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Symmetry breaking in $[\mathrm{SU}(6)]^3 \times \mathrm{Z}_3$

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ABSTRACT. We analize the different ways for the spontaneous breaking of the gauge symmetry, for the $[\mathrm{SU}(6)]^3 \times \mathrm{Z}_3$ family unification model. In particular we study the consequences of a previous selection for the vacuum expectation values of the Higgs fields, showing that such set predicts unwanted flavor changing neutral currents at the $m_\mathrm{Z} = 91$ GeV mass scale. A new set of vacuum expectation values which solves this problem is proposed.

RESUMEN. Se analizan los diferentes caminos para el rompimiento espontaneo de la simetría de norma en el modelo de unificación de familias $[\mathrm{SU}(6)^3] \times \mathrm{Z}_3$. En particular se estudian las consecuencias de una elección previa de los valores esperados en el vacío de los campos de Higgs, mostrando que tal conjunto predice corrientes neutras que cambian de sabor a la escala de masa $m_\mathrm{Z} = 91$ GeV. Un nuevo conjunto de valores esperados en el vacío el cual resuelve este problema es propuesto.

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1. INTRODUCTION

Although the standard model (SM) is a successful theory which is in good agreement with the experimental results [1], it leaves several primordial aspects unanswered. Outstanding among them is the so called flavor problem which is the lack of predictions for the fermion mass spectrum, the number of families in nature and the small values for the quark mixing angles. In order to get an answer to this problem we believe that there is a more fundamental theory, not far away from the present experimental energies. This is one of the motivation for grand unified theories (GUTs) [2] which are extensions of the SM gauge structure $\mathrm{SU}(3)_\mathrm{c} \otimes \mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_\mathrm{Y}$, into larger groups with a single gauge coupling constant.

In Ref. 3 it was presented a variant of the three family extension of the Pati-Salam [4] model which does not have mirror fermions and is renormalizable. In this model the known families belong to a single irreducible representation of the local gauge group

$[\mathrm{SU}(6)]^3 \times \mathrm{Z}_3$, each family being defined by the dynamics of the left-right symmetric extension (LRSE) [5] $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_R \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_{(B-L)}$ of the SM.

In Ref. 6 the new model was systematically studied, paying special attention to its particle content, the symmetry breaking (SB) pattern, the mass scales, the free Lagrangean for all the gauge bosons (including their mass terms), the bare masses for all the exotic fermion fields in the model, and the interacting Lagrangean with all the known and predicted gauge interactions. Even though the results presented in Ref. 6 are important for the analysis of the mass spectrum of the known quarks and leptons in the context of the model, there are several problems in the SB scheme proposed, because the set of Higgs fields and vacuum expectation values (vevs) used do not break the local symmetry $[\mathrm{SU}(6)]^3 \times \mathrm{Z}_3$ down to the SM symmetry. As a matter of fact, an extra U(1) symmetry is predicted by this SB with a nonuniversal coupling to the standard matter at the m_Z scale. Moreover, the spontaneous breaking of this symmetry mixes the Z gauge field of the SM with the field associated to the extra symmetry, giving Flavor Changing Neutral Currents (FCNC) mediated by a gauge boson field with a mass of the order of m_Z. Since this neutral currents are not allowed by the low energy experimental results, a careful analysis of the SB pattern for the $[\mathrm{SU}(6)]^3 \times \mathrm{Z}_3$ is needed. That is the aim of the work presented in this paper.

The paper is organized in the following way: in the next section we briefly review the model $[\mathrm{SU}(6)]^3 \times \mathrm{Z}_3$. In the first part of Sect. 3 we analyze general ways for implementing the spontaneous SB in the context of the model, in the second part of Sect. 3 we discuss the problem with the SB scheme used in Ref. 6, and we propose a new pattern which solves the puzzle. The renormalization group equation analysis for the new SB scheme is presented in Sect. 4 and the mass scales for the new model are estimated. We write our conclusions and some comments in the last section. An Appendix with technical information is included at the end.

2. THE MODEL

The model under consideration is based on the local gauge group

$$G \equiv \mathrm{SU}(6)_L \otimes \mathrm{SU}(6)_c \otimes \mathrm{SU}(6)_R \times \mathrm{Z}_3 \tag{1}$$

and unifies non-gravitational forces with transitions among three families. In Eq. (2.1) $\otimes$ indicates a direct product, $\times$ a semidirect one, and Z_3 is a three-element cyclic group acting upon $[\mathrm{SU}(6)]^3$ such that if (A,B,C) is a representation of $[\mathrm{SU}(6)]^3$ with A a representation of the first factor, B of the second and C of the third, then $\mathrm{Z}_3(\mathrm{A,B,C}) \equiv (\mathrm{A,B,C}) \oplus (\mathrm{B,C,A}) \oplus (\mathrm{C,A,B})$ is an irreducible representation (irrep) of G. $\mathrm{SU}(6)_c$ is a vector-like group which includes three hadronic and three leptonic colors, and has as a subgroup the $\mathrm{SU}(3)_c \otimes \mathrm{U}(1)_{B-L}$ group of the LRSE model. $\mathrm{SU}(6)_L \otimes \mathrm{SU}(6)_R$ includes the $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ gauge group of the LRSE model. Among the special properties of this model we may recall that its gauge group, G, is the most economical unifying group for three families, with left-right symmetry and with (extended) vector color; it leads to (perturbative) stability of the proton [7]. Furthermore, all the known elementary fermions belong to an irrep of G. On the other hand the presence of the horizontal group in $\mathrm{SU}(6)_{\mathrm{L}} \otimes \mathrm{SU}(6)_{\mathrm{R}}$ allows for the possibility of obtaining predictions for the fermion mass spectrum [3, 6].

