Revista Mexicana de Física **43**, *No. 2 (1997) 240–247*

On Wolter's vortex in total reflection

H.C. Rosu
Instituto de Física de la Universidad de Guanajuato
Apartado postal E–143, León, Gto, México
and
Institute of Gravitation and Space Sciences, Bucharest, Romania

Recibido el 2 de julio de 1996; aceptado el 12 de noviembre de 1996

Abstract. The "circulatory wave" ("die zirkulierende Welle") put into evidence in 1949 by Wolter (Wolter's vortex) in total reflection is interpreted as a phase defect in the scalar theory of Green and Wolf of 1953, which is the Madelung (hydrodynamic) representation of the optical field. Some comments are added on its possible relevance for the Hamamatsu experiment aimed to clarify the wave-particle duality at the "single photon" level of down-converted laser beams.

Resumen. La "onda circulante" descubierta en 1949 por Wolter (vórtice de Wolter) en la reflexión total, es interpretada como un defecto de fase en la teoría escalar de Green y Wolf de 1953, la cual es la representación de Madelung del campo óptico. Algunos comentarios son vertidos sobre su posible relevancia en el experimento Hamamatsu intentado para clarificar la dualidad onda-partícula al nivel de un solo fotón obtenido de rayos láser en conversión descendente.

PACS: 42.10.Fa; 03.65.-w

1. Introduction

At the present time optical phase defects are quite well known especially due to their connection with transverse laser patterns [1]. Less well known is an old phase singularity discovered by Wolter [2] in total reflection (TR). Even though Wolter's vortex plays an important role in the realm of TR, it has not drawn much attention over the years. Checking the literature one will find out that this vortex has not been discussed since 1970, in the thesis of Lotsch [3]. It is true however that very similar waves can be encountered in other research fields, *e.g.*, in Madelung's quantum mechanics where more detailed analyses have been made [4].

TR is one of the experimental discoveries of Newton, so it has a long history of 300 years [3, 5] and a long list of authors. For a recent review, though on the frustrated version, see Ref. 6.

This phenomenon (or more exactly frustrated internal TR) is similar to the tunnel effect in quantum mechanics, an analogy presented in many textbooks [7]. This is so, because, despite the name, there is some light in the rarer medium even at the limiting angle of total reflection and beyond it. Fresnel called this faint field *evanescent light.*

The evanescent light is not really a wave, not even a wave packet but only part of a wave packet. Thus, in TR experiments it is possible to reveal some remarkable inhomogeneities of wave packets which escape observation in the usual propagation.

On the experimental side, only in 1947 were Goos and Hänchen [8] able to observe the longitudinal shift of the totally reflected light calculated by Picht [9] already in 1929. Another shift, a transverse one, has been predicted (with some prehistory) by Fedorov [10] in 1955 and discussed either in the Poynting-vector method or the stationary-phase one by a number of authors. It was first investigated experimentally by Imbert [11] in 1969. Very recently, Dutriaux, Le Floch, and Bretenaker [12] made important progress by measuring the transverse displacement of the helicoidal eigenstates of a He-Ne laser beam for various angles of incidence, providing in this way a first check of the various formulas for the transverse shift for circular polarization.

A simple understanding of the longitudinal shift may be obtained in terms of the propagation of light in the second rarer medium very close to the surface in such a way (Newton's parabola) that apparently the incident ray is reflected by a surface located at some small depth in the second medium. This depth gives the exponential rate of decrease of the field amplitude. For the transverse shift, Costa de Beauregard [13] has given a quite unconventional explanation, which he dubbed the "inertial spin effect", essentially the noncolinearity of the velocity and momentum of the photon in TR conditions. The problem of the angular and spin momentum conservation at reflection is still debatable [14]. Both shifts are polarization dependent and their order of magnitude is about half a wavelength in the case of the transverse shift and usually ten wavelengths for the longitudinal one.

