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Size effect in high-temperature superconductors at unilateral electromagnetic excitation

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ABSTRACT. The size effect in a high-T_c superconducting plate at unilateral electromagnetic excitation is theoretically studied. This effect manifests itself, for example, in the temperature dependences of the surface impedance and absorptivity. It is established that the plate absorptivity *versus* temperature has always a maximum independently on the dielectric permeability of its substrate. At the same time the size effect in the surface resistance is quite sensitive to the substrate properties. Calculations are carried out within the frame of the theory of thermally assisted flux flow (TAFF) using only one parameter of the superconductor, namely, the temperature-dependent effective conductivity σ.

RESUMEN. Se estudia teóricamente el efecto de tamaño en una placa superconductora de alta T_c bajo excitación electromagnética unilateral. Este efecto se manifiesta, por ejemplo, en las dependencias de temperatura de la impedancia superficial y la absorción. Se establece que la absorción de la placa contra temperatura siempre tiene un máximo independientemente de la permeabilidad dieléctrica de su substrato. Al mismo tiempo el efecto de tamaño en la resistencia superficial es bastante sensible a las propiedades del substrato. Los cálculos se realizan dentro del marco de la teoría de transporte de flujo magnético térmicamente activado, utilizando sólo un parámetro del superconductor, es decir, la conductividad efectiva σ dependiente de la temperatura.

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1. INTRODUCTION

The size effect is long-known in the electrodynamics of metals (see, for example, Ref. 1) and superconductors [2–8]. At bilateral electromagnetic excitation of a plate this effect manifests itself as a step-like change in the imaginary part and as a maximum in the real part of the surface impedance as a function of the temperature, frequency, external magnetic field and other parameters of the problem. These peculiarities in the surface impedance of high-temperature superconductors (HTS) were investigated experimentally, for instance, in the works of Refs. 9 and 10. Many authors tried to relate the behavior observed in the electromagnetic response to dynamic characteristics of the vortex array in the superconductor [11–14]. However, Geshkenbein *et al.* [16] demonstrated that the results of the above mentioned experiments can be easily interpreted as a typical manifestation of the size effect. In any case it is clear that the size effect should be considered in the analysis of any experiment with bilateral excitation.

Frequently the size effect is explained in the following way. Let us consider the bilateral, symmetric in the ac-magnetic field (antisymmetric in the electric field), excitation of a conducting plate. If the penetration depth δ of the electromagnetic signal is much smaller than the sample thickness d, then the surface impedance Z is small as $\delta/d \ll 1$ and increases with δ. Otherwise, when $d \ll \delta$, the electric field of the wave is small since it has opposite signs at the plate boundaries and does not vary, practically, in the interior of the sample. Hence, the value of the surface resistance $R = \mathrm{Re}\, Z$ turns out to be again small and decreases with augmenting δ. This means that the real part of the surface impedance should have a maximum in the vicinity $\delta \sim d$. Therefore, to observe the size effect it is necessary to have two waves which irradiate the plate at both sides. For this reason there exists a prevalent idea that the size effect can take place only at bilateral, symmetric in the ac-magnetic field excitation.

Recently, we have demonstrated [15] that the size effect is manifest and plays an important role in the electromagnetic response of hard superconductor not only at bilateral, but also at unilateral excitation. The cause of the appearance of the size effect at unilateral excitation is connected to the fact that even under these conditions the sample is, in practice, irradiated at both sides. Besides the incident wave, there is a second wave which is reflected from the interface superconductor-substrate. It is important to emphasize that many experiments over the radio-frequency range, trying to avoid the size effect, are carried out with unilateral excitation. However, the result of our previous work [15] show that the size effect must be considered in experiments where the penetration depth of the alternating signal can be of the same order as the sample thickness.

For the theoretical analysis of the size effect one have to select a model which describes the relation between the current density and the radio-wave field inside the superconductor. It is known (see, for example, Ref. 16), that the form of the current-voltage characteristics of HTS materials qualitatively changes at certain line $T = T^*(H)$ on the plane H-T. Here T is the temperature, H is the magnitude of the external constant magnetic field. In the region $T < T^*$ the vortex system is in the pinning state, and the electrodynamic properties of HTS are well described by the critical-state model [17, 18]. The size effect in that temperature range was studied in Ref. 15. At temperatures $T > T^*$ when the vortex system passes to liquid phase, the current-voltage characteristics of the superconductor are determined by the effect of thermally assisted flux flow (TAFF). The theory of this effect was developed by Kes and co-authors in Ref. 14. They established that in the temperature interval $T^* < T < T_c$ the relation of the current density $\mathbf{j}$ with the electric field $\mathbf{E}$ is well described by Ohm's law,