The 105 gauge fields (GFs) in G can be divided into two sets: 70 of them belonging to $\mathrm{SU}(6)_{\mathrm{L}} \otimes \mathrm{SU}(6)_{\mathrm{R}}$ and 35 being associated with $\mathrm{SU}(6)_{\mathrm{c}}$. The first set includes $W_{\mathrm{L}}^{\pm}$ and W_{L}^{0} (the GFs of $\mathrm{SU}(2)_{\mathrm{L}}$ in the SM), the GFs associated with $\mathrm{SU}(2)_{\mathrm{R}}$; the GFs of the horizontal interactions, and new GFs of nonuniversal charged and neutral interactions. All of them have electric charges 0 or ± 1. The generators of $\mathrm{SU}(6)_{L(R)}$ may be written in a $\mathrm{SU}(2)_{\mathrm{L(R)}} \otimes \mathrm{SU}(3)_{\mathrm{HL(HR)}}$ basis as

$$\sigma_i \otimes I_3/2\sqrt{3}\,, \qquad I_2 \otimes \lambda_\alpha/2\sqrt{2}\,, \qquad \sigma_i \otimes \lambda_\alpha/2\sqrt{2}\,, \tag{2}$$

where $\sigma_i, i = 1,2,3$ are the three 2×2 Pauli matrices, $\lambda_\alpha, \alpha = 1,\dots,8$ are the eight 3×3 Gell-Mann matrices, and I_2 and I_3 are 2×2 and 3×3 identity matrices respectively.

The second set of gauge fields includes the eight gluon fields of $\mathrm{SU}(3)_{\mathrm{c}}$, nine lepto-quark GFs ($\mathrm{X}_i$, Y_i and $\mathrm{Z}_i = 1,2,3$ with electric charges $-\frac{2}{3}$, $\frac{1}{3}$, and $-\frac{2}{3}$ respectively), their nine charge conjugated field, six dilepton GFs ($P_a^{\pm}, P^0$, and $\tilde{P}^0, a = 1,2$, with electric charges as indicated), and GFs associated with diagonal generation in $\mathrm{SU}(6)_{\mathrm{c}}$ and not take into account already in $\mathrm{SU}(3)_{\mathrm{c}}$, including among them the gauge field associated with the $(\mathrm{B}-\mathrm{L})$ abelian generator of the LRSE model.

The fermion field of the models are in the irrep

$$\psi(180)_{\mathrm{L}} = \mathrm{Z}_3\psi(6,1,\overline{6})_{\mathrm{L}} = \psi(6,1,\overline{6})_{\mathrm{L}} \oplus \psi(\overline{6},6,1)_{\mathrm{L}} \oplus \psi(1,\overline{6},6)_{\mathrm{L}}\,, \tag{3}$$

with quantum numbers with respect to the SM factors [$\mathrm{SU}(3)_{\mathrm{c}}$, $\mathrm{SU}(2)_{\mathrm{L}}$, $\mathrm{U}(1)_{\mathrm{Y}}$] given by

$$\begin{aligned}
\psi(6,\overline{6},1)_{\mathrm{L}} \equiv \psi_a^\alpha :&\quad 3(3,2,1/3) \oplus 6(1,2,-1) \oplus 3(1,2,1)\,,\\
\psi(1,\overline{6},6)_{\mathrm{L}} \equiv \psi_a^A :&\quad 3(\overline{3},1,-4/3) \oplus 3(\overline{3},1,2/3) \oplus 6(1,1,2)) \oplus 9(1,1,0) \oplus 3(1,1,-2)\,,\\
\psi(6,1,\overline{6})_{\mathrm{L}} \equiv \psi_A^a :&\quad 9(1,2,1) \oplus 9(1,2,-1)\,,
\end{aligned}$$

where $a, b, \dots, \mathrm{A}, \mathrm{B}, \dots, \alpha, \beta, \dots = 1, \dots, 6$ label L, R and c tensor indices, respectively. The known fermions are contained in $\psi(\overline{6},6,1)_{\mathrm{L}} \oplus \psi(1,\overline{6},6)_{\mathrm{L}} \subset \psi(180)$.

The electric charge operator in the model is given by

$$Q = T_{ZL} + \frac{Y}{2}\,, \tag{4}$$

where the hypercharge $Y/2 = T_{ZL} + (1/2)Y_{(\mathrm{B-L})}$ and $T_{ZL,R} = diag\{1,-1,1,-1,1,-1\}/2$ and $Y_{(\mathrm{B-L})} = \mathrm{diag}\{1/3,1/3,1/3,-1,1,-1\}$ which act on the subspaces of the fundamental irreps of $\mathrm{SU}(6)_{L(R)}$ and $\mathrm{SU}(6)_{\mathrm{c}}$ respectively.

3. THE SPONTANEOUS SYMMETRY BREAKING PATTERN.

3.1. GENERAL ANALYSIS.

In order to achieve the spontaneous SB we introduce appropriate Higgs scalars. Using the branching rules

$$
\begin{array}{rcl}
\mathrm{SU}(6)_{L(R)} & \to & \mathrm{SU}(2)_{L(R)} \otimes \mathrm{SU}(3)_{HL(R)} \\
6 & \to & (2,3) \\
15 & \to & (1,6)+(3,\overline{3}) \\
21 & \to & (1,\overline{3})+(3,6) \\
35 & \to & (3,8)+(3,1)+(1,8)
\end{array}
\qquad
\begin{array}{rcl}
\mathrm{SU}(6)_{\mathrm{c}} & \to & \mathrm{SU}(3)_{\mathrm{c}} \\
6 & \to & (3)+3(1) \\
15 & \to & (\overline{3})+3(3)+3(1) \\
21 & \to & (6)+6(1)+3(3) \\
35 & \to & (8)+3(3)+3(\overline{3})+9(1),
\end{array}
\tag{5}
$$

we can see that the vacuum expectation values (vevs) of a 6 of $\mathrm{SU}(6)_{\mathrm{L}}$ necessarily break $\mathrm{SU}(2)_{\mathrm{L}}$; besides a Higgs field $\phi(18) = \mathrm{Z}_3\phi(6,1,1)$ is not sufficient to give tree-level masses to ordinary fermion fields. We therefore assume, as it was done in Ref. [6], that the last step of the SB chain is due to the vevs of a Higgs field $\phi_4 = \phi(108) = \mathrm{Z}_3\phi(1,\overline{6},6)$, and that these vevs lie only in the electrically neutral directions of the $\mathrm{SU}(6)_{\mathrm{L}} \otimes \mathrm{SU}(6)_{\mathrm{R}}$ subspace, in such a way that the *modified horizontal survival hypothesis* [3, 6] holds (which states that at tree level the top quark is the only standard matter fermion field acquiring mass, with the masses of the other known fermions being generated as radiative corrections).