The organization of the paper is as follows. Section 2 is a brief summary of Wolter's results on the "circulatory wave". In the next section, after an outline of the Green-Wolf (GW) scalar theory, Wolter's vortex is explained within the GW theory as a result of the 'quantization' of the 'velocity' circulation. The next section contains some comments on the possible relevance of Wolter's vortex in the Hamamatsu wave-particle experiment with laser beams at the single-photon level, and I end up with concluding remarks.

2. THE CIRCULATORY WAVE

The circulatory wave was discovered when Wolter [2] performed accurate measurements of the Goos-Hanchen shift by means of his powerful method called "minimum ray characteristic". Wolter observed the displacement of interference *minima* formed by *two* slightly inclined (plane) waves (and the two reflected ones) which have undergone multiple reflections, on one hand at the interface glass-air, and on the other hand at the interface glass-silver. A plane parallel plate was used on which a silver strip of $\lambda/4$ thickness was deposited. The advantages of this technique over the diffraction maximum method were presented by Wolter himself [16]. The "minimum ray characteristic" allowed Wolter to reveal clearly the details of the electromagnetic field in the immediate neighborhood of the interface between the two media. He has also calculated the phase surfaces, energy flux lines and time averaged Poynting vectors, obtaining a full picture of the field. Wolter was particularly careful with the region close to the geometric point of TR (denoted as the origin in his paper), where he put into evidence the circulatory wave of the energy streamlines, both experimentally and in theory, as a special solution that comes out when integrating a Bernoulli differential equation of order $n = -1$, *i.e.*, of the type $y' = a(z)y^{-1} + b(z)y$, with $y' = dy/dz$, $a(z) = z(1 + 2\pi f z)$ and $b(z) = 1/F = \text{const.}$, in almost Wolter's notations; y, which is the coordinate along the surface in the thin medium, the transverse coordinate z, and $f \approx 2\pi F$ are related to the mean angle of incidence (angle of observation of the interference minima) and the mean shift.

The circulatory wave is very similar to an ordinary vortex in the turbulent motion of a classical fluid. In the rarer medium there is always a saddle point (or a stagnation point in hydrodynamical language), which is essential for the energy flow in TR. At that point the incoming wave is broken in two parts: one is going through the glass and after one period is again crossing the surface in order to meet the other part which remained in air. The splitting and meeting are repeated every period. Therefore the permanence of the evanescent wave in the rarer medium is due to the circulatory wave, which is carrying energy back and forth across the interface. We have here a clear explanation for the fact that although the incident energy is equal to the reflected one, there is also a time averaged Poynting vector parallel to the surface propagating very close to the surface ($d \approx \lambda$) in the rarer medium. Before Wolter, the energy problem in TR has been considered by many authors, *e.g.*, by Drude [17] for the plane wave case, and by Picht [9], Noether [18], and Schaefer and Pich [19] for more realistic situations. These authors have shown that due to the limited extent of the beam field, some amount of energy is extracted from one side of the incident beam and added to the other side of the reflected beam, in the same plane of incidence, after having propagated parallel to the interface in the less dense medium. At the saddle point the energy flux is zero. Between the stagnation point and the interface, in the region near the origin, we enter the vortex rings of the circulatory wave. Most of the core of the vortex wave is to be found in the denser medium and in the middle of the vortex the amplitude of the wave is naught, but in the rarer medium the amplitude never goes to zero on the scale of a wavelength.

Wolter obtained the circulatory wave by the interference of four plane waves (two incident and two reflected ones). Vortex configurations never show up in the TR of a single plane wave. Braunbek [15] proved that the field of only three plane waves displays vortex configurations.

3. Wolter's vortex in the GW scalar theory

The GW scalar theory of electromagnetic field has been developed in the 1950's [20], and is nothing else but the electromagnetic counterpart of the Madelung quantum theory. The GW theory is based on a single, generally complex scalar wave function $V(\mathbf{x}, t)$ and is valid in regions free of charges and currents. This theory was extended to the full generality by Roman [21].

