1 = -\left[\frac{\omega^2}{c^2}\epsilon_1\left(1 - \frac{t^2}{\epsilon_1}\right)\sin^2\theta - \frac{\omega^2}{c^2}\epsilon_3\left(1 - \frac{t^2}{\epsilon_3}\right)\cos^2\theta\right], \tag{31}$$

$$a_2 = 0, \tag{32}$$

$$\begin{aligned} a_3 = {} & \frac{w^4}{c^4}\left[\epsilon_1\epsilon_3\left(1 - \frac{t^2}{\epsilon_1}\right)\left(1 - \frac{t^2}{\epsilon_3}\right)\right] \\ & + \frac{w^4}{c^4}\left[\epsilon_1\epsilon_3\left(1 + \frac{3t^2}{\epsilon_1}\right)\left(1 - \frac{t^2}{\epsilon_3}\right)\right]\cos^2\theta \\ & + \frac{w^4}{c^4}\left[\left(\epsilon_1\left(1 + \frac{t^2}{\epsilon_1}\right)\right)^2 - \epsilon_2^2 - 4t^4\right]\sin^2\theta, \end{aligned} \tag{33}$$

$$a_4 = -4\frac{w^5}{c^5}t\epsilon_2\epsilon_3\left(1 - \frac{t^2}{\epsilon_3}\right)\cos\theta, \tag{34}$$

$$\begin{aligned} a_5 = {} & \frac{w^6}{c^6}\left[\epsilon_3\left(\epsilon_2^2 - \epsilon_1^2\right)\left(1 - \frac{t^2}{\epsilon_3}\right)\right. \\ & \left. + 2t^2\epsilon_1\left(\epsilon_3 - t^2\right) - t^4\left(\epsilon_3 - t^2\right)\right]. \end{aligned} \tag{35}$$

Here, there are four different eigenmodes for $\vec{k}$ as implied by Eq. (30). The components of the permitivity tensor are obtained using the constitutive equations (14) and (15), and are given by

$$\epsilon_1 = 1 - \frac{w_{\mathrm{p}}^2}{w^2 - w_c^2}, \tag{36}$$

$$\epsilon_2 = \frac{w_{ce}}{w}\,\frac{w_{\mathrm{p}}^2}{w^2 - w_c^2}, \tag{37}$$

$$\epsilon_3 = 1 - \frac{w_{\mathrm{p}}^2}{w^2}, \tag{38}$$

where w_{p} is the plasma frequency and w_{c} is the electron gyrofrequency given by

$$\omega_{\mathrm{p}}^2 = \frac{4\pi n_e e^2}{m_e}, \omega_{\mathrm{c}} = \frac{eB_0}{m_e c}. \tag{39}$$

We can observe that for $t = 0$ we obtain the same expressions given by Krall and Trivelpiece [4] for a plasma media.

5. HIGH-FREQUENCY WAVES WITH $\vec{k} \parallel \vec{B}_0$ AND $\vec{k} \perp \vec{B}_0$

Setting $\theta = 0$, it is possible from Eq. (28) to find circularly polarized waves, writing the $\vec{E}$-field vector equation in the form

$$\begin{aligned} (n^2 - \epsilon_{\mathrm{R}})E_{\mathrm{R}} &= 0, \\ (n^2 - \epsilon_{\mathrm{L}})E_{\mathrm{L}} &= 0, \\ \epsilon_3\left(1 - \frac{t^2}{\epsilon_3}\right)E_z &= 0, \end{aligned} \tag{40}$$

where

$$\epsilon_{\mathrm{R,L}} = \epsilon_1\left(1 - \frac{t^2}{\epsilon_1}\right) \pm \epsilon_2\left(1 - \frac{2tn}{\epsilon_2}\right) \tag{41}$$

and

$$E_{\mathrm{R,L}} = E_x \pm iE_y. \tag{42}$$

It is useful to explore these solutions in terms of the wavenumbers k:

$$k_{\mathrm{R}} = \frac{t\omega}{c} \pm \frac{\omega}{c}\sqrt{\epsilon_1 - \epsilon_2} \tag{43}$$

and

$$k_L = -\frac{t\omega}{c} \pm \frac{\omega}{c}\sqrt{\epsilon_1 + \epsilon_2} \tag{44}$$