In order to comply with the *survival hypothesis* (which states that when a gauge group G is broken down to $G_1 \subset G$ at a mass scale M_1, all the fermion fields belonging to real representations of G_1 must acquire masses of order M_1 [8]) we demand that the first steps of the SB chain arise from vevs of Higgs fields of the type $\mathrm{Z}_3\phi(\overline{n},1,n)$, where n may be 15 or 21. For this kind of fields their vevs have the general form

$$
\langle\phi\rangle = m\left[\langle\phi_{Lc}\rangle \oplus \langle\phi_{cR}\rangle \oplus \langle\phi_{LR}\rangle\right], \tag{6}
$$

where the subindices indicate the subspaces involved in each term and m is the mass scale of the breaking implement by $\langle\phi\rangle$. The covariant derivative acting on a representation of the form $a_{\mathrm{L}} \otimes a_{\mathrm{c}} \otimes a_{\mathrm{R}}$ of $[\mathrm{SU}(6)]^3$, with a_{L} (a_{c} or a_{R}) a fundamental irrep of the factor $\mathrm{SU}(6)_{\mathrm{L}}$ ($\mathrm{SU}(6)_{\mathrm{c}}$ or $\mathrm{SU}(6)_{\mathrm{R}}$) is

$$
\mathcal{D} = D_{\mathrm{L}} \otimes \mathbf{1} \otimes \mathbf{1} \;\oplus\; \mathbf{1} \otimes D_{\mathrm{c}} \otimes \mathbf{1} \;\oplus\; \mathbf{1} \otimes \mathbf{1} \otimes D_{\mathrm{R}}, \tag{7}
$$

being D_i (i = L,c,R) the corresponding covariant derivative on the irrep a_i defined by $D_i^\mu = \partial^\mu + ig\mathbf{A}_i^\mu$, with $\mathbf{A}_i^\mu = \frac{1}{2}\lambda_i^a A_{a,i}^\mu$ $a = 1,\ldots,35$, where λ_i^a are the generators of $\mathrm{SU}(6)_i$ normalized to $Tr\lambda_i^a\lambda_i^b = 2\delta^{ab}$. Also $A_{a,i}^\mu$ are the gauge bosons associated to the generators λ_i^a and g is the gauge coupling constant of G. For fields Φ in irreps 15 or 21 of an SU(6) factor, the action of the covariant derivative is

$$
D_i^\mu(\Phi) = \partial^\mu\Phi + ig\left(\mathbf{A}_i^\mu\Phi + \Phi\mathbf{A}_i^{\mu,T}\right), \tag{8}
$$

where the last equation is stated in a 6×6 matrix form. The mass Lagrangean for the gauge bosons produced by $\langle\phi\rangle$ is of the form

$$
\mathcal{L}_{\mathrm{mass}} = \mathrm{Tr}[\mathcal{D}(\langle\phi\rangle)]^\dagger[\mathcal{D}(\langle\phi\rangle)] = \mathcal{L}_{\mathrm{Lc}} + \mathcal{L}_{\mathrm{cR}} + \mathcal{L}_{\mathrm{LR}},
$$

where $\mathcal{L}_{\mathrm{Lc,cR,LR}}$ are the corresponding contributions to the Lagrangean by $\langle \phi_{\mathrm{Lc,cR,LR}} \rangle$ respectively. They may be written as

$$\mathcal{L}_{ij} = 2g^2 \mathrm{Tr}\Big(\langle \phi_{ij} \rangle \mathbf{A}_i{}^2 \langle \phi_{ij} \rangle \,+\, \langle \phi_{ij} \rangle \mathbf{A}_i \langle \phi_{ij} \rangle \mathbf{A}_i{}^T \,+\, (i \to j) \,-\, 4 \langle \phi_{ij} \rangle \mathbf{A}_i \langle \phi_{ij} \rangle \mathbf{A}_j \Big), \quad (9)$$

with $ij = \mathrm{Lc, cR, LR}$. While the first terms in (9) implement the spontaneous breaking of the corresponding factor of $\mathrm{SU}(6)_i$, giving masses to the associated bosons, the last term mixes the bosonic fields of both sectors involved, in such a way that the breaking of $\mathrm{SU}(6)_i \otimes \mathrm{SU}(6)_j$ via $\langle \phi_{ij} \rangle$ is of the form $\mathrm{SU}(6)_i \otimes \mathrm{SU}(6)_j \to \mathrm{G}_i \otimes \mathrm{G}_j \otimes \mathrm{G}_{\mathrm{mix}}$ where the specific groups $\mathrm{G}_{i(j)}$ depend only on the particular direction of the vevs in the i (j) subspace, but the mixing symmetry is given by the combined action of directions in both subspaces.

According to the branching rules stated in (5), there are six $\mathrm{SU}(2)_{\mathrm{L,R}}$ singlets in 15 of $\mathrm{SU}(6)_{\mathrm{L(R)}}$ and three in irrep 21; they are along the directions

$$\begin{array}{lll} 15 & : & [1,4]-[2,3], \quad [1,6]-[2,5], \quad [3,6]-[4,5], \\ & & [1,2], \quad [3,4], \quad [5,6], \\ 21 & : & \{1,4\}-\{2,3\}, \quad \{1,6\}-\{2,5\}, \quad \{3,6\}-\{4,5\}. \end{array} \quad (10)$$

where the notation is such that $[a,b] = ab - ba$, and $\{a,b\} = ab + ba$. The analysis shows [9] that if the Higgs fields get vevs along these direction in the subspaces L or R, the corresponding SU(6) factor breaks down to the following subgroups:

$$\mathrm{SU}(6) \left\{ \begin{array}{ll} \xrightarrow{[1,6]-[2,5]+[3,4]} & Sp(6) \\ \xrightarrow{[1,2]} & \mathrm{SU}(4) \otimes \mathrm{SU}(2) \end{array} \right. \quad \text{and } \mathrm{SU}(6) \left\{ \begin{array}{ll} \xrightarrow{[1,4]-[2,3]} & Sp(4) \otimes \mathrm{SU}(2) \\ \xrightarrow{\{1,4\}-\{2,3\}} & [\mathrm{SU}(2)]^3 \end{array} \right. \quad (11)$$