$$\mathbf{j} = \sigma \mathbf{E}. \tag{1}$$

In the present work we study the size effect within the frame of the TAFF model, *i.e.*, by employing Eq. (1). In this study the value of the conductivity σ is considered as the sole phenomenological parameter which determines the electrodynamics properties of the superconductor. With varying the temperature and external constant magnetic field, the conductivity can change in a wide interval of values: $1 < \sigma(T,H)/\sigma_n < 10^5$ (σ_n is the conductivity of the sample in normal state). The results obtained with the model (1) are of interest since many experiments in the radio-wave frequency range are carried out at temperatures $T \sim T_c$ in order to observe such effects as the Berezinskii-Kosterlitz-Thouless transition, the fluctuation conductivity, etcetera (see, for example, Ref. 19).

2. Formulation of the problem

Let us consider an infinite plane-parallel superconducting plate with a thickness d which overlies a substrate characterized by a dielectric permeability ε. Axis x is oriented perpendicularly to the plate boundaries and the origin of coordinate $x = 0$ is at the free boundary of the superconductor. The interface superconductor-substrate coincides with the plane $x = d$. The free plate surface $x = 0$ is irradiated by an electromagnetic wave whose magnetic vector $\mathbf{H}_0(t)$ is parallel to axis z and varies with time as

$$H_0(t) = H_0 \cos(\omega t). \tag{2}$$

The magnetic induction $\mathbf{B}(x,t)$ and the electric field $\mathbf{E}(x,t)$ in the superconductor depend only on the spatial coordinate x. In this case vector $\mathbf{B}(x,t)$ has only a z-component and vector $\mathbf{E}(x,t)$ contains exclusively a y-component

$$\mathbf{B}(x,t) = \{0,0,B(x,t)\}, \qquad \mathbf{E}(x,t) = \{0,E(x,t),0\}. \tag{3}$$

The system of Maxwell's equations together with boundary conditions for functions $B(x,t)$, $E(x,t)$ and the material Eq. (1) are linear. For this reason it is convenient to search the fields $B(x,t)$ and $E(x,t)$ in the complex form

$$B(x,t) = B(x)\exp(-i\omega t), \qquad E(x,t) = E(x)\exp(-i\omega t). \tag{4}$$

Thus, Maxwell's equations can be written as follows:

$$B'(x) = -(4\pi/c)\sigma E(x), \qquad E'(x) = (i\omega/c)B(x), \tag{5}$$

where primes indicate derivative with respect to coordinate x. The boundary conditions for system (5) are

$$B(0) = H_0, \qquad B(d) = \varepsilon^{1/2}E(d). \tag{6}$$

We are interested in the manifestation of the size effect in the temperature dependences of the real and imaginary parts of the surface impedance

$$Z = \frac{4\pi}{c}\,\frac{E(0)}{H_0}, \tag{7}$$

as well as of the absorptivity of the superconducting plate

$$A = \frac{4}{H_0}\left[\operatorname{Re} E(0) - \frac{|B(d)|^2}{H_0 \varepsilon^{1/2}}\right]. \tag{8}$$

The quantity A represents the ratio of the mean wave energy in a period, which is absorbed in the superconductor at the time unit, to the flux of the electromagnetic energy of the wave that irradiates the system. Let us emphasize that in the considered case of the unilateral excitation the quantity $R = \operatorname{Re} Z$ characterizes the total energy that passes to the system plate-substrate, whereas the quantity A determines only the fraction of this energy which is dissipated inside the superconductor.

We introduce dimensionless variables:

$$\begin{aligned} \zeta = r - i\chi = Zc^2/2\pi\omega d, \qquad & a = Ac/2\omega d, \\ u = \delta/d, \qquad & \beta = \varepsilon^{1/2}\omega d/2c. \end{aligned} \tag{9}$$

Here

$$\delta = (c^2/2\pi\sigma\omega)^{1/2} \tag{10}$$

denotes the penetration depth of the wave in the sample, which is well known from the theory of the normal skin effect. Parameter β represents, essentially, the ratio of the surface impedance of a semi-infinite superconductor with δ replaced by d to the substrate impedance. This parameter determines the role of the substrate in the size effect.

