

The radiative charm decay in the minimal supersymmetric standard model

R. MARTÍNEZ AND J. ALEXIS RODRÍGUEZ

*Departamento de Física, Universidad Nacional de Colombia
Apartado aéreo 14490, Bogotá, Colombia*

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ABSTRACT. The $c \rightarrow u\gamma$ decay induced through loop processes is calculated in the framework of the minimal supersymmetric standard model (MSSM). We compare the $c \rightarrow u\gamma$ and $b \rightarrow s\gamma$ processes and we find that the MSSM can enhance the standard model branching ratio $c \rightarrow u\gamma$ by 7 to 10 orders of magnitude.

RESUMEN. El decaimiento $c \rightarrow u\gamma$ inducido a un lazo es calculado en el marco del modelo estándar supersimétrico mínimo. Comparamos los procesos $c \rightarrow u\gamma$ y $b \rightarrow s\gamma$ y encontramos que el MSSM puede ensanchar la fracción del decaimiento de $c \rightarrow u\gamma$ del modelo estándar entre 7 y 10 órdenes de magnitud.

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1. INTRODUCTION

The standard model, based on the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge group [1], is up to now the most experimentally supported theory of the strong and electroweak interactions. However, signals of new physics in future experiments could clear up the view of physics beyond the standard model. One of these alternatives is LEP II which could explore supersymmetry in its minimal version, search for SUSY-particles like fermions, charginos and neutralinos, and explore the Higgs sector. A salient feature of the standard model (SM) is that flavour changing neutral currents (FCNC) are forbidden at the tree level, although they can occur at the one loop level. These transitions are of the form $q_i \rightarrow q_j + N$, where N is a neutral state such as γ or $l\bar{l}$, and q_i is the quark with flavour i [2]. Their rather small decay rates have received much theoretical attention recently [3, 4], mainly because they serve not only as precision tests of the SM but also as a window to look for possible new physics.

2. THE DECAY $c \rightarrow u\gamma$ IN THE MSSM

In the present paper, we shall concentrate on the $c \rightarrow u\gamma$ inclusive decay which corresponds to a FCNC process at one loop. This decay would manifest itself through the $D \rightarrow \rho + \gamma$ decay. A new competitive source of D -mesons, like the proposed tau-charm

factory could be sensitive to the rare D -decays and would allow us to test other sectors of the SM [5]. We start by discussing the dominant FCNC contributions in the framework of the minimal supersymmetric standard model (MSSM), and their effects on the inclusive $c \rightarrow u\gamma$ decay rate. Then, we shall compare this rate with the similar transition $b \rightarrow s\gamma$ to obtain the branching ratio of the $c \rightarrow u\gamma$ decay in the MSSM.

Bigi, Gabbiani and Masiero [6] have calculated the branching ratio for the $c \rightarrow u\gamma$ decay in a non-minimal supersymmetric standard model. They have examined an extension of the MSSM that includes two additional Higgs doublets. The Higgs doublets are chosen so that the source of FCNC is proportional to m_t instead of m_b , thus enhancing the branching ratio for $c \rightarrow u\gamma$.

In the SM, the $c \rightarrow u\gamma$ transition is dominated by one-loop contributions with the exchange of a virtual W boson. In Ref. 5 the branching ratio for $c \rightarrow u\gamma$ decay was estimated in the context of the SM and the tiny value $BR(c \rightarrow u\gamma) \approx 10^{-15}$ was found. In the MSSM there are new sources of FCNC which are due to the supersymmetrization of the loops (there are new contributions from squarks and from the partners of the gauge bosons) [4].

The inclusive width for $c \rightarrow u + \gamma$ decay can be written as

$$\Gamma(c \rightarrow u\gamma) = \frac{m_c^5}{16\pi} |F_2^R|^2, \quad (1)$$

where F_2^R is the form factor coming from the FCNC-loop calculation and is associated with the second rank tensor of the current proportional to $\sigma_{\mu\nu} q^\mu \epsilon_\nu^\nu$. Duncan [7] pointed out that the most relevant contribution coming from SUSY models is due to the gluino, because it involves the strong coupling constant. Recently, Bertolini and Vissani [8] have pointed that chargino contribution is more important than gluino contribution in the case of $b \rightarrow s\gamma$ decay. Nevertheless, in the $c \rightarrow u\gamma$ decay, the gluino contribution is more important than the chargino contribution, because in $b \rightarrow s\gamma$ the top squark is running in the loop and its coupling with chargino-bottom is enlarged by the top quark mass. On the other hand, in $c \rightarrow u\gamma$ it is the bottom squark that is running, and m_b appears in the coupling instead of m_t . Thus, we can assume that the gluino contribution gives us the order of magnitude of the branching ratio for $c \rightarrow u\gamma$ in the MSSM. The neutralino contribution is neglected [4].

