Revista Mexicana de Física **43**, *No. 1 (1997) 108–120*

Effects of suction and free convection current on the oscillating flow of an elastico-viscous fluid past an infinite vertical plate with constant heat flux

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Recibido el 10 de noviembre de 1995; aceptado el 4 de julio de 1996

ABSTRACT. An approximate solution to the laminar flow of an elastico-viscous fluid past a vertical infinite porous plate is presented under following conditions: 1) Constant suction velocity, 2) Free-stream oscillating about a non-zero constant mean, 3) Presence of free-convection currents. The transient velocity and temperature are shown on graphs. The numerical values of the mean skin-friction, the mean Nusselt number, the amplitude and the phase of the skin-friction and the amplitude and the phase of the first and second harmonic of the Nusselt number are listed in Tables. The effects of ω (frequency), G (the Grashof number), E (the Eckert number), P (the Prandtl number), κ (the elastic parameter) and S (suction parameter) are discussed.

RESUMEN. Se presenta una solución aproximada al flujo de un fluido viscoelástico a través de una placa porosa vertical e infinita, con las siguientes condiciones: 1) velocidad de succión constante, 2) corriente libre oscilante e infinita alrededor de una media constante distinta de cero, 3) presencia de corrientes de convección libre. Se presentan gráficas de la velocidad transitoria y la temperatura. Se presentan en tablas los valores numéricos de la fricción superficial media, el número de Nusselt medio, y las amplitudes y fases de la fricción superficial media y del primero y segundo armónicos del número de Nusselt. Se discuten los efectos de ω (frecuencia), G (número de Grashof), E (número de Eckert), P (número de Prandtl), κ (parámetro elástico) y S (parámetro de succión).

PACS: 47.50.+d; 83.50.Sp

1. INTRODUCTION

Oscillatory flows past semi-infinite bodies were first studied by Lighthill [1], Lin [2] on the assumption of small and finite amplitude, respectively. Stuart [3] considered the oscillatory flow past an infinite horizontal porous plate with constant suction. But the suction effect was not considered explicitly. Hence Soundalgekar [4] reconsidered Stuart's problem by considering the effects of suction parameter explicitly. Lighthill's theoretical predictions were confirmed through experiments by Hill and Stenning [5]. The effects of free-stream oscillations and free-convection currents on the flow of an incompressible viscous fluid past an infinite vertical porous plate were studied by Soundalgekar [6, 7] and by Soundalgekar, Vighnesan and Murty [8]. In Refs. [6, 7] the effect of suction parameter was not considered, while in Ref. [8], the effect of the suction parameter was considered in an explicit manner.

In modern technology many new fluids, not obeying Newtonian laws, are being studied by different researchers. One such fluid whose constitutive equations were formulated by Walters' [9] are known as Walters' fluid A' and B'. In case of Walters' fluid B', the oscillatory flows past an infinite horizontal porous-plate were studied by Kaloni [10], Soundalgekar and Puri [11] without taking the suction parameter in an explicit way. So Soundalgekar and Bhat [12] reconsidered this problem by taking into account the suction parameter.

But in many industrial applications the plate is not always in an isothermal condition. The plate is heated from an external source of heat which is supplied at constant or variable rate. Such a situation is known as constant heat flux or variable heat flux case. We now consider here the oscillatory flow of an elastico-viscous fluid past an infinite vertical porous plate with constant heat flux. In Sect. 2 the mathematical analysis is presented, and in Sect. 3 the conclusions are set out.

2. Mathematical analysis

Consider the unsteady flow of an elastico-viscous fluid past an infinite vertical porous plate with x'-axis taken along the plate in the vertically upward direction and the y'-axis is taken normal to it. The constitutive equations characterizing the elastico-viscous liquid (Walters' liquid B') are

$$p_{ik} = -pg_{ik} + p'_{ik}, \tag{1}$$

$$p'^{ik}(x,t) = 2\int_{-\infty}^{t} \psi(t-t')\frac{\partial x^i}{\partial x'^m}\cdot\frac{\partial x^k}{\partial x'^r}e^{(l)mr}(x',t')\,dt', \tag{2}$$

where p_{ik} is the stress-tensor, p an arbitrary isotropic pressure, g_{ik} the metric tensor of a fixed coordinate system x^i, x'^i the position at time t' of the element which is instantaneously at the point x' at time t, $e_{ik}^{(l)}$ the rate of strain tensor and

$$\psi(t-t') = \int_0^\infty \frac{N(\tau)}{\tau}\exp\left[-(t-t')/\tau\right]\,d\tau;$$

$N(\tau)$ being the distribution function of relaxation time τ. Walters [9] has shown that in the case of liquids with short memories (*i.e.*, short relaxation times) the equation of state can be written in a simplified form

$$p'^{ik} = 2\eta_0 e^{(l)ik} - 2\kappa_0\frac{\delta}{\delta t}e^{(l)ik}, \tag{3}$$

where

$$\eta_0 = \int_0^\infty N(\tau)\,d\tau$$

is the limiting viscosity at small rates of shear

$$\kappa_0 = \int_0^\infty \tau N(\tau)\, d\tau$$

and $\delta/\delta t$ denotes the convected differentiation of a tensor quantity, which for any contravariant tensor b^{ik} is given by

$$\frac{\delta b^{ik}}{\delta t} = \frac{\partial b^{ik}}{\partial t} + v^m \frac{\partial b^{ik}}{\partial x^m} - b^{im}\frac{\partial V^k}{\partial x^m} - b^{mk}\frac{\partial V^i}{\partial x^m}, \tag{4}$$

where v^i is the velocity vector.