Omitting simple calculations, we shall present final results for the quantities ζ and a:

$$\zeta = (1-i)u\,\frac{\operatorname{ch}[(1-i)/u] + (1-i)\beta\, u\, \operatorname{sh}[(1-i)/u]}{\operatorname{sh}[(1-i)/u] + (1-i)\beta\, u\, \operatorname{ch}[(1-i)/u]}, \tag{11}$$

$$\begin{aligned} a = \operatorname{Re}\left\{(1-i)u\,\frac{\operatorname{ch}[(1-i)/u] + (1-i)\beta\, u\, \operatorname{sh}[(1-i)/u]}{\operatorname{sh}[(1-i)/u] + (1-i)\beta\, u\, \operatorname{ch}[(1-i)/u]}\right\} \\ - 2\beta u^2 \Big|\operatorname{sh}[(1-i)/u] + (1-i)\beta\, u\, \operatorname{ch}[(1-i)/u]\Big|^{-2}. \end{aligned} \tag{12}$$

3. Results and discussion

Let us analyze expressions (11) and (12) as the functions of the parameter u for a fixed value of β. This means that we are investigating the influence of the conductivity σ upon the size effect [see definitions of the parameters u, β (9) and depth δ (10)]. In turn, the conductivity σ depends on the sample temperature and the magnitude of the external constant magnetic field which is not included in the explicit form into the formulation of the boundary problem (5) and (6). It is clear that the size effect can manifest itself also in the dependences on the frequency ω and plate thickness d. Nevertheless, here we concentrate our attention on the dependence of the size effect on the parameter σ in

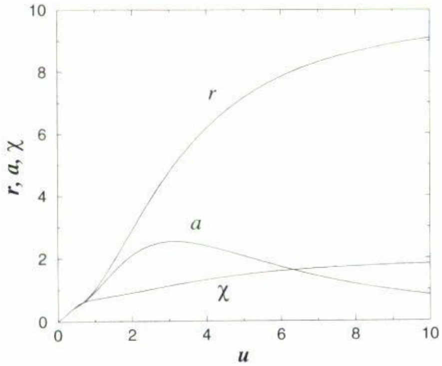

FIGURE 1. Dependence of $r(u)$, $a(u)$, and $\chi(u)$ with $\beta = 0.1$.

accordance with the performance of the majority of experiments with unilateral excitation (see, for example, Ref. 19).

Our analysis is carried out separately for the cases of optically soft ($\beta \ll 1$) and optically dense ($\beta \gg 1$) substrates. The case

$$\beta \ll 1 \tag{13}$$

is realized practically at all reasonable frequencies, if a dielectric serves as the substrate. At relatively small values of u,

$$u \leq 1, \tag{14}$$

when the penetration depth δ is either smaller or of the same order as the sample thickness d, Eqs. (11) and (12) are simplified and in the leading approximation in the parameter β acquire the form

$$r = a = \mathrm{Re}\,\zeta = u\,\frac{\mathrm{sh}(2/u) + \sin(2/u)}{\mathrm{ch}(2/u) - \cos(2/u)}, \qquad \chi = -\mathrm{Im}\,\zeta = u\,\frac{\mathrm{sh}(2/u) - \sin(2/u)}{\mathrm{ch}(2/u) - \cos(2/u)}. \tag{15}$$

In the region

$$u \gg 1 \tag{16}$$

the following asymptotics are valid:

$$r = \frac{u^2}{1 + \beta u^2}, \qquad \chi = \frac{2}{3}\,\frac{1 + 3\beta u^2 + 3\beta^2 u^4}{(1 + \beta u^2)^2}, \qquad a = \frac{u^2}{(1 + \beta u^2)^2}. \tag{17}$$

For small values of u the quantities r and a coincide because under condition $\delta \ll d$ the fraction of energy, passing into the substrate, is exponentially small. In the region $u > 1$ ($\delta > d$) the size effect is manifest: an abrupt enhancement of function $r(u)$ and $a(u)$ as well as a slowdown in the increase of $\chi(u)$ occur. The absorptivity $a(u)$ has a maximum $a_m = 1/4\beta$ at $u = u_m = \beta^{-1/2}$. In Fig. 1 graphs $r(u)$, $a(u)$ and $\chi(u)$ with $\beta = 0.1$ show the size effect in the case of an optically soft substrate.

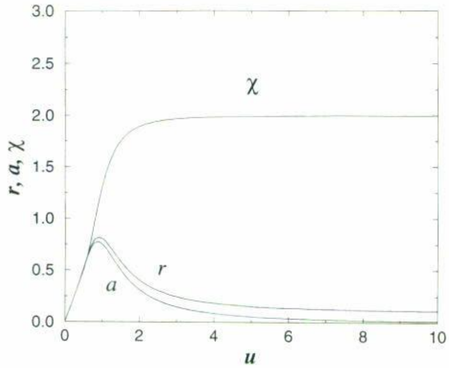

FIGURE 2. Dependence of $r(u)$, $a(u)$, and $\chi(u)$ with $\beta = 10$.