where we have written just the isomorphic residual symmetry group, with the specific structure of each subgroup depending on the particular direction for the vevs. Other combinations of the vevs above produce similar SB patterns; for example, $[1,6]-[2,5]-[3,4]$ also breaks SU(6) down to $Sp(6)$, $[3,4]$ or $[5,6]$ (instead of $[1,2]$) break SU(6) down to $\mathrm{SU}(4) \otimes \mathrm{SU}(2)$, $[1,6]-[2,5]$ or $[3,6]-[4,5]$ (instead of $[1,4]-[2,3]$) break SU(6) down to $Sp(4) \otimes \mathrm{SU}(2)$, $\{1,6\}-\{2,5\}$ or $\{3,6\}-\{4,5\}$ instead of $\{1,4\}-\{2,3\}$ break SU(6) down to $[\mathrm{SU}(2)]^3$, etc..

The reason for choosing these channels for the first step of the SB is due to the following facts: they contain the $\mathrm{SU}(2)_{\mathrm{L(R)}}$ structures of the LRSE group as subgroups; the vevs in the directions of the singlets in irreps 15 and 21 assure an unbroken $\mathrm{SU}(2)_{\mathrm{L(R)}}$ factor; and finally they comply properly with the survival hypothesis.

In order to break $\mathrm{SU}(2)_{\mathrm{R}}$, the only constraint on the vevs directions come from the demand that the generator associated to the hypercharge Y must not be broken before the last step of the SB chain. In other words, the vevs of the Higgs fields ϕ_3 which breaks $\mathrm{SU}(2)_{\mathrm{R}}$ must satisfy

$$Y\left(\langle \phi_3 \rangle\right) = 0, \quad (12)$$

where again we choose ϕ_3 of the form $\mathrm{Z}_3\phi_3(\overline{n}, 1, n)$ with $n = 15, 21$; so it is also of the form given by (6). The simplest way to achieve the constraint (12) is imposing that

$$T_{iR}\left(\langle\phi_{3\mathrm{LR}}\rangle\right) = 0, \qquad T_{iR}\left(\langle\phi_{3\mathrm{Lc}}\rangle\right) = 0 \tag{13}$$

and

$$T_{iR}\left(\langle\phi_{3\mathrm{cR}}\rangle\right) \neq 0. \tag{14}$$

where $T_{i\mathrm{R}}, i = 1, 2, 3$ are the three generators for $\mathrm{SU}(2)_{\mathrm{R}}$. These constraints are achieved in general for the combinations (α, β) odd-odd (even-even) only if (A, B) are odd-odd (even-even). In the subspace c there are just 3 directions in 21 with even-even indices which are $\{4, 4\}$, $\{4, 6\}$ and $\{6, 6\}$; while there is just one in 15 which is $[4, 6]$. For the odd-odd combination there is only one possibility, the $\{5, 5\}$ in irrep 21.

For completeness let us mention finally that any of the diagonal directions $\{i, i\}$, $i = 1 \ldots 6$ in 21 break SU(6) down SU(5), while other directions implement the breakings

$$\mathrm{SU}(6) \xrightarrow{[2,4],[2,6],[4,6]} \mathrm{SU}(4) \otimes \mathrm{SU}(2) \quad \text{and} \quad \mathrm{SU}(6) \xrightarrow[\{2,4\},\{2,6\},\{4,6\}]{\{1,3\},\{1,5\},\{3,5\}} \mathrm{SU}(4) \otimes \mathrm{U}(1). \tag{15}$$

3.2. Problems with the old SB scheme.

In a previous paper [6] the SB for this model was implemented by introducing four different sets of scalar fields:

$$\phi_1 = \phi(675) = \mathrm{Z}_3\phi(\overline{15}, 1, 15)$$

with vevs at the scale M in the directions $[a, b], [A, B] = [1, 6] = -[2, 5] = -[3, 4]$ and $[\alpha, \beta] = [5, 6]$;

$$\phi_2 = \phi(1323) = \mathrm{Z}_3\phi(\overline{21}, 1, 21)$$

with vevs at the scale M' in the directions $\{a, b\}, \{A, B\} = \{1, 4\} = -\{2, 3\}$ and $[\alpha, \beta] = \{4, 5\}$;

$$\phi_3 = \phi(675) = \mathrm{Z}_3\phi(\overline{15}, 1, 15)$$

such that $\langle\phi^{[a,b]}_{3[A,B]}\rangle = \langle\phi^{[\alpha,\beta]}_{3[a,b]}\rangle = 0$, and $\langle\phi^{[A,B]}_{3[\alpha,\beta]}\rangle \neq 0$ with vevs at the scale M_{R} in the directions $[\alpha, \beta], [A, B] = [4, 6]$.

The last step of the SB which breaks the SM symmetry was implemented by introducing the scalar fields

$$\phi_4 = \phi(108) = \mathrm{Z}_3\phi(6, 1, \overline{6})$$

with vevs $\langle\phi^{\alpha}_{4A}\rangle = \langle\phi^{a}_{4\alpha}\rangle = 0$ and $\langle\phi^{A}_{4a}\rangle = m_{\mathrm{Z}}$ for $A, a = 2, 4, 6$. As it was shown in Refs. 6 and 7, the model with only two different mass scales M_G and m_{Z}, such that

$$G \xrightarrow{M_G} \mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{SU}(3)_{\mathrm{c}} \otimes \mathrm{U}(1)_Y \xrightarrow{m_{\mathrm{Z}}} \mathrm{SU}(3)_{\mathrm{c}} \otimes \mathrm{U}(1)_{\mathrm{em}}$$