Because of the flow is past an infinite vertical plate in the upward direction, the physical variables of the fully developed flow are functions of y' and t' only and are independent of x' and hence the two-dimensional unsteady flow of an elastico-viscous fluid with constant physical properties can be shown to be governed by the following equations:

$$\frac{\partial V'}{\partial y'} = 0, \tag{5}$$

$$\rho\frac{\partial u'}{\partial t'} + \rho' V' \frac{\partial u'}{\partial y'} = -\frac{\partial p'}{\partial x'} - \rho' g_x + \eta_0 \frac{\partial^2 u'}{\partial y'^2} - \kappa_0 \left(\frac{\partial^3 u'}{\partial y'^2 \partial t} + V'\frac{\partial^3 u'}{\partial y'^3} - 3\frac{\partial u'}{\partial y'}\frac{\partial^2 V'}{\partial y'^2} - 2\frac{\partial V'}{\partial y'}\frac{\partial^2 u'}{\partial y'^2}\right), \tag{6}$$

$$\rho\frac{\partial V'}{\partial t'} + \rho' V' \frac{\partial V'}{\partial y'} = -\frac{\partial p'}{\partial y'} + 2\eta_0 \frac{\partial^2 V'}{\partial y'^2} - 2\kappa_0 \left(\frac{\partial^3 V'}{\partial y^2 \partial t} + V'\frac{\partial^3 V'}{\partial y'^2} - 3\frac{\partial V'}{\partial y'}\frac{\partial^2 V'}{\partial y'^2}\right). \tag{7}$$

Here u', V' are the velocity components in the x' and y' directions, respectively, t' is the time, η_0 is the limiting viscosity at small rates of shear, and κ_0 is the elastic constant of the Walters' liquid B' assumed to be very small ($\ll 1$). Also ρ' is the density and g_x is the acceleration due to gravity. As the suction-velocity at the plate is assumed to be constant, Eqs. (6) and (7) reduce to following:

$$\rho\left(\frac{\partial u'}{\partial t'} + V'\frac{\partial u'}{\partial y'}\right) = -\frac{\partial p'}{\partial x'} - \rho g_x + \eta_0 \frac{\partial^2 u'}{\partial y'^2} - \kappa_0\left(\frac{\partial^3 u'}{\partial y'^2 \partial t} + V'\frac{\partial^3 u'}{\partial y'^3}\right), \tag{8}$$

$$0 = -\frac{\partial p'}{\partial y'}. \tag{9}$$

Then the energy equation in view of constant suction is given by

$$\rho' c_p\left(\frac{\partial T'}{\partial t'} + V'\frac{\partial T'}{\partial y'}\right) = K\frac{\partial^2 T'}{\partial y'^2} + \eta_0\left(\frac{\partial u'}{\partial y'}\right)^2 - \kappa_0\left(\frac{\partial^2 u'}{\partial y' \partial t} + V'\frac{\partial^2 u'}{\partial y'^2}\right)\frac{\partial u'}{\partial y'}. \tag{10}$$

Here c_p is the specific heat at constant pressure, K the thermal conductivity of the fluid, assumed constant, T' the temperature of the fluid near the plate, and the last two terms on the right-hand side represent heat due to viscous dissipation.

If $U'(t')$ is the free-stream velocity and ρ' is the density of the fluid far away from the plate, then from Eq. (8), we have in the free-stream,

$$\rho' \frac{\partial U'}{\partial t'} = -\frac{\partial p'}{\partial x'} - \rho'_\infty g_x. \tag{11}$$

Eliminating $-\partial p'/\partial x'$ between (8) and (11), we have

$$\rho \left(\frac{\partial u'}{\partial t'} + V' \frac{\partial u'}{\partial y'} \right) = \rho' \frac{\partial U'}{\partial t'} + g_x (\rho'_\infty - \rho') + \eta_0 \frac{\partial^2 u'}{\partial y'^2} - \kappa_0 \left(\frac{\partial^3 u'}{\partial y'^2 \partial t'} + V' \frac{\partial^3 u'}{\partial y'^3} \right). \tag{12}$$

But from the equation of state, we have

$$g_x(\rho'_\infty - \rho') = g_x \beta \rho' (T' - T'_\infty), \tag{13}$$

where β is the coefficient of volume expansion.