For the case of the squark mass matrix it is convenient to notice that the corresponding mass eigenstates are obtained in mixed form from the left and right sectors. We are considering the unmixed case where $\theta = 0$ [9]. After calculation of the loops is done, the gluino contribution is

$$F_2^R = 2g_s^2 ee_u C(R) \left\{ \sum_k \Gamma_{UL}^{kc} \Gamma_{UL}^{*ku} (C_{11} - C_{21} + C_{23} - C_{12}) + \frac{m_{\tilde{g}}}{m_c} \Gamma_{UR}^{kc} \Gamma_{UL}^{*ku} (C_{11} - C_0) \right\}, \quad (2)$$

where $C(R) = \frac{4}{3}$, $C_{ij}(-p, q, m_{\tilde{g}}, m_{\tilde{q}}, m_{\tilde{q}})$ are the Veltman-Passarino functions [10] and g_s is the strong coupling constant. In Eq. (2) we have adopted the notation of the first

paper in the Ref. 4. The integrals $C_i(-p, q, m_{\tilde{g}}, m_{\tilde{q}}, m_{\tilde{q}})$ are evaluated with $q^2 = 0$ for the photon on-mass-shell, and the external fermion masses approximated to zero. In this approximation, we get for the gluino contribution

$$F_2^R = -\frac{2}{3}\alpha_s\sqrt{\frac{\alpha}{\pi}}C(R)\frac{1}{\tilde{m}_u^2}\left[\Gamma_{UL}^{kc}\Gamma_{UL}^{*ku}f_1 - \frac{m_{\tilde{g}}}{m_c}\Gamma_{UR}^{kc}\Gamma_{UL}^{*ku}f_2\right], \tag{3}$$

where f_1 and f_2 are defined by

$$\begin{aligned} f_1 &= \frac{1}{12(\hat{m}_g - 1)^4} [1 - 6\hat{m}_g + 3\hat{m}_g^2 + 2\hat{m}_g^3 - 6\hat{m}_g^2 \ln(\hat{m}_g)], \\ f_2 &= 12(\hat{m}_g - 1)^3 [-1 + \hat{m}_g^2 - 2\hat{m}_g \ln(\hat{m}_g)], \end{aligned} \tag{4}$$

with $\hat{m}_g = m_{\tilde{g}}^2/m_u^2$.

The chargino contribution to the $c \rightarrow u\gamma$ process (in the notation of the first paper in the Ref. 4), is given by

$$\begin{aligned} F_R^2 &= -\frac{\alpha_w}{2}\sqrt{\frac{\alpha}{\pi}}\frac{1}{m_d^2}\left[(G_{DL}^{*jku} - H_{DR}^{*jku})(G_{DL}^{jkc} - H_{DR}^{jkc})(f_3 + e_d f_4) \right. \\ &\quad \left. - \frac{m_{\tilde{\chi}}}{m_c}(G_{DL}^{*jku} - H_{DR}^{*jku})H_{DL}^{jkc}(f_5 + e_d f_6)\right], \end{aligned} \tag{5}$$

and the functions f_i are now given by

$$\begin{aligned} f_3 &= \frac{1}{12(\hat{m}_\chi - 1)^4}(3\hat{m}_\chi + \hat{m}_\chi^3 - 6m_\chi^2 + 2 + 6\hat{m}_\chi \log(m_\chi)), \\ f_4 &= \frac{1}{12(\hat{m}_\chi - 1)^4}(3\hat{m}_\chi^2 + 2\hat{m}_\chi^3 - 6m_\chi + 1 - 6\hat{m}_\chi^2 \log(\hat{m}_\chi)), \\ f_5 &= \frac{1}{2(\hat{m}_\chi - 1)^3}(\hat{m}_\chi^2 - 4\hat{m}_\chi + 3 + 2 \log(\hat{m}_\chi)), \\ f_6 &= \frac{1}{2(\hat{m}_\chi - 1)^3}(\hat{m}_\chi^2 - 1 - 2\hat{m}_\chi \log(\hat{m}_\chi)), \end{aligned} \tag{6}$$

where $\hat{m}_\chi = m_{\tilde{\chi}}^2/m_d^2$. In the approach established by Bertolini and Vissani [8], the chargino contribution is reduced to

$$F_2^R = 2G_F\sqrt{\frac{2}{(2\pi)^3}}\frac{1}{\sin 2\beta}\left[K^{ic}K^{*iu}\frac{m_i}{\mu_R}g(\hat{m}_\chi)\right]_{SGIM}, \tag{7}$$

where the function $g(\hat{m}_\chi)$ is defined as

$$g(\hat{m}_\chi) = -\frac{1}{6(1 - \hat{m}_\chi)^3}(10\hat{m}_\chi - 12\hat{m}_\chi^2 + 2\hat{m}_\chi^3 + 2\hat{m}_\chi(3 + \hat{m}_\chi)\ln(\hat{m}_\chi)). \tag{8}$$