is excluded by the analysis of the renormalization group equations, and the experimental values for $\alpha_i(m_Z), i = 1,2,3$. The breaking pattern with three different mass scales, where the first step is $G \to \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_R \otimes \mathrm{SU}(2)_L \otimes U(1)_{(B-L)}$ is also forbidden [6]. Then the hierarchy $M_R \gg M_H \equiv M \sim M' \gg m_Z$ is suggested, and therefore the first step of the SB should be implemented via $\langle\phi_3\rangle$. From the analysis presented in Eq.(15) we note that $\langle\phi_3\rangle$ breaks $\mathrm{SU}(6)_L \otimes \mathrm{SU}(6)_c \otimes \mathrm{SU}(6)_R$ down to $G_M \equiv \mathrm{SU}(6)_L \otimes \mathrm{SU}(4)_c \otimes \mathrm{SU}(2)_c \otimes \mathrm{SU}(4)_R \otimes \mathrm{SU}(2)_R \otimes U(1)_\Sigma$, where $U(1)_\Sigma$ is an abelian symmetry generated as a mixing in the c-R subspaces, where

$$\Sigma = \frac{1}{\sqrt{2}}\left(\Sigma_c + \Sigma_R\right) \tag{16}$$

with

$$\Sigma_{c(R)} = \mathrm{diag}\big(1,1,1,-2,1,-2\big)/\sqrt{6}. \tag{17}$$

The algebra shows that $\Sigma\langle\phi_3\rangle = 0$, even though $\Sigma_{c(R)}\langle\phi_3\rangle \neq 0$. In the unbroken group the factor $\mathrm{SU}(4)_{c(R)}$ ($\mathrm{SU}(2)_{c(R)}$) acts on the indices 1, 2, 3, 5 (4,6) of the fundamental irrep of $\mathrm{SU}(6)_{c(R)}$.

The next step of the SB was implemented in Ref. [6] by $\langle\phi_1 + \phi_2\rangle$. The analysis of the bosonic mass Lagrangean [9] shows now that $G_M \to \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_R \otimes \mathrm{SU}(2)_L \otimes U(1)_{(B-L)} \otimes \mathrm{U}(1)'$; where we can notice that, unlike the statement in reference [6], $\langle\phi_1 + \phi_2 + \phi_3\rangle$ does not break G down the SM gauge group. As a matter of fact, there is an extra Abelian symmetry at the m_z scale. The gauge boson associated with this $\mathrm{U}(1)'$ symmetry is

$$B' = \left(9\sqrt{3}B_{(B-L)} - 15\sqrt{6}W_R^0 - 28\sqrt{5}B_{Y'} + 140H_L + 140H_R\right)/10\sqrt{469}, \tag{18}$$

where the fields involved are the gauge bosons associated to the generators $Y_{(B-L)}$, T_{ZR}, $Y' = \mathrm{diag}(1,1,1,-3,-2,2)/\sqrt{10}$, and $T_{HL,HR} = \mathrm{diag}(1,1,0,0,-1,-1)/\sqrt{2}$. As it is easy to check, the generator T' of $U(1)'$ satisfies $T'\langle\phi_1\rangle = T'\langle\phi_2\rangle = T'\langle\phi_3\rangle = 0$, but $T'\langle\phi_4\rangle \neq 0$. Then because $Q\langle\phi_4\rangle = 0$, the symmetry is properly broken at the m_Z scale, predicting the correct low energy unbroken symmetry $\mathrm{SU}(3)_c \otimes U(1)_{em}$. Nevertheless, since $T_{ZL}\langle\phi_4\rangle \neq 0$, the last step of the SB mixes B' with the Z standard field producing two new neutral fields Z_1 and Z_2 of the form

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos\varepsilon & -\sin\varepsilon \\ \sin\varepsilon & \cos\varepsilon \end{pmatrix} \begin{pmatrix} B' \\ Z \end{pmatrix}. \tag{19}$$

Considering the mass Lagrangean produced by $\langle\phi_4\rangle$, decoupling all the fields with high masses, and introducing explicitly the Z standard and the photon fields, we obtain the mixing terms

$$\mathcal{L}_{\langle\phi_4\rangle} = \frac{3}{28}g^2M_Z^2\left\{\frac{811}{134}B'^2 + 3\sqrt{\frac{69}{67}}\,B'\,Z + \frac{23}{2}Z^2\right\} + \cdots. \tag{20}$$

From here it is simple to compute the mixing angle $\sin\varepsilon$ which is

$$\sin\varepsilon = \frac{3\sqrt{67}\sqrt{69}}{2(223)^{1/4}\sqrt{14}\sqrt{\alpha_0}} \simeq 0.2520. \tag{21}$$

where $\alpha_0 = 365 + 28\sqrt{223}$. Then the mixing is large and its effects for the low energy phenomenology are important.

From the interaction Lagrangean of the fermion fields with the gauge bosons given in appendix B of the Ref. 6, we have the following terms corresponding to the fields $H_{\mathrm{L,R}}$

$$\mathcal{L}_H = -\frac{q}{2\sqrt{2}}\left[H_{\mu,\mathrm{R}}\sum_{\delta=1}^{3}\left(\bar{d}^0_{\delta,\mathrm{R}}\gamma^\mu d^0_{\delta,\mathrm{R}} + \bar{u}^0_{\delta,\mathrm{R}}\gamma^\mu u^0_{\delta,\mathrm{R}} - \bar{b}^0_{\delta,\mathrm{R}}\gamma^\mu b^0_{\delta,\mathrm{R}} - \bar{t}^0_{\delta,\mathrm{R}}\gamma^\mu t^0_{\delta,\mathrm{R}}\right) + (\mathrm{R}\to\mathrm{L})\right],$$

where δ is a color index, and the fields u^0, d^0, t^0, and b^0 (together with c^0 and s^0) constitute a basis for the quark fields, which must be rotated in order to get the physical quarks, but since the couplings of $H_{\mathrm{L,R}}$ are not universal as can be seen from $\mathcal{L}_{\mathrm{H}}$, FCNC mediated by $H_{\mathrm{L(R)}}$ (for both Z_1 and Z_2) will appear at the m_{Z} scale, in contradiction with the experimental bounds related to the non-existence of low energy FCNC.