Then from (12) and (13), we get

$$\rho' \left(\frac{\partial u'}{\partial t'} + V' \frac{\partial u'}{\partial y'} \right) = \rho' \frac{\partial U'}{\partial t'} + g_x \beta \rho' (T' - T'_\infty) + \eta_0 \frac{\partial^2 u'}{\partial y'^2} - \kappa_0 \left(\frac{\partial^3 u'}{\partial y'^2 \partial t'} + V' \frac{\partial^3 u'}{\partial y'^3} \right). \tag{14}$$

The boundary conditions of the problem are

$$u' = O, \quad \frac{dT'}{dy'} = -\frac{q'}{K} \quad \text{at} \quad y' = 0;$$
$$u' = U'(t) = U_0 \left(1 + \varepsilon e^{i\omega' t'} \right), \qquad T' \to T'_\infty \quad \text{as} \quad y' \to \infty. \tag{15}$$

The condition on temperature at $y' = 0$ indicates the constant supply of heat at the plate where q' is the quantity of heat supplied per unit area. Also, the velocity in the free-stream is assumed to be periodic in nature, oscillating about a constant mean velocity U_0 with frequency ω' and ε is a constant small quantity (Fig. 1).

From Eq. (5), for constant suction,

$$V' = -V'_0, \tag{16}$$

where V_0 is the constant suction velocity and the negative sign in (16) indicates that it is directed towards the plate.

On introducing following non-dimensional quantities:

$$y = \frac{y' U_0}{\nu}, \quad t = \frac{t' U_0^2}{4\nu}, \quad \omega = \frac{4\nu\omega'}{U_0^2}, \quad u = \frac{u'}{U_0},$$
$$\theta = \frac{T' - T_\infty}{\nu q'/K U_0}, \quad G = \frac{\nu g \beta q'}{K U_0^3}, \quad P = \frac{\eta_0 c_p}{K}, \tag{17}$$
$$E = \frac{K U_0^3}{q' \nu c_p}, \quad \kappa = \frac{\kappa_0 U_0^2}{\rho \nu^2}, \quad S = V'_0 / U_0,$$

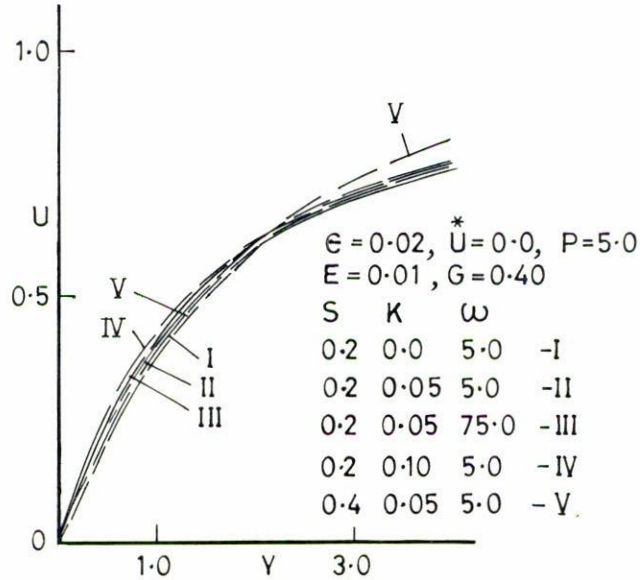

FIGURE 1. Transient velocity profiles for $\omega t = \pi/2$.

in Eqs. (14), (10) and (15), we have

$$\frac{1}{4}\frac{\partial u}{\partial t} - S\frac{\partial u}{\partial y} = \frac{1}{4}\frac{\partial U}{\partial t} + G\theta - \kappa\left(\frac{1}{4}\frac{\partial^3 u}{\partial y^2 \partial t} - S\frac{\partial^3 u}{\partial y^3}\right), \tag{18}$$

$$\frac{P}{4}\frac{\partial \theta}{\partial t} - SP\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + PE\left(\frac{\partial u}{\partial y}\right)^2 - \kappa PE\left(\frac{1}{4}\frac{\partial^2 u}{\partial t \partial y} - S\frac{\partial^2 u}{\partial y^2}\right)\frac{\partial u}{\partial y}, \tag{19}$$

with the following boundary conditions:

$$u = 0, \quad \frac{d\theta}{dy} = -1, \quad \text{at} \quad y = 0;$$
$$u = 1 + \varepsilon e^{i\omega t} = U(t), \quad \theta = 0, \quad \text{as} \quad y \to \infty. \tag{20}$$

Here P, G, E are the Prandtl, Grashof and Eckert numbers, respectively, whose physical meanings are well-discussed in standard text-books like Schlischting [13] and Gebhart [14]; κ is an elastic-parameter and S is the suction parameter.