Let us consider now the situation when the substrate is an optically dense medium,

$$\beta \gg 1. \tag{18}$$

This case can be realized if, for example, the superconducting plate overlies a metallic substrate (see Ref. 20 and references therein) whose conductivity is considerably larger than σ.

In region (14) Eqs. (11) and (12) give

$$r = a = u\,\frac{\mathrm{sh}(2/u) - \sin(2/u)}{\mathrm{ch}(2/u) + \cos(2/u)}, \qquad \chi = u\,\frac{\mathrm{sh}(2/u) + \sin(2/u)}{\mathrm{ch}(2/u) + \cos(2/u)}, \tag{19}$$

and for interval (16) we get

$$r = 1/\beta + 4/3u^2, \qquad a = 4/3u^2, \qquad \chi = 2(1 - 1/\beta u^2 - 1/u^4). \tag{20}$$

The functions $r(u)$, $a(u)$ and $\chi(u)$ have linear behavior at $u \ll 1$. In the region $u \sim 1$ (at $\delta \sim d$) $r(u)$ and $a(u)$ exhibit maxima. Note that Eqs. (19) and (20) for r and a, with $\beta \to \infty$, coincide with the results of Ref. 6, dedicated to the analysis of the size effect at the bilateral excitation. This is not surprising since the case $\beta \to \infty$ corresponds to the boundary condition $E(d) = 0$. There exists a similar condition in the problem of the bilateral excitation, where the electric field vanishes at the plate middle. This means that the size effect in a plate with thickness $2d$ at bilateral excitation follows the same way as in a plate with thickness d at unilateral excitation with $\beta \to \infty$. Graphs of functions $r(u)$, $a(u)$, and $\chi(u)$ with $\beta = 10$ are shown in Fig. 2.

Let us compare our results for the surface impedance with those obtained within the critical-state model [15]. It turns that the size effect is observed in the dependences of the surface impedance and absorptivity on the parameter $u = \delta/d$ for the TAFF model, whereas for the critical-state model it manifests in the dependences of $Z = R - iX$ and A on the parameter $h_0 = H_0/H_p$ (H_p is the ac-amplitude at which the radiowave reaches the substrate). Both parameters u and h_0 characterize the penetration depth of the ac field. In the case of optically dense substrates the graphs $R(h_0)/R(1)$ and $r(u)$, $X(h_0)/X(1)$ and $\chi(u)$, $A(h_0)/A(1)$ and $a(u)$ qualitatively agree. However, when the substrate is optically soft ($\beta \ll 1$) there are differences between them. Thus, the maximum of $a(u)$ is in the region $u \sim 1/\beta^{1/2} \gg 1$ ($\delta \gg d$), while $A(h_0)/A(1)$ reaches its maximum at $h_0 \sim 1$ ($\delta \sim d$). Similarly, the surface resistance is saturated (*i.e.*, it becomes comparable with the surface resistance of the substrate) at $u \gg 1/\beta^{1/2} \gg 1$ within TAFF approach, and at $h_0 \gg 1$ within the critical-state model [15]. The differences detected in the case of optically soft substrates ($\beta \ll 1$) may be explained as follows. In the critical-state model the condition $\delta \gg d$ is sufficient in order that the superconducting plate ceases to obstruct the propagation of the wave. On the other hand, in the TAFF model (as in the case of a metallic plate) that condition is not enough. Under conditions $1 \ll u \ll 1/\beta^{1/2}$, that is,

$$\delta^2 \epsilon^{1/2} \omega / c \ll d \ll \delta, \tag{21}$$

the impedance of the system plate-substrate is much higher than the impedance of a semi-infinite superconductor, but remains to be much smaller than the substrate impedance. In other words, the plate with $\mathbf{j} = \sigma \mathbf{E}$ shields strongly even under conditions $\delta \gg d$. To eliminate the influence of the plate it is necessary to reduce more its thickness. Only in the region $u \gg 1/\beta^{1/2}$, *i.e.*,

$$d \ll \delta^2 \epsilon^{1/2} \omega / c \ll \delta, \tag{22}$$

the impedance of the system plate-substrate is comparable with the substrate impedance.

We have demonstrated within the TAFF model that the size effect plays a fundamental role in the electromagnetic response of high-temperature superconductors not only at bilateral excitation, but also at unilateral one. The manifestation of the size effect is similar to that obtained withing other models, for example, the critical-state approach.

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