Hence the SB scheme in Ref. 6 should be changed in order to make the model consistent with the low energy phenomenology. In order to do it we choose a more economical set of Higgs fields to implement the SB properly. We keep ϕ_3 and ϕ_4 as in Ref. 6, but instead of ϕ_1 and ϕ_2 we use ϕ'_1 and ϕ'_2 both in irreps $\mathrm{Z}_3\phi(\overline{15},1,15)$, with vevs along the following directions

$$\begin{aligned}
\langle\phi'^{[a,b]}_{1[\alpha,\beta]}\rangle &= \sqrt{3}M_{\mathrm{H}}; && \text{for } [a,b] = -[1,4] = [2,3] = [5,6];\quad [\alpha,\beta] = [5,6],\\
\langle\phi'^{[\alpha,\beta]}_{1[A,B]}\rangle &= \sqrt{3}M_{\mathrm{H}}; && \text{for } [a,b] = -[1,4] = [2,3] = [5,6];\quad [\alpha,\beta] = [4,5],\\
\langle\phi'^{[A,B]}_{1[a,b]}\rangle &= M_{\mathrm{H}}; && \text{for } [a,b],[A,B] = -[1,4] = [2,3] = -[5,6];
\end{aligned}$$

and

$$\begin{aligned}
\langle\phi'^{[a,b]}_{2[\alpha,\beta]}\rangle &= \sqrt{3}M_{\mathrm{H}}; && \text{for } [a,b] = [1,2] = -[3,6] = [4,5];\quad [\alpha,\beta] = [5,6],\\
\langle\phi'^{[\alpha,\beta]}_{2[A,B]}\rangle &= \sqrt{3}M_{\mathrm{H}}; && \text{for } [a,b] = [1,2] = -[3,6] = [4,5];\quad [\alpha,\beta] = [4,5],\\
\langle\phi'^{[A,B]}_{2[a,b]}\rangle &= M_{\mathrm{H}}; && \text{for } [a,b],[A,B] = -[12] = -[3,6] = [4,5].
\end{aligned}$$

where the $\sqrt{3}$ factor is included just for convenience. The algebra shows that $\langle\phi'_1\rangle + \langle\phi'_2\rangle$ break G down the LRSE model, and together with $\langle\phi_3\rangle$ break it down to the SM, solving the problem discussed above.

4. The mass scales.

The symmetry breaking chain is constrained by the requirement that the evolution of the gauge coupling constants associated with the factor groups of the SM, from the m_{Z} scale to the unification scale, agree with the experimental values [1] $sin^2\theta_W(m_Z) = 0.2315$, $\alpha_{EM}^{-1}(m_Z) = 127.9$, and $\alpha_3(m_Z) = 0.113$. Then for the use of the renormalization group equations (rge) we assume the validity of the *survival hypothesis* [8] as well as the validity of the *extended survival hypothesis* (which claims that when the vevs of a scalar field ϕ break a group G down to $\mathrm{G}_1 \subset \mathrm{G}$ at a mass scale M_1, only those components of ϕ which acquire vevs get a mass of the order of M_1, with the rest of the components getting masses at the G scale [10]).

When the symmetry is broken in three steps at the scales M_{R}, M_{H} and m_{Z}, the coupling constants satisfy, up to one loop, the rge

$$\alpha_i^{-1}(m_{\mathrm{Z}}) = f_i\, \alpha^{-1} - b_i^0 \ln\left(\frac{M_{\mathrm{H}}}{m_{\mathrm{Z}}}\right) - b_i^1 \ln\left(\frac{M_{\mathrm{R}}}{M_{\mathrm{H}}}\right), \tag{22}$$

where $\alpha_i = g_i^2/4\pi$, $i = 1,\ 2,\ 3$, and g_i are the gauge coupling constants of the $U(1)_Y$, $\mathrm{SU}(2)_{\mathrm{L}}$ and $\mathrm{SU}(3)_{\mathrm{c}}$ subgroups of the SM respectively. The factors f_i are constants and define the relation at the unification scale M between g, the coupling constant of $[\mathrm{SU}(6)]^3 \times \mathrm{Z}_3$ and g_i. The numerical values of these factors $f_1 = 14/3$, $f_2 = 3$ and $f_3 = 1$ [3, 6], arise from the normalization conditions imposed on the generators of G.

In Eq. (22) the beta functions b_i^k are given by

$$b_i^k = \tfrac{1}{4\pi}\left\{\tfrac{11}{3}C_i^k(\text{vectors}) - \tfrac{2}{3}C_i^k(\text{Weyl fermions}) - \tfrac{1}{6}C_i^k(\text{scalars})\right\}, \tag{23}$$

where $k = 0, 1$ and $C_i^k(\dots)$ are the index of the representation to which the $(\dots)$ particles are assigned. For a complex field the value of $C_i^k(\text{scalars})$ should be doubled. Also, the following relationships

$$\alpha_{EM}^{-1} \equiv \alpha_1^{-1} + \alpha_2^{-1} \quad \text{and} \quad \tan^2\theta_W = \frac{\alpha_1}{\alpha_2}, \tag{24}$$

where θ_W is the weak mixing angle, hold at all energy scales. From this expressions we get

$$\frac{\sin^2\theta_W(m_{\mathrm{Z}})}{\alpha_{EM}(m_{\mathrm{Z}})} - \alpha_3^{-1}(m_{\mathrm{Z}}) = \left(b_3^0 - \frac{1}{3}b_2^0\right)\ln\left(\frac{M_{\mathrm{H}}}{m_{\mathrm{Z}}}\right) + \left(b_3^1 - \frac{1}{3}b_2^1\right)\ln\left(\frac{M_{\mathrm{R}}}{M_{\mathrm{H}}}\right) \tag{25}$$

and

$$\begin{aligned}\alpha_{EM}^{-1}(m_{\mathrm{Z}}) - \frac{23}{3}\alpha_3^{-1}(m_{\mathrm{Z}}) &= \left(\frac{23}{3}b_3^0 - b_1^0 - b_2^0\right)\ln\left(\frac{M_{\mathrm{H}}}{m_{\mathrm{Z}}}\right) \\ &\quad + \left(\frac{23}{3}b_3^1 - b_1^1 - b_2^1\right)\ln\left(\frac{M_{\mathrm{R}}}{M_{\mathrm{H}}}\right)\end{aligned} \tag{26}$$

The analysis shows again that there is not a consistent set of solutions for the last two equations (the same problem was found in Refs. 6 and 7). In order to get consistent solutions we have to slightly modify the Higgs sector, adding two more Higgs fields at the scale M_{H}, ϕ_1^0 and ϕ_2^0 in irreps $\mathrm{Z}_3\phi_i^0(\overline{15}, 1, 15), i = 1, 2$, with vevs in the same directions than ϕ_1' and ϕ_2' respectively. With this new set of Higgs fields and vevs Eqs.(25) and (26) can be solved easily, giving $M_{\mathrm{R}} \sim 10^{11}$ GeV and $M_{\mathrm{H}} \sim 10^8$GeV, which are consistent with the seesaw neutrino mass analysis presented in Ref. 12, and with the bounds on low energy FCNC.