where k_R is the wave number for a circularly polarized wave which drive electrons in the direction of their cyclotron motion, *i.e.*, right-hand circularly polarized waves (RCP) and k_L is the wave number for a circularly polarized wave which drive electrons in the direction opposite of their cyclotron motion, *i.e.*, left-hand circularly polarized waves (LCP). The t parameter modifies the typical plot of $\omega(k)$ shown by Krall and Trivelpiece, where the cutoff frequencies are shifted. When $\epsilon_3 + t^2 = 1 - \frac{w_p^2}{w^2}$ is zero, we obtain the longitudinal electron plasma oscillations modified by the chiral parameter t. Since there is not wave propagation along the magnetic field, these chiroplasma oscillations do not constitute a propagation mode. Also the reflection points of the RCP and LCP are shifted. However the resonance which occurs when the wave phase velocity goes to zero is not modified by the chiral parameter. In Fig. 1 we present the modifications introduced by the parameter t in the dispersion relations of the right and left polarized waves. In this figure the dispersion relations of the right and left circularly polarized waves are indicated by circles and stars, respectively. When $t \neq 0$, ϵ_1 and ϵ_3 depend on t and k_R and k_L have a linear term, $t\omega/c$, as can be seen in Eqs. (43) and (44). In this way, further than to modify the curves that exist for $t = 0$, the parameter t permits that the wave propagates in a region of frequencies that is forbidden in the case $t = 0$. The RCP wave mode, in the lower branch, also known as the electron cyclotron wave is weakly modified by the chiral parameter, but the upper branch is strongly shifted when t increases. We can observe in the Fig. 1 that for $t = 0$, there is no intersection of the dispersion relations of the right and left circularly polarized waves. Here the frequency bands for which there is no wave propagation can be identified in the plots. When $t \neq 0$ we can observe that there is an intersection of these curves, indicating that the presence of the t parameter permits that a wave changes its polarization. In the Fig. 1 for $t = 0$ we can also observe that there is a region where only right circularly polarized waves propagate, a region where only left circularly polarized waves propagate and a region where both propagate. If their amplitude are equal, the effect of the superposition of a left and right circularly polarized wave is to produce a plane wave with a particular plane of polarization. Because the two polarizations propagate at different velocities, the plane of polarization rotates as the wave propagates along the magnetic field. This effect is called Faraday rotation.

The global rotation of the plane of polarization as a function of distance in the direction of propagation is given by

$$\frac{E_x}{E_y} = \cot\left(\frac{k_L - k_R}{2}\right) z, \tag{45}$$

which means that the presence of the t parameter affects also the Faraday rotation. This chiral-Faraday rotation can be used as a plasma probe. In a laboratory experiment this would be done by launching a planewave along the magnetic field in a chiroplasma. Considering that the plane of polarization of this wave can be determined by an antenna