The transition to the GW scalar theory is especially simple in the case of the electromagnetic field in vacuum, that can be completely specified by the vector potential, $\mathbf{A}(\mathbf{x}, t)$ satisfying the divergence free condition

$$\nabla \cdot \mathbf{A}(\mathbf{x}, t) = 0. \tag{1}$$

In Fourier integral form **A** reads

$$\mathbf{A}(\mathbf{x},t) = \int [\mathbf{a}(\mathbf{k},t)\cos(\mathbf{k},\mathbf{x}) + \mathbf{b}(\mathbf{k},t)\sin(\mathbf{k},\mathbf{x})]\, d\mathbf{k}. \tag{2}$$

The integration is taken over the half plane $k_z > 0$. Eq. (1) implies that for each **k** we have

$$\mathbf{k}\cdot\mathbf{a} = \mathbf{k}\cdot\mathbf{b} = 0. \tag{3}$$

One may introduce two real, mutually orthogonal unit vectors $\mathbf{l}_1(\mathbf{k})$ and $\mathbf{l}_2(\mathbf{k})$, both at right angles to **k**

$$\mathbf{l}_1(\mathbf{k}) = \frac{\mathbf{n}\times\mathbf{k}}{|\mathbf{n}\times\mathbf{k}|}, \tag{4}$$

$$\mathbf{l}_2(\mathbf{k}) = \frac{\mathbf{k}\times\mathbf{l}_1}{|\mathbf{k}\times\mathbf{l}_1|}, \tag{5}$$

where **n** is a real, arbitrary but fixed vector. The two vectors **a** and **b** may be expressed in the form $\mathbf{a}(\mathbf{k},t) = a_1\mathbf{l}_1 + a_2\mathbf{l}_2$ and $\mathbf{b}(\mathbf{k},t) = b_1\mathbf{l}_1 + b_2\mathbf{l}_2$. One may pass to the complex combinations $\alpha = a_1 + ia_2$ and $\beta = b_1 + ib_2$, which are considered as the Fourier coefficients of a new function, known as the complex potential of the field

$$V(\mathbf{x},t) = \int [\alpha(\mathbf{k},t)\cos(\mathbf{x},t) + \beta(\mathbf{k},t)\sin(\mathbf{x},t)]\, d\mathbf{k}. \tag{6}$$

Once the constant vector **n** has been chosen, the complex potential V is uniquely specified by the Fourier components **a** and **b** of the vector potential **A** and hence by **A** itself. In this way the vector potential **A** is uniquely specified by the complex potential V. Green and Wolf have shown that the relationship between **A** and V is a linear one

$$V(\mathbf{x},t) = \int \mathbf{A}(\mathbf{y},t)\cdot\mathbf{M}(y-x)\, dy. \tag{7}$$

The kernel **M** is the Fourier transform of the set of the complex base vectors $\mathbf{L}(\mathbf{k})$ given as follows:

$$\mathbf{L}(\mathbf{k}) = \mathbf{l}_1(\mathbf{k}) + i\mathbf{l}_2(\mathbf{k}), \quad k_z > 0. \tag{8}$$

The form of the momentum density $\mathbf{g}(\mathbf{x},t)$ and the energy density $w(\mathbf{x},t)$, in terms of the complex potential are similar to the probability current and probability density in quantum mechanics

$$\mathbf{g}(\mathbf{x},t) = -\frac{1}{8\pi}(\dot{V}^*\nabla V + \dot{V}\nabla V^*), \tag{9}$$

$$w(\mathbf{x},t) = \frac{1}{8\pi}(VV^* + \nabla V\,\nabla V^*), \tag{10}$$

where the dot denotes the time derivative and the star is the complex conjugate operation. The close connection to the quantum potential representation of quantum mechanics is natural if we think that both frameworks are hydrodynamical-like theories.

The energy density and the energy flow vector satisfy an equation of continuity of the usual form $\partial w/\partial t + \nabla \cdot \mathbf{g} = 0$.