5. Concluding remarks.

We show with our analysis that there is not much freedom for the SB channels of the unified model of flavors and forces based upon the local gauge group G. We saw that a step of the SB implemented by vevs of Higgs fields in irreps $\mathrm{Z}_3\phi(\overline{n},1,n)$, with $n = 15, 21$, constraint, in the L sector, to break $\mathrm{SU}(6)_{\mathrm{L}}$ down only to some of the subgroups $\mathrm{SU}(4)_{\mathrm{L}} \otimes \mathrm{SU}(2)_{\mathrm{L}}$, $Sp(6)_{\mathrm{L}}$, $\mathrm{SU}(4)_{\mathrm{L}} \otimes \mathrm{SU}(2)_{\mathrm{L}}$ or $[\mathrm{SU}(2)_{\mathrm{L}}]^3$.

We also calculated the general form of the vevs for Higgs fields, in such a way that the hypercharge Y of the SM does not get broken by them, and we studied the possible direction for the vevs, and their breakings induced on the different SU(6) factors of G.

The analysis enabled us to give an economical set of Higgs fields and vevs which implement a SB pattern without the problems contained in the SB proposed in Ref. 6, and in agreement with the renormalization group equation analysis and the experimental data.

An important result is the existence of at least three different mass scales, that in our case have the hierarchy

$$M_{\mathrm{R}} \sim 10^{11}\mathrm{GeV} > M \sim 10^{8}\mathrm{GeV} \gg m_z \sim 10^{2}\mathrm{GeV},$$

with the FCNC present only at the scale M, in perfect agreement with the low energy constraints.

The set of Higgs fields used (as also the set in Ref. [6]) do not break spontaneously the baryon number $\mathbf{B}$, which in the fundamental irrep if $\mathrm{SU}(6)_{\mathrm{c}}$ is of the form $\mathbf{B} \sim \mathrm{diag}(\frac{1}{3},\frac{1}{3},\frac{1}{3},0,0,0)$. So, the proton remains stable with the Higgs fields we have introduced in the present analysis, and therefore we do not expect experimental conflicts with the upper mass scale M_{R} calculated.

Even though the mass hierarchy calculated here is in agreement with the one used in the analysis of the generational seesaw mechanism, which provide small masses to the three light neutrinos [12], we mention that the quantitative predictions of the seesaw analysis may depend on the particular set of Higgs fields and vevs used to break the symmetry, specially those used for the second step of the SB because there are not right handed neutrino mass terms from $\langle\phi_3\rangle$ (they come from the Yukawa couplings between $\psi(108)$ and the scalars involved in the second step of the SB, ϕ_1' and ϕ_2'). In this way the changes in the scalar content of the model should affect the neutrino mass analysis, and it should be repeated in order to check the consistence of the previous results. Nevertheless, since the modified horizontal survival hypothesis [3, 6] is not violated by our new SB pattern (it is realized by the vevs of ϕ_4, unchanged here), and $\langle\phi_1' + \phi_2'\rangle$ produce masses of order M_{H} for all the exotic fermions in $\psi(6,1,\overline{6})$ and for all the vector-like particles with respect to the LRSE model as it should be according to the survival hypothesis [8], we expect that a new seesaw analysis gives essentially similar results.

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APPENDIX A.

In this appendix we present some aspects related to the branching rules of the SU(6) irreps in terms of those of their maximal subgroups. We are interested here in the breaking of SU(6) via the irreps 6, 15, 21, 35 and their conjugates. We consider also a general SU(6) group which could be identified with whatever factor of G.

Considering all the possible decomposition of irrep 6 of SU(6) into irreps of other groups with less dimensions (6 = 5+1,4+2 and 3+3), it is a simple matter to obtain the regular maximal subalgebras of SU(6), they are $SU(5) \otimes U(1)$, $SU(4) \otimes SU(2) \otimes U(1)$ and $SU(3) \otimes SU(3) \otimes U(1)$. Besides, SU(6) also has four special maximal subalgebras [11] which are SU(3), SU(4), $Sp(6)$ and $SU(3) \otimes SU(2)$. From them the only ones containing the subgroup $SU(2)_{L(R)}$ are $SU(3) \otimes SU(2)$, $SU(4) \otimes SU(2) \otimes U(1)$ and $Sp(6)$. The branching rules for $SU(3) \otimes SU(2)$ were given in Section 3.1; the branching rules for the last two groups are

$$
\begin{aligned}
&SU(6) \to SU(4) \otimes SU(2) \otimes U(1) \\
&\quad 6 \to (4,1)(-1) + (1,2)(2) \\
&\quad 15 \to (1,1)(4) + (6,1)(-2) + (4,2)(1) \\
&\quad 21 \to (10,1)(2) + (1,3)(4) + (4,2)(1) \\
&\quad 35 \to (1,1)(0) + (15,1)(0) + (1,3)(0) + (4,2)(-3) + (\bar{4},2)(3);
\end{aligned}
\tag{27}
$$

and

$$
\begin{aligned}
&SU(6) \to Sp(6) \\
&\quad 6 \to 6 \\
&\quad 15 \to 14 + 1 \\
&\quad 21 \to 21 \\
&\quad 35 \to 14 + 21.
\end{aligned}
\tag{28}
$$

Therefore only the vevs of the scalar field along the singlet of a 15 may break SU(6) down to $SU(4) \otimes SU(2) \otimes U(1)$ or to $Sp(6)$ (there is no way to implement the breaking to those subgroups using irrep 21, because for the first subgroup there is not a (1,1) branching, and for the second there is not a $Sp(6)$ singlet). From the main text we see that irrep 21 may be used only to break SU(6) down to $SU(3) \otimes SU(2)$.