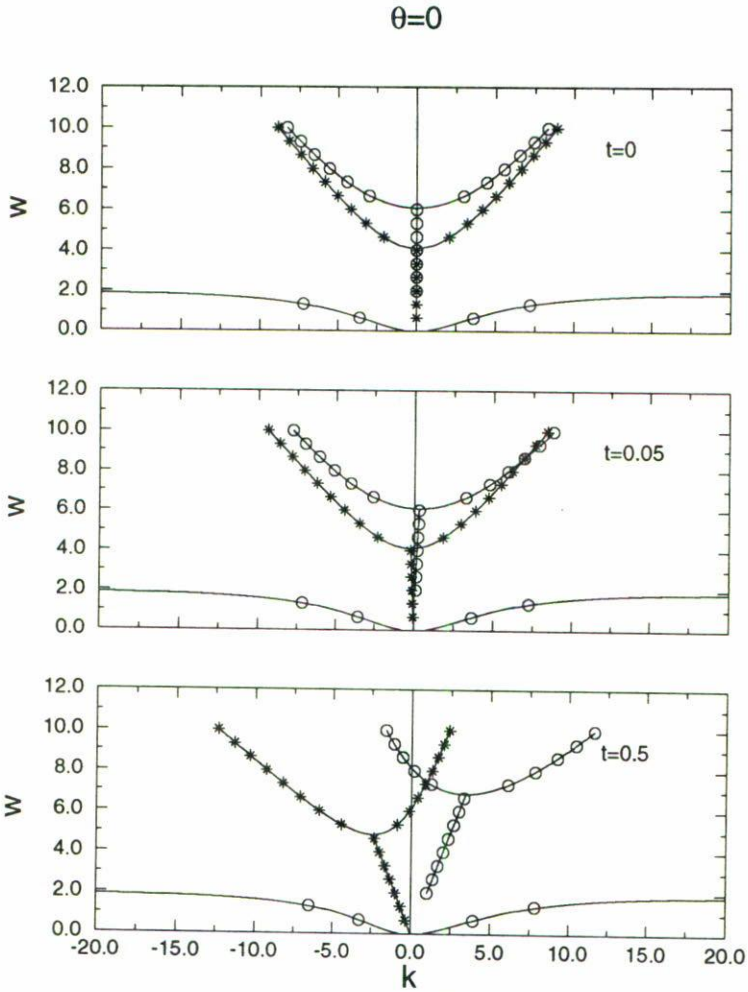

FIGURE 1. Dispersion relations for various values of the parameter t when the direction of propagation is parallel to the magnetic field ($\theta = 0$). The curves indicated by circles and stars correspond to the right and left circularly polarized waves, respectively.

and that we know the magnetic field, the density of the plasma and the frequency of the launched wave, the measurement of the plane of polarization away from the source can determine the value of the parameter t. For instance, considering for the plasma frequency, $w_{\mathrm{p}} = 5.10^7\ \mathrm{s}^{-1}$, for the electron gyrofrequency, $w_{\mathrm{c}} = 2.10^7\ \mathrm{s}^{-1}$, and for the launched wave, $w = 6.5.10^7\ \mathrm{s}^{-1}$, the value of the plane of polarization 1 cm away from the source is $E_x/E_y = 85.76$, $E_x/E_y = 118.17$ and $E_x/E_y = 186.11$ for $t = 0$, $t = 0.05$ and $t = 0.1$, respectively. Another important effect caused by the presence of the parameter t is the conversion of modes. In Fig. 1 for $t = 0.5$ we can also observe that there is a region where both RCP and LCP propagate. For $k = 1.0$ we have a mode conversion from RCP to LCP wave This means that the energy of the electrons, obtained from the RCP electromagnetic wave at the electron cyclotron frequency can be transferred to the ions and this mode conversion can be used as a means of heating of plasma. Note that the RCP wave rotates in the same direction as the electrons about the magnetic field and near of resonance the energy is transferred from the wave field to the electrons but the mode conversion allows the absorption of energy by the ions.

Setting $\theta = \pi/2$, we obtain the following dispersion relations:

$$k_X = \pm \frac{\sqrt{A - \sqrt{B}}}{\sqrt{2(\epsilon_1 - t^2)}} \tag{46}$$

and

$$k_O = \pm \frac{\sqrt{A + \sqrt{B}}}{\sqrt{2(\epsilon_1 - t^2)}}, \tag{47}$$

where

$$A = \frac{\omega^2}{c^2} \left[\epsilon_1^2 - \epsilon_2^2 + \epsilon_1\epsilon_3 + t^2(\epsilon_1 - \epsilon_3) - 2t^4\right] \tag{48}$$

and

$$\begin{aligned} B = \frac{\omega^4}{c^4} \Big[& \left((\epsilon_1^2 - \epsilon_2^2) - \epsilon_1\epsilon_3\right)^2 \\ & + t^2(6\epsilon_1^3 - 6\epsilon_1\epsilon_2^2 - 2\epsilon_1\epsilon_3^2 + 12\epsilon_1^2\epsilon_3 - 2\epsilon_2^2\epsilon_3) \\ & + t^4(-15\epsilon_1^2 + 8\epsilon_2^2 - 18\epsilon_1\epsilon_3 + \epsilon_3^2) + 8t^6(\epsilon_1 + \epsilon_3)\Big]. \end{aligned} \tag{49}$$

It should be pointed out that the electric field of the extraordinary wave, k_X is perpendicular to the magnetic field and the electric field of the ordinary wave, k_O, is parallel to the magnetic field.