If the complex potential is written down in the polar form $V(\mathbf{x}, t) = A(\mathbf{x}) \exp[iB(\mathbf{x}, t)]$, from Eq. (9) one gets

$$\mathbf{g}(\mathbf{x}, t) = -\frac{1}{4\pi}\left(\dot{A}\nabla A + A^2 \dot{B} \nabla B\right) \tag{11}$$

and by making use of the eikonal phase ansatz $B(\mathbf{x}, t) = kS(\mathbf{x}) - \omega t$, one will find

$$\mathbf{g}(\mathbf{x}, t) = \frac{k^2}{4\pi} A^2 \, \nabla S. \tag{12}$$

Thus, there is an orthogonal flow of the energy onto the surfaces $S(\mathbf{x}) = \text{const}$, which are the wavefronts. The momentum density may be looked upon as curves (rays) orthogonal to wavefronts. By means of the hydrodynamical definition of the velocity field $\mathbf{v} = \nabla S$, one can find out the irrotational condition $\nabla \times \mathbf{v} = 0$ everywhere except at the nodal regions $A = 0$, where S cannot be defined.

Introducing in the usual manner the circulation as the line integral $\oint_L \mathbf{v}\, d\mathbf{r}$ on a closed path L, we obtain the well-known condition of its discrete valuedness provided the loop is not passing directly through any of the nodal regions of the scalar potential. The quantization of the circulation

$$\oint_L \mathbf{v}\, d\mathbf{r} = 2\pi n, \quad n = 0, 1, 2, \dots , \tag{13}$$

is an old hydrodynamical fact due to the continuity and single-valuedness character of the velocity potential. More general circulations may be found in superconductivity and superfluidity as well as in various types of quantum field models. In the framework of the simple scalar (hydrodynamical) theory, the discrete values of the velocity circulation expresses the fact that the phase $S(\mathbf{x})$ of the scalar potential is defined up to an integer multiple of 2π.

If within the loop there is only one node of the scalar potential, as in the case of Wolter's wave, the circulation will be 2π corresponding exactly to a time delay of one period in the denser medium. The nodal regions of the scalar potential occur because of the diffractive conditions at the surface.

There are obvious similarities between the GW theory and the known superfluids (superconducting electrons, superfluid He^4, and superfluid He^3), as we already remarked. The macroscopic quantum-mechanical wave function of superfluids should be single valued. This implies the quantization of circulation in a neutral superfluid and of magnetic flux in a charged superfluid. A complete microscopic theory of superfluid He^4 does not exist, although the standard interpretation is in terms of Bose-Einstein condensation and the postulate of quantized circulation in units of h/m_4. Some phases of liquid He^3 are considered to be Cooper-paired, neutral superfluids with the circulation quantized in units of h/m_3. Recently, in Ref. 22 the quantized circulation of the B-phase of He^3 has been observed in the laboratory. On the other hand, Wolter's vortex is equivalent to a quantized circulation, however occuring in a rather special kind of electromagnetic (zero-mass) superfluid [23].

4. WOLTER'S VORTEX AND HAMAMATSU EXPERIMENT

Wolter's vortex in TR might be relevant for the progress in the field of wave-particle duality in the case of light, especially for the Hamamatsu anticoincidence experiment of Mizobuchi and Ohtaké [24] in which both behaviors are revealed simultaneously at "single-photon" levels of the tested beam as suggested by Ghose, Home, and Agarwal [25]. The experiment can be considered as a modern version of the double-prism experiment on frustrated TR first done by Bose in 1897, and in fact a Bose's double-prism is used as the beam splitter. The physical aspect to be understood is the nonlocality of the photon field in this particular type of experiment. In our opinion, for anticoincidence experiments performed with Bose beam splitters, Wolter's vortex is of direct relevance to the non-locality responsible for the logic *and* of the wave-particle duality. This is so because the Bose beam splitter is basically operating on the principle of the optical tunneling (frustrated TR) in which Wolter's vortex is part of the phenomenology. Moreover, one should be aware of the network of vortices existing within any laser beam [26]. Interestingly, in a paper of Elitzur, Popescu and Rohrlich [27] the nonlocality is shown to be present in each pair of an ensemble of spins when the spins are coupled in singlet states only. They comment on a formal analogy between local and nonlocal photons and normal and superfluid components of helium II.

Other types of measurements at the "single-photon" level emphasize the non-local nature of a "single photon" and the relationship with the EPR paradox and Bell's inequalities, for a review see [28].