Finally, the neutralino contribution is given by

$$F_2^R m_c = \frac{2}{3} g^2 e \left[m_c (\sqrt{2} G_{0UL}^{*jku} + H_{0UR}^{*jku}) (G_{0UL}^{jkc} + H_{0UR}^{jkc}) (C_{11} - C_{21} + C_{23} - C_{12}) \right. \\ \left. + m_{\tilde{\chi}^0} (\sqrt{2} G_{0UL}^{*jku} + H_{0UR}^{*jku}) (\sqrt{2} G_{0UR}^{jkc} - H_{0UL}^{jkc}) (C_{11} - C_0) \right]. \quad (9)$$

The above functions are similar to the functions presented in appendix B of the first paper in Ref. 4. This is due to the similarity between the $b \rightarrow s\gamma$ and $c \rightarrow u\gamma$ processes; the only difference is in their couplings, which are proportional to the CKM matrix parameters and the quarks involved in the loops. The branching ratio for the process $b \rightarrow s\gamma$ is [4]

$$BR(b \rightarrow s\gamma) = \frac{\Gamma(b \rightarrow s\gamma) BR(b \rightarrow ce\nu)}{\Gamma(b \rightarrow ce\nu)}, \quad (10)$$

where

$$\Gamma(b \rightarrow ce\nu) = \frac{G_F^2 m_b^5}{192\pi^3} \rho(m_c^2/m_b^2) |K_{bc}|^2, \quad (11)$$

and the phase-space factor $\rho(m_c^2/m_b^2) \approx 0.447$, K_{bc} is the CKM matrix entry, and $BR(b \rightarrow ce\nu) \approx 0.11$ [11]. Similarly, the $c \rightarrow u\gamma$ decay can be written as

$$BR(c \rightarrow u\gamma) = \frac{\Gamma(c \rightarrow u\gamma) BR(D^0 \rightarrow K^- e^+ \nu)}{\Gamma(c \rightarrow se\nu)}, \quad (12)$$

where the branching ratio of the process $D^0 \rightarrow K^- e^+ \nu \approx 0.033$ is proportional to the branching ratio associated to $c \rightarrow se\nu$ [11]. The width of the semileptonic decay $c \rightarrow se\nu$ is given by

$$\Gamma(c \rightarrow se\nu) = \frac{G_F^2 m_c^5}{192\pi^3} \rho(m_s^2/m_c^2) |K_{cs}|^2, \quad (13)$$

with $\rho(m_s^2/m_c^2) \approx 0.48$ and $K_{cs} \approx 1$.

We can compare the branching ratios of $b \rightarrow s\gamma$ and $c \rightarrow u\gamma$ because they are analogous processes, and we can use the experimental value obtained by CLEO [12] for $b \rightarrow s\gamma$ to estimate the order of magnitude of $c \rightarrow u\gamma$ in the MSSM. We can see that the f_1 function has the same form as the function defined in Eq. (B.2) of the first paper in Ref. 4, and we observe that this function depends on the free parameters of the MSSM, like the squark and gluino masses. Therefore, the ratio between $BR(c \rightarrow u\gamma)$ and $BR(b \rightarrow s\gamma)$ can be written as

$$\frac{BR(c \rightarrow u\gamma)}{BR(b \rightarrow s\gamma)} = \frac{4BR(D \rightarrow Ke\nu) \rho(m_c^2/m_b^2) \alpha_s(m_c^2)}{BR(b \rightarrow ce\nu) \rho(m_s^2/m_c^2) \alpha_s(m_b^2)} \times \\ \left| \frac{K_{bc}}{K_{cs}} \right|^2 \left| \frac{m_d^2 K_{cs} K_{su} f_1(m_g^2/m_u^2)}{m_u^2 K_{tb} K_{st} f_1(m_g^2/m_d^2)} \right|^2 \quad (14)$$

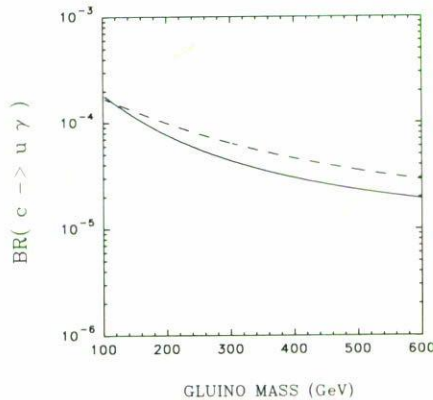


FIGURE 1. The branching ratio $BR(c \rightarrow u\gamma)$ from Eq. (14) versus the gluino mass for two values of $(m_{\tilde{b}}, m_{\tilde{c}})$: (300, 100) GeV (solid line), (500, 200) GeV (dashed).