Equations (18) and (19) are coupled non-linear equations and hence their exact solutions are not possible. So we derive approximate solutions. We now assume in the neighbourhood of the plate

$$u = u_0 + \frac{\varepsilon}{2}\left(e^{i\omega t}u_1 + e^{-i\omega t}\bar{u}_1\right), \tag{21}$$

$$\theta = \theta_0 + \frac{\varepsilon}{2}\left(e^{i\omega t}\theta_1 + e^{-i\omega t}\bar{\theta}_1\right) + \frac{\varepsilon^2}{2}\left(e^{2i\omega t}\theta_2 + e^{-2i\omega t}\bar{\theta}_2\right). \tag{22}$$

We substitute (21) and (22) in Eqs. (18) to (20), equate harmonic and non-harmonic terms to zero, neglect the coefficient of ε^2 in (18) and those of ε^3 in (19) and we have the following:

$$\kappa S u_0''' + u_0'' + S u_0' = -G\theta_0, \tag{23}$$

$$\kappa S u_1''' + \left(1 - \frac{i\omega\kappa}{4}\right) u_1'' + S u_1' - \frac{i\omega}{4} u_1 = -\frac{i\omega}{4} - G\theta_1, \tag{24}$$

$$\theta_0'' + PS\theta_0' = -PE\left[(u_0')^2 + \frac{\varepsilon^2}{2} u_1' \bar{u}_1'\right] - \kappa PES\left[u_0' u_0'' + \frac{\varepsilon^2}{4}\left(\bar{u}_1' u_1'' + u_1' \bar{u}_1''\right)\right], \tag{25}$$

$$\theta_1'' + PS\theta_1' - \frac{i\omega P}{4}\theta_1 = -2PEu_0' u_1' + \kappa PE\left[\frac{i\omega}{4} u_0' u_1' - S\left(u_0' u_1'' + u_1' u_0''\right)\right], \tag{26}$$

$$\theta_2'' + PS\theta_2' - \frac{i\omega P}{2}\theta_2 = -\frac{1}{2} PE\left(u_1'\right)^2 + \kappa PE\left[\frac{i\omega}{4}\left(u_1'\right)^2 - \frac{S}{2}\cdot\left(u_1' u_1''\right)\right], \tag{27}$$

where primes denote derivatives with respect to y.

The corresponding boundary conditions are

$$\begin{aligned} &u = 0, \quad u_1 = 0, \quad \theta_0' = -1, \quad \theta_1' = \theta_2' = 0, \quad \text{at} \quad y = 0; \\ &u_0 = 1, \quad u_1 = 1, \quad \theta_0 = 0, \quad \theta_1 = \theta_2 = 0, \quad \text{as} \quad y \to \infty. \end{aligned} \tag{28}$$

Here and henceforth, primes denote the derivative with respect to y.

Equations (23) and (24) are of third order with two boundary conditions which is mathematically ill-posed problem. To overcome this difficulty, we now expand, u_0, u_1, ..., etc. in powers of κ as κ for such fluids is always very small ($\ll 1$). Hence we now assume,

$$\begin{aligned} &u_0 = u_{01} + \kappa u_{02}, \quad u_1 = u_{11} + \kappa u_{12}, \\ &\theta_0 = \theta_{01} + \kappa\theta_{02}, \quad \theta_1 = \theta_{11} + \kappa\theta_{12}, \quad \theta_2 = \theta_{21} + \kappa\theta_{22}. \end{aligned} \tag{29}$$

Substituting (29) in Eqs. (23 to (28), equating the coefficients of different powers of κ, neglecting those of κ^2, we have the following set of equations:

$$u_{01}'' + S u_{01}' = -G\theta_{01}, \tag{30}$$

$$u_{02}'' + S u_{02}' = -G\theta_{02} - S u_{01}''', \tag{31}$$

$$u_{11}'' + S u_{11}' - \frac{i\omega}{4} u_{11} = -\frac{i\omega}{4} - G\theta_{11}, \tag{32}$$

$$u_{12}'' + S u_{12}' - \frac{i\omega}{4} u_{12} = -G\theta_{12} - S u_{11}'' + \frac{i\omega}{4} u_{11}'', \tag{33}$$

$$\theta_{01}'' + PS\theta_{01}' = -PE\left[\left(u_{01}'\right)^2 + \frac{\varepsilon^2}{2}u_{11}'\bar{u}_{11}'\right], \tag{34}$$

$$\theta_{02}'' + PS\theta_{02}' = -PE\left[2u_{01}'u_{02}' + \frac{\varepsilon^2}{2}\left(u_{11}'\bar{u}_{12}' + \bar{u}_{11}'u_{12}'\right)\right]$$
$$- PES\left[u_{01}'u_{01}'' + \frac{\varepsilon^2}{2}\left(\bar{u}_{11}'u_{11}'' + u_{11}'\bar{u}_{11}''\right)\right], \tag{35}$$

$$\theta_{11}'' + PS\theta_{11}' - \frac{i\omega P}{4}\theta_{11} = -2PEu_{01}'u_{11}', \tag{36}$$

$$\theta_{12}'' + PS\theta_{12}' = -\frac{i\omega P}{4}\theta_{12} - PE\left[\frac{i\omega}{4}u_{11}'u_{01}' - S\left(u_{01}'u_{11}'' + u_{11}'u_{01}''\right)\right]$$
$$- 2PE\left(u_{01}'u_{12}' + u_{02}'u_{11}'\right), \tag{37}$$