Now, from the special embedding of $Sp(4)$ and the regular one of $SU(2) \otimes SU(2)$ in SU(4) which have the branching rules

$$\begin{aligned} \mathrm{SU}(4) &\to \mathrm{SU}(2)\otimes\mathrm{SU}(2) & \mathrm{SU}(4) &\to Sp(4) \\ 4 &\to (2,1)+(1,2) & 4 &\to 4 \\ 6 &\to (3,1)+(1,3) & 6 &\to 5+1 \\ 10 &\to (3,3)+(1,1) & 10 &\to 10 \\ 15 &\to (3,1)+(1,3)+(3,3) & 15 &\to 5+10, \end{aligned} \tag{29}$$

we see that there is a singlet in irrep 21 of SU(6) for the group $[\mathrm{SU}(2)]^3$ which contains the $\mathrm{SU}(2)_{L(R)}$ subgroup in an special embedding. Also there is a singlet of $Sp(4)$ in irrep 15 of SU(6), and then the SB of SU(6) down to $[\mathrm{SU}(2)]^3$ or $\mathrm{SU}(4)\otimes\mathrm{SU}(2)$ using a single Higgs (in one step) is always possible.

The breaking of SU(6) to any of the other maximal subalgebras necessarily break the $\mathrm{SU}(2)_\mathrm{L}$ group structure, therefore there may be paths allowed only for vevs with indices in the c and R spaces.

The branching rules for the other two regular maximal subalgebras are

$$\begin{aligned} \mathrm{SU}(6) &\to \mathrm{SU}(5)\otimes U(1) \\ 6 &\to 5(1)+1(-5) \\ 15 &\to 10(2)+5(-4) \\ 21 &\to 15(2)+5(-4)+1(-10) \\ 35 &\to 24(0)+1(0)+5(6)+\bar{5}(-6); \end{aligned} \tag{30}$$

and

$$\begin{aligned} \mathrm{SU}(6) &\to \mathrm{SU}(3)\otimes\mathrm{SU}(3)\otimes U(1) \\ 6 &\to (3,1)(1)+(1,3)(-1) \\ 15 &\to (\overline{3},1)(2)+(1,\overline{3})(-2)+(3,3)(0) \\ 21 &\to (6,1)(2)+(1,6)(-2)+(3,3)(0) \\ 35 &\to (8,1)(0)+(1,8)(0)+(1,1)(0)+(3,\overline{3})(2)+(\overline{3},3)(-2). \end{aligned} \tag{31}$$

So, the breaking down to SU(5) is possible only by the vevs in irrep 21, while there is not a lower dimension scalar field able to break SU(6) down to its regular subgroup $\mathrm{SU}(3)\otimes\mathrm{SU}(3)$.

For the other special subalgebras of SU(6) we have the branching rules

$$\begin{array}{cc} SU(6) \to SU(4) & SU(6) \to SU(3) \\ 6 \to 6 & 6 \to 6 \\ 15 \to 15 & 15 \to 15 \\ 21 \to 20 + 1 & 21 \to 15 + 6 \\ 35 \to 15 + 20 & 35 \to 8 + 27. \end{array} \tag{32}$$

Again, the breaking down to the special subgroup SU(4) may be implemented only via vevs in irrep 21, while neither 15 nor 21 could do the breaking down to the special SU(3) subgroup.

To conclude, notice that the breaking of SU(6) down to the non maximal subalgebra $SU(4) \times U(1)$ is possible by vevs along the irreps $(1, 3)$ of $SU(4) \otimes SU(2)$ in irrep 21, and also that the breaking of SU(6) down to the special maximal subgroup $SU(3) \otimes SU(2)$ is not possible via irreps 15, 21 or 35 as it can be seen from Eq. (5). [A further analysis shows that it is possible only via irrep 105 in SU(6)].

REFERENCES

1. Particle Data Group: L. Montanet *et al.*, *Phys. Rev. D* **50** (1994) 1173; R.M. Barnett *et al.*, *Phys. Rev. D* **54**, (1996) 1.
2. For a Review see G.G. Ross *Grand Unified Theories.* Frontiers in physics, **60** (Benjamin-Cumings Pub. Co., 1984); R. Mohapatra, *Unification and Supersymmetry.* second edition, (Springer-Verlag, Berlin, 1992); P. Langacker, *Phys. Rep.* **72** (1981) 185, and references therein.
3. W.A. Ponce, in *Proceedings of the Third Mexican School of Particles and Fields*, edited by J.L. Lucio and A. Zepeda, (World Scientific, Singapore, 1989), pp. 90-129; A.H. Galeana, R. Martínez, W.A. Ponce, and A. Zepeda, *Phys. Rev. D* **44** (1991) 2166.
4. J.C. Pati and A. Salam, *Phys. Rev. D* **10** (1974) 275; V. Elias and S. Rajpoot, *Phys. Rev. D* **20** (1979) 2445.
5. R.N. Mohapatra and J.C. Pati, *Phys. Rev. D* **11** (1975) 566, 2558; G. Senjanovic and R.N. Mohapatra, *Phys. Rev. D* **12** (1975) 1502.
6. W.A. Ponce and A. Zepeda, *Phys. Rev. D* **48** (1993) 240.
7. J.B. Flórez, W.A. Ponce, and A. Zepeda, *Phys Rev. D* **49** (1994) 4958; A.H. Galeana and R. Martínez, *Phys. Rev. D* **51** (1995) 3962.
8. H. Georgi, *Nucl. Phys. B* **156** (1979) 126; R. Barbieri and D.V. Nanopoulos, *Phys. Lett. B* **91** (1980) 369.
9. Abdel Pérez Lorenzana, *Masters Tesis*, CINVESTAV, 1995, unpublished.
10. F. del Aguila and L. Ibañez, *Nucl. Phys. B* **177** (1981) 60; H. Georgi and S. Dimopoulos, *Phys. Lett. B* **140** (1984) 67.
11. R. Slansky, *Phys. Rep.* **79** (1981) 1.
12. W.A. Ponce, A. Zepeda and R. Gaitán, *Phys. Rev. D* **49** (1994) 4954.