In Fig. 2 we present the effect of the parameter t on the dispersion relations for the case $\theta = \pi/2$. In this figure the ordinary and extraordinary waves are indicated by circles and stars, respectively. When $t = 0.05$, for $\theta = \pi/2$, the effect of the parameter is very small. We can observe that the dispersion relations are a little modified, but the parameter is not able to break up the forbidden regions that exist when $t = 0$. When $t = 0.5$, the dispersion relations present very different curves with respect the curves for $t = 0$. The difference in the way the t parameter acts in the parallel and perpendicular directions is due to the kind of equations we have. In the Eqs. (43) and (44) the t parameter appears as a linear term and in the Eqs. (46) and (47) the t parameter appears just inside a square root [6] Also, we see that for $\theta = \pi/2$ the parameter t does not lead to the conversion of modes, as it happens for $\theta = 0$. However, at $t = 0.5$ the dispersion relation for the ordinary wave mode is flat and the reflection point is shifted. Now the ordinary and extraordinary wave mode have not the same value of ω when k is very large. The more important effect is on the lower branch of the extraordinary wave mode because there is no more bands of forbidden frequencies, and the reflection point vanishes.

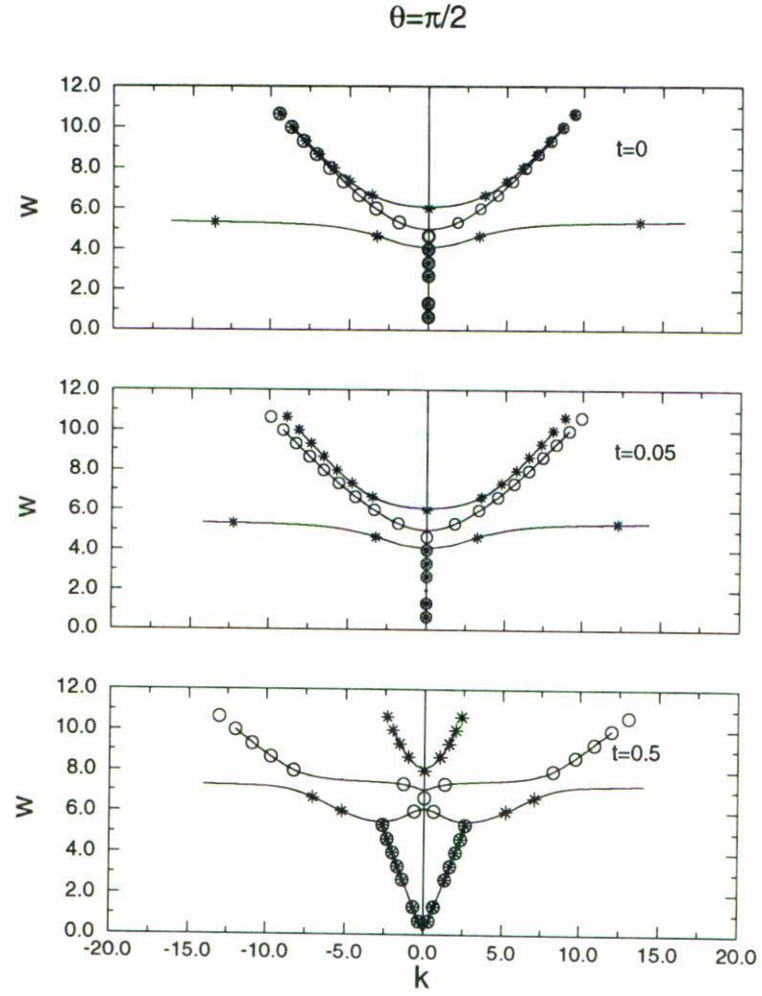

FIGURE 2. Dispersion relations for various values of the parameter t when the direction of propagation is perpendicular to the magnetic field ($\theta = \pi/2$). The curves indicated by circles and stars correspond to the ordinary and extraordinary waves, respectively.

6. CONCLUSIONS

In this paper we have studied the electromagnetic properties of a magnetized chiroplasma which is doubly anisotropic. For the case of waves which propagate parallel to the magnetic field, we show that the cold magnetized chiral-plasma presents a new mode conversion. We compare our results with the typical results obtained for a cold plasma [4]. Also we obtain the chiral-faraday rotation which can be compared with the typical Faraday rotation for a pair of right- and left-handed circularly polarized waves. For waves which propagate perpendicular to the magnetic field, we show that there is no mode conversion but a lower band of forbidden frequencies disappears when the chiral parameter increases.

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