$$\theta_{21}'' + PS\theta_{21}' - \frac{i\omega P}{2}\theta_{21} = -\tfrac{1}{2}PE\left(u_{11}'\right)^2, \tag{38}$$

$$\theta_{22}'' + PS\theta_{22}' - \frac{i\omega P}{2}\theta_{22} = -PEu_{11}'u_{12}' + PE\left[\frac{i\omega}{4}\left(u_{11}'\right)^2 - \frac{S}{2}u_{11}'u_{11}''\right], \tag{39}$$

and the boundary conditions are

$$\text{at } y = 0 \quad \begin{cases} u_{01} = 0, & u_{02} = 0, & u_{11} = 0, & u_{12} = 0, & \bar{u}_{11} = 0, & \bar{u}_{12} = 0, \\ \theta_{01}' = -1, & \theta_{02}' = 0, & \theta_{11}' = 0, & \theta_{12}' = 0, & \bar{\theta}_{12}' = 0, & \bar{\theta}_{12}' = 0, \\ \theta_{21}' = 0, & \theta_{22}' = 0, & \bar{\theta}_{21}' = 0, & \bar{\theta}_{22}' = 0, \end{cases} \tag{40}$$

$$\text{at } y \to \infty \quad \begin{cases} u_{01} = 1, & u_{02} = 0, & u_{11} = 1, & u_{12} = 0, & \bar{u}_{11} = 1, & \bar{u}_{12} = 0, \\ \theta_{01} = 0, & \theta_{02} = 0, & \theta_{11} = 0, & \theta_{12} = 0, & \bar{\theta}_{11} = 0, & \bar{\theta}_{12} = 0, \\ \theta_{21} = 0, & \theta_{22} = 0, & \bar{\theta}_{11} = 0, & \bar{\theta}_{22} = 0. \end{cases} \tag{41}$$

Equations (30) to (39) are still coupled and non-linear differential equations whose exact solutions are not possible. Hence we again expand $u_{01}, u_{02}, \ldots$, in powers of E, the Eckert number, as the Eckert number for incompressible fluids is always small. Hence we assume

$$\begin{aligned} u_{01} &= u_{011} + Eu_{012}, & u_{02} &= u_{021} + Eu_{022}, \\ u_{11} &= u_{111} + Eu_{112}, & u_{12} &= u_{121} + Eu_{122}, \\ \theta_{01} &= \theta_{011} + E\theta_{012}, & \theta_{02} &= \theta_{021} + E\theta_{022}, \\ \theta_{11} &= \theta_{111} + E\theta_{112}, & \theta_{12} &= \theta_{121} + E\theta_{122}, \\ \theta_{21} &= \theta_{211} + E\theta_{212}, & \theta_{22} &= \theta_{221} + E\theta_{222}, \end{aligned} \tag{42}$$

and substitute (42) in Eqs. (30) to (41), equate the coefficients of different powers of E, neglect those of E^2 and we get

$$u''_{011} + Su'_{011} = -G\theta_{011}, \tag{43}$$

$$u''_{012} + Su'_{012} = -G\theta_{012}, \tag{44}$$

$$u''_{021} + Su'_{021} = -G\theta_{021} - Su'''_{011}, \tag{45}$$

$$u''_{022} + Su'_{022} = -G\theta_{22} - Su'''_{012}, \tag{46}$$

$$u''_{111} + Su'_{111} - \frac{i\omega}{4}u_{111} = -\frac{i\omega}{4} - G\theta_{111}, \tag{47}$$

$$u''_{112} + Su'_{112} - \frac{i\omega}{4}u_{112} = -G\theta_{112}, \tag{48}$$

$$u''_{121} + Su'_{121} - \frac{i\omega}{4}u_{121} = -G\theta_{121} - Su''_{111} + \frac{i\omega}{4}u''_{111}, \tag{49}$$

$$u''_{122} + Su'_{122} - \frac{i\omega}{4}u_{122} = -G\theta_{122} - Su'''_{112} + \frac{i\omega}{4}u''_{122}, \tag{50}$$

$$\theta''_{011} + PS\theta'_{011} = 0, \tag{51}$$

$$\theta''_{012} + PS\theta'_{012} = -P\left[\left(u'_{011}\right)^2 + \frac{\varepsilon^2}{2}u'_{111}\bar{u}'_{111}\right], \tag{52}$$

$$\theta''_{021} + PS\theta'_{021} = 0, \tag{53}$$

$$\begin{aligned}\theta''_{022} + PS\theta'_{022} = &-P\left[2u'_{011}u'_{021} + \frac{\varepsilon^2}{2}\left(u'_{111}\bar{u}'_{121} + \bar{u}''_{111}u'_{121}\right)\right]\\ &- PS\left[u'_{011}u''_{011} + \frac{\varepsilon^2}{4}\left(\bar{u}'_{111}u''_{111} + u'_{111}\bar{u}''_{111}\right)\right],\end{aligned} \tag{54}$$