It would be interesting to perform experiments dealing with "single photons" in the sense of one-photon Fock (number) state [29] in order to investigate the nonlocal properties of the "single-photon" Fock field.

5. CONCLUDING REMARKS

In this paper I focused on Wolter's vortex, a forgotten phase singularity of the optical field in TR, clearly put into experimental evidence already in 1949.

The vorticity-type singularities are special details of all sorts of wave packets: electromagnetic, acoustic and/or quantum mechanical ones [4]. They can be created at the moment of formation of the wave packet or by diffraction during propagation. Their space scale is of the order of the wavelength.

We also gave some hints on possible implications of the Wolter's vortex in the Hamamatsu experiment on wave-particle duality claiming a logic *and* for this duality at the "single-photon" level of the tested beam. It might be possible that in the case of Bose (double-prism) splitter, the nonlocality of the "single photon" and the Wolter's vortex are interrelated. Moreover, the Hamamatsu experiment must be repeted in more definite conditions and making use of the recent technological progress in high-efficiency "single-photon" detectors. A detailed treatment of low-intensity photon beams is needed.

It should also be emphasized that vortical singularities, being classical interference patterns endowed with a circulation constraint, are not included in quantization procedures for the evanescent waves [30]. Very recently, Lugiato and Grynberg elaborated on the effect of quantum noise on the optical vortices [31].

Acknowledgments

This work was partially supported by the CONACyT Project 4868-E9406.

References

1. See, *e.g.*, M. Brambilla *et al.*, *Phys. Rev. A* **43** (1991) 5090, 5114; *Phys. Rev. A* **49** (1994) 1427.
2. H. Wolter, *Z. Naturforschg.* **5a** (1950) 276.
3. H.K.V. Lotsch, *Optik* **32** (1970) 116.
4. J.O. Hirschfelder, A.C. Christoph, and W.E. Palke, *J. Chem. Phys.* **61** (1974) 5435, 5456; (One of the most exhaustive treatments of the Madelung quantum hydrodynamics, discussing also the optical tunneling analogy); See also J.O. Hirschfelder, *J. Chem. Phys.* **67** (1977) 5477, where Hirschfelder discusses two types of vortices of the flux of probability density corresponding to a complex (quantum) wave function, namely *axial vortices* and *toroidal vortices*. There exist also discontinuities in the surfaces of constant S which occur at nodal points, similar to dislocations and disclinations in crystals, first discussed by J.F. Nye and M.V. Berry, *Proc. R. Soc.* **A336** (1974) 165.
5. A. Sommerfeld, *Optics* (Academic Press Inc., 1954).
6. S. Zhu *et al.*, *Am. J. Phys.* **54** (1986) 601; A compact historical account of reflection and refraction may be found in H.K.V. Lotsch, *J. Opt. Soc. Am.* **58** (1968) 551.
7. *Berkeley Physics Course, Quantum Physics* (McGraw-Hill, 1967), Vol. 4; For the inverse analogies, see for example, J.J. Hupert and G. Ott, *Am. J. Phys.* **34** (1966) 260. These authors argue that of the two analogies to quantum-mechanical tunneling, the frustrated TR and the microwave signal into a TE_{10} waveguide loaded with dielectric material everywhere except for a narrow gap left air-filled, the latter is the complete one. The microwave 'tunneling' is studied by many authors and there are many interesting results. In our opinion, both analogies are equally right. The TR analogy for two-dimensional square potential barrier problems is exact mathematically, as shown by Hirschfelder *et al.*, when one is considering the polarization perpendicular to the plane of incidence. In that case, the Maxwell equations turn into the Helmholtz equation for the electric field, for which the boundary conditions and the equation of continuity across the surface produce reflection and transmission coefficients similar to the quantum-mechanical ones when the identities $k = \omega n/c = [2m(E-V)]^{1/2}$ and $\omega = E/\hbar$ are fulfilled. See also, L.I. Schiff, *Am. J. Phys.* **25** (1957) 207.
8. F. Goos and H. Hänchen, *Ann. Physik* **1** (1947) 333.
9. J. Picht, *Ann. Physik* **3** (1929) 433.
10. F.I. Fedorov, *Dokl. Akad. Nauk SSSR* **105** (1955) 465; For recent papers, see V.G. Fedoseyev, *J. Phys. A: Math. Gen.* **21** (1988) 2045; Liu Xin Ya, *ibid.* **24** (1991) L323.
11. C. Imbert, *C. R. Acad. Sci.* **269** (1969) B1227.
12. L. Dutriaux, A. Le Floch, and F. Bretenaker, *Europhys. Lett.* **24** (1993) 345.
13. O. Costa de Beauregard, *Cahiers de Physique* **18** (1964) 471.
14. M.A. Pleyer, *J. Phys. A: Math. Gen.* **20** (1987) 3667; W.N. Hugrass, *J. Mod. Optics* **3** (1990) 339; S. Huard, *Opt. Commun.* **30** (1979) 8; J.L. Agudín, A.M. Platzeck, and J.R. Albano, *Phys. Lett.* **65A** (1978) 77; R. Dutheil, *Annales Fond. Louis de Broglie* **13** (1993) 239; L. de Broglie and J.P. Vigier, *Phys. Rev. Lett.* **28** (1972) 1001.
15. W. Braunbek, *Z. Naturforschg.* **6a** (1951) 12.
16. H. Wolter, in *Progress in Optics* edited by E. Wolf, vol. **1** (1961) see pp. 183–186.