$$\theta''_{111} + PS\theta'_{111} - \frac{i\omega P}{4}\theta_{111} = 0, \tag{55}$$

$$\theta''_{112} + PS\theta'_{112} - \frac{i\omega P}{4}\theta_{112} = -2Pu'_{011}u'_{111}, \tag{56}$$

$$\begin{aligned}\theta''_{122} + PS\theta'_{122} - \frac{i\omega P}{4}\theta_{122} = &P\left[\frac{i\omega}{4}u'_{111}u'_{011} - S\left(u'_{011} - u''_{111} + u'_{111}u''_{011}\right)\right]\\ &- 2P\left(u'_{011}u'_{121} + u'_{111}\right),\end{aligned} \tag{57}$$

$$\theta''_{211} + PS\theta'_{211} - \frac{i\omega P}{2}\theta'_{211} = 0, \tag{58}$$

$$\theta''_{212} + PS\theta'_{212} - \frac{i\omega P}{2}\theta'_{212} = -\frac{1}{2}P\left(u'_{111}\right)^2, \tag{59}$$

$$\theta''_{221} + PS\theta'_{221} - \frac{i\omega P}{2}\theta_{221} = 0, \tag{60}$$

$$\theta''_{222} + PS\theta'_{222} - \frac{i\omega P}{2}\theta_{222} = P\left[\frac{i\omega}{4}\left(u'_{111}\right)^2 - \frac{S}{2}u'_{111}u''_{111} - u'_{111}u'_{121}\right], \tag{61}$$

with following boundary conditions:

$$\left.\begin{array}{llllll} u_{011}=0, & u_{012}=0, & \theta'_{011}=1, & \theta'_{112}=0, & \theta_{021}=0, & \theta_{022}=0, \\ u_{021}=0, & u_{022}=0, & \theta_{111}=0, & \theta'_{112}=0, & \theta_{121}=0, & \theta_{122}=0, \\ u_{111}=0, & u_{112}=0, & \theta_{211}=0, & \theta_{212}=0, & \theta_{221}=0, & \theta_{222}=0, \\ u_{121}=0, & u_{122}=0, & \bar\theta_{111}=0, & \bar\theta_{112}=0, & \bar\theta_{121}=0, & \bar\theta_{122}=0, \\ \bar u_{111}=0, & \bar u_{112}=0, & \bar\theta_{211}=0, & \bar\theta_{212}=0, & \bar\theta_{221}=0, & \bar\theta_{222}=0, \\ \bar u_{121}=0, & \bar u_{122}=0; & & & & \end{array}\right\} \text{ at } y=0$$

$$\left.\begin{array}{llllll} u_{011}=1, & u_{012}=0, & \theta'_{011}=0, & \theta'_{112}=0, & \theta_{021}=0, & \theta_{022}=0, \\ u_{021}=0, & u_{022}=0, & \theta_{111}=0, & \theta'_{112}=0, & \theta_{121}=0, & \theta_{122}=0, \\ u_{111}=1, & u_{112}=0, & \theta_{211}=0, & \theta_{212}=0, & \theta_{221}=0, & \theta_{222}=0, \\ u_{121}=0, & u_{122}=0, & \bar\theta_{111}=0, & \bar\theta_{112}=0, & \bar\theta_{121}=0, & \bar\theta_{122}=0, \\ \bar u_{111}=1, & \bar u_{112}=0, & \bar\theta_{211}=0, & \bar\theta_{212}=0, & \bar\theta_{221}=0, & \bar\theta_{222}=0, \\ \bar u_{121}=0, & \bar u_{122}=0; & & & & \end{array}\right\} \text{ at } y\to\infty \tag{62}$$

These are coupled linear equations and their closed form solutions are derived. These are straightforward and hence to save space, these expressions are not mentioned.

Substituting for u_{011}, u_{012}, ... , etc. in Eq. (29), we get the expression for u_0 and θ_0 which represent the mean velocity and the mean temperature. The effect of different parameters on the mean velocity and temperature being same as that on the transient velocity and temperature, we have discussed the same in case of the transient velocity and temperature.

The skin-friction is given by

$$p'_{x'y'} = \eta_0\frac{\partial u'}{\partial y'} - \kappa_0\left(\frac{\partial^2 u'}{\partial y'\partial t'} + V'\frac{\partial^2 u'}{\partial y'^2}\right), \tag{63}$$

which in virtue of (17), reduce to

$$p_{xy} = \frac{p'_{x'y'}}{\rho U_0^2} = \left.\frac{du}{dy}\right|_{y=0} - \kappa\left(\frac{1}{4}\frac{\partial^2 u}{\partial t\partial y} - S\frac{\partial^2 u}{\partial y^2}\right). \tag{64}$$

Substituting for u in terms of u_{01}, u_{02}, we can show that the mean skin-friction is given by

$$P_{xy_m} = u'_{01}\big|_{y=0} + \kappa\left(u'_{02} + Su''_{01}\right)_{y=0}, \tag{65}$$

where primes now denote derivative with respect to y. We have calculated the numerical values of P_{xy_m} and these are listed in Table I.

We observe from this table that due to the presence of elastic-property in the fluid, there is a rise in the mean skin-friction and it increases with increasing the value of the elastic-parameter κ. Greater viscous dissipative heat or an increase in G, the Grashof

TABLE I. Values of P_m, $(Nu)_m$, $\varepsilon = 0.02$.

P	E	S	G	κ	ω	$P_{xy}\|_m$	$(Nu)_m$
5.0	0.01	0.20	0.40	0.00	5.0	0.60389	0.98846
5.0	0.01	0.20	0.40	0.05	5.0	0.60393	0.98834
5.0	0.01	0.20	0.40	0.05	75.0	0.60393	0.98832
5.0	0.01	0.20	0.40	0.10	5.0	0.60397	0.98822
5.0	0.01	0.20	0.60	0.05	5.0	0.89631	0.98264
5.0	0.01	0.40	0.40	0.05	5.0	0.60220	1.9758
5.0	0.02	0.20	0.40	0.05	5.0	0.60786	0.97695
10.0	0.01	0.20	0.40	0.05	5.0	0.40432	1.9744

number, leads to an increase in the value of the mean skin-friction. But the mean skin-friction decreases with increasing the Prandtl number P or the suction parameter S.

The rate of heat transfer expressed in terms of the Nusselt number is given by

$$Nu = -\frac{1}{\theta(0)}\left.\frac{du}{dy}\right|_{y=0} = \frac{1}{\theta(0)} \quad \text{as} \quad \left.\frac{du}{dy}\right|_{y=0} = -1. \tag{66}$$

Substituting for θ in terms of θ_0 we can show that the mean Nusselt number is given by

$$Nu_m = \frac{1}{\theta_0(0)}. \tag{67}$$

The numerical values of Nu_m calculated from (67) are listed in Table I. We conclude from this table that due to the presence of elastic property in the fluid, there is a decrease in the mean Nusselt number. Nu_m is also found to decrease due to increasing the frequency or the Grashof number G, or due to greater viscous dissipative heat. But due to an increase in S or P, there is an increase in the mean Nusselt number.

2.1. Unsteady flow

On substituting for u_{111}, θ_{111} etc. in the expression for u_1 and θ_1, we can get the expression for the unsteady components. Then the expressions for the velocity and temperature are given by Eqs. (21) and (22) which can be written in terms of the oscillating parts as

$$u = u_0 + \frac{\varepsilon}{2}\left[(M_r + iM_i)\, e^{i\omega t} + (M_r - iM_i)\, e^{-i\omega t}\right], \tag{68}$$

$$\theta = \theta_0 + \frac{\varepsilon}{2}\left[(\theta_{1r} + i\theta_{1i})\, e^{i\omega t} + (\theta_{1r} - i\theta_{1i})\, e^{-i\omega t}\right] + \frac{\varepsilon}{2}\left[(\theta_{2r} + i\theta_{2i})\, e^{2i\omega t} + (\theta_{2r} - i\theta_{2i})\, e^{-2i\omega t}\right]. \tag{69}$$

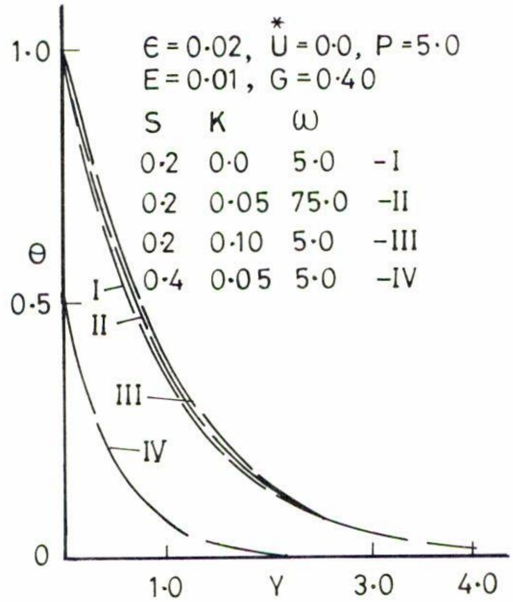

FIGURE 2. Transient temeperature profiles for $\omega t = \pi/2$.

TABLE II. Values of P_m, $(Nu)_m$, $\varepsilon = 0.02$.