17. P. Drude, *Precis d'Optique* tome 2 (1912).
18. F. Noether, *Ann. Physik* **11** (1931) 141.
19. C. Schaefer and R. Pich, *Ann. Physik* **30** (1937) 245.
20. H.S. Green and E. Wolf, *Proc. Phys. Soc.* **A66** (1953) 1129.
21. P. Roman, *Acta Phys. Hung.* **4** (1955) 209.
22. J.C. Davis *et al.*, *Phys. Rev. Lett.* **66** (1991) 329.
23. A superfluid model of the aether/elm. vacuum has been discussed during the 1970's by K.P. Sinha, C. Sivaram, and E.C.G. Sudarshan, *Found. Phys.* **6** (1976) 65; K.P. Sinha and E.C.G. Sudarshan, *Found. Phys.* **8** (1978) 823; See also, H. Inoue, T. Tajima, and S. Tanaka, *Prog. Theor. Phys.* **48** (1972) 1338.
24. Y. Mizobuchi and Y. Ohtaké, *Phys. Lett.* **A168** (1992) 1; There are other experimental investigations of the duality problem at the single-photon level, mainly on the lines of Dirac's paradigm, "a photon interferes with itself", see P. Grangier, G. Roger, and A. Aspect, *Europhys. Lett.* **1** (1986) 173, and the review by A. Aspect and P. Grangier, *Hyperfine Interact.* **37** (1987) 3. Time-sequential data of temporarily-interfering photon arrivals in the single-photon region have been investigated by T. Okoshi, A. Hirose, and K. Kimura, *Opt. Commun.* **72** (1989) 7.
25. P. Ghose, D. Home, and G.S. Agarwal, *Phys. Lett.* **A153** (1991) 403; For popular discussions see D. Home and J. Gribbin, "What is light ?", *New Scientist* (2 November 1991) and K. Croswell, "When is a wave also a particle?", *New Scientist* (22 August 1992)
26. I. Freund, *Phys. Lett.* **A198** (1995) 139.
27. A.C. Elitzur, S. Popescu, and D. Rohrlich, *Phys. Lett.* **A162** (1992) 25.
28. D. Home and F. Selleri, *Riv. Nuovo Cimento* **14** (1991) No. 9.
29. C.K. Hong and L. Mandel, *Phys. Rev. Lett.* **56** (1986) 58.
30. C.K. Carniglia and L. Mandel, *Phys. Rev. D* **2** (1971) 280.
31. L. Lugiato and G. Grynberg, *Europhys. Lett.* **29** (1995) 675.