P	E	S	G	κ	ω	$\|B\|$	$\tan\alpha$
5.0	0.01	0.20	0.40	0.00	5	1.1908	0.8780
5.0	0.01	0.20	0.40	0.05	5	1.1900	0.8779
5.0	0.01	0.20	0.40	0.05	75	1.3982	0.9678
5.0	0.01	0.20	0.40	0.10	5	1.1891	0.8777
5.0	0.01	0.20	0.60	0.05	5	1.1900	0.8761
5.0	0.01	0.40	0.40	0.05	5	1.2677	0.7687
5.0	0.02	0.20	0.40	0.05	5	1.1889	0.8747
10.0	0.01	0.20	0.40	0.05	5	1.1888	0.8778

By putting $\omega t = \pi/2$, we can derive the expressions for transient velocity and temperature profiles. The numerical values of u and θ are calculated and these are shown in Figs. 1 and 2, respectively. We observe from Fig. 1 that due to the presence of elastic-property in the fluid, the transient velocity is found to increase and it also increases with increasing the frequency ω. An increase in κ, the elastic parameter, leads to more increase in the transient velocity. The effect of suction-parameter S is observed near the plate only and the transient velocity is found to decrease with increasing suction velocity which is physically possible because more and more fluid from the fluid-region near the plate is sucked towards the plate and hence the said effect. However, little away from the plate, suction velocity parameter helps to increase the transient velocity. The effects of

TABLE III. Values of $|B_1|$, $\tan\alpha_1$, $|B_2|$, $\tan\alpha_2$.

P	E	S	G	κ	ω	$\|B_1\|$	$\tan\alpha_1$	$\|B_2\|$	$\tan\alpha_2$
5.0	0.01	0.20	0.40	0.00	5.0	6.1101×10^{-3}	−0.70982	1.5940×10^{-3}	−0.056898
5.0	0.01	0.20	0.40	0.05	5.0	6.0670×10^{-3}	−0.82173	1.5834×10^{-3}	−0.12843
5.0	0.01	0.20	0.40	0.05	75.0	2.4513×10^{-3}	−0.22793	2.00220×10^{-3}	−1.0321
5.0	0.01	0.20	0.40	0.10	5.0	6.0544×10^{-3}	−0.94861	1.5808×10^{-3}	−0.20185
5.0	0.01	0.20	0.60	0.05	5.0	7.8649×10^{-3}	−0.79400	1.5652×10^{-3}	−0.12843
5.0	0.01	0.40	0.40	0.05	5.0	2.2085×10^{-2}	−0.75660	6.6695×10^{-3}	−0.18200
5.0	0.02	0.20	0.40	0.05	5.0	1.1856×10^{-2}	−0.82173	3.0942×10^{-3}	−0.12843
10.0	0.01	0.20	0.40	0.05	5.0	1.7518×10^{-2}	−0.01364	7.1275×10^{-3}	−0.12577

κ and ω on the transient temperature are the same as in case of transient velocity, but the transient temperature is found to fall, causing cooling of elastico-viscous fluid.

We now study the effects of these parameters on the amplitude and phase of the skin-friction and the rate of heat transfer. It is given by

$$P_{xy} = \left.\frac{du_0}{dy}\right|_{y=0} + |B|\cos(\omega t + \alpha), \tag{70}$$

$$Nu = \frac{1}{\theta_0(0)} + |B_1|\cos(\omega t + \alpha_1) + |B_2|\cos(2\omega t + \alpha_2), \tag{71}$$

where

$$B = B_r + iB_i = \left.\frac{du}{dy}\right|_{y=0}, \qquad \tan\alpha = \frac{B_i}{B_r}, \tag{72}$$

$$B_1 = B_{1r} + iB_{1i} = \left|\frac{\theta_1(0)}{\theta_0^2(0)}\right|, \qquad B_2 = B_{2r} + iB_{2i} = \left|\frac{\theta_2(0)}{\theta^2(0)}\right|,$$

$$\tan\alpha_1 = B_{1i}/B_{1r}, \qquad \tan\alpha_2 = B_{2i}/B_{2r}. \tag{73}$$

The expressions for $|B|$, $|B_1|$, and $|B_2|$ are very lengthy and hence to save space these are not mentioned here. The numerical values of $|B|$, tan α are listed in Table II. We observe from this Table II that an increase ω or S leads to an increase in the values of the amplitude $|B|$. Amplitude $|B|$ is found to decrease owing to an increase in G, E, P. Also we observe that there is always a phase lead.

In Table III, the numerical values of $|B_1|, |B_2|$, tan α_1 tan α_2 are entered. We observe from this Table that the amplitude of the first and second harmonic decreases due to the presence of elastic property in the fluid. An increase in ω or κ leads to a decrease in both the amplitudes. But an increase in S, G or E leads to an increase in the values of both the amplitudes. We also observe that the values of tan α_1 and tan α_2 being negative, we conclude that there is always a phase-lag.

3. CONCLUSIONS

1. The mean skin-friction increases with increasing G or E but decreases due to increasing P or S.
2. Nu_m decreases due to increasing ω or G or E and increases owing to increasing S or P.
3. Amplitude $|B|$, tends to increase with an increase in ω or S but it is found to decrease owing to an increase in G, E, P. Also we observe that there is always a phase-lead.
4. Amplitudes $|B_1|$, $|B_2|$ increase owing to increasing S, G or E and decrease due to the presence of the elastic-property in the fluid.
5. There is always a phase-lag in case of both the first and second harmonic in the rate of heat transfer.

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