Some remarks about this ratio are in order: first, we have used Eq. (1) for the width $\Gamma(c \rightarrow u\gamma)$ and Eq. (43) of the first paper in Ref. 4 for the $b \rightarrow s\gamma$ decay; second, we can drop the functions $f_1(m_{\tilde{g}}^2/m_{\tilde{u}}^2)$ and $f_1(m_{\tilde{g}}^2/m_{\tilde{d}}^2)$ when we calculate their ratio, because they depend on free parameters like $m_{\tilde{u}}$ and $m_{\tilde{d}}$ which can be of the same order of magnitude. This approximate degeneracy is predicted by supergravity models [13] and is required by $K^0 - \bar{K}^0$ phenomenology [14]. This requirement is not very stringent for the third-generation of squarks because there is a small mixing. Implications of this fact have been discussed in the literature [15]. Finally, we observe that in the $b \rightarrow s\gamma$ decay we focus our attention upon the bottom squark, whereas in the $c \rightarrow u\gamma$ decay we focus on the charm squark.

If we take the experimental value for the branching ratio of the decay $b \rightarrow s\gamma \approx 10^{-4}$ [12] and $f_1(m_{\tilde{g}}^2/m_{\tilde{u}}^2)/f_1(m_{\tilde{g}}^2/m_{\tilde{d}}^2) \approx 1$, we obtain the value

$$BR(c \rightarrow u\gamma) \approx 6.74 \times 10^{-6} . \quad (15)$$

In Fig. 1 we display the branching ratio $c \rightarrow u\gamma$ from Eq. (14) versus the gluino mass with $\alpha_s(m_c^2) = 0.33$, $\alpha_s(m_b^2) = 0.27$ for two different values of $(m_{\tilde{b}}, m_{\tilde{c}})$: (300, 100) GeV and (500, 200) GeV.

In Fig. 2 we display the gluino contribution given by Eq. (12) and the results obtained from the loop calculations, [Eq. (3)]. We have plotted the branching ratio $c \rightarrow u\gamma$ versus the squark mass and we have considered different values (100, 200 and 400 GeV) for the gluino mass. We note that in this range of parameters the branching ratio varies from 10^{-5} to 10^{-7} , which are the same orders of magnitude obtained in Eq. (15). In Fig. 3, we display the chargino contribution given by Eq. (7), where we have used $\tan\beta = 20$ and chargino masses of 100, 200 and 400 GeV. The orders of magnitude of this contribution vary from 10^{-8} to 10^{-11} . Also, we have considered the neutralino contribution in Fig. 4, where we have used the input parameters $M = M' = 50$, $\mu = 30$, $\tan\beta = 1$ and $\sin^2\theta_w = 0.23$, $m_Z = 91.187$ GeV from Ref. 11. We note that neutralino contribution could be neglected in the framework of the MSSM.

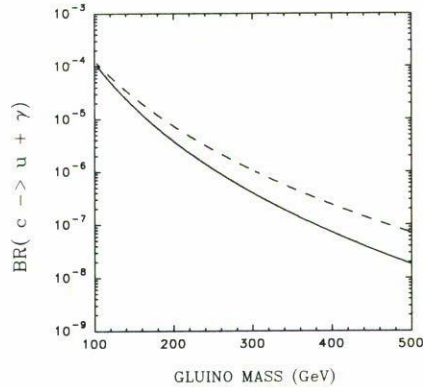


FIGURE 2. The gluino contribution to the branching ratio $BR(c \rightarrow u\gamma)$ as a function of gluino mass for three values of squark mass, 100 GeV (solid line), 200 GeV (dashed) and 400 GeV (dot-dashed).

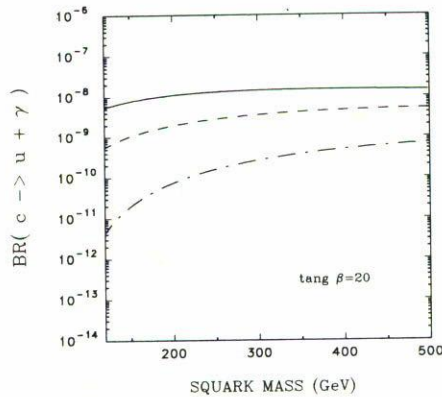


FIGURE 3. The chargino contribution to the branching ratio $BR(c \rightarrow u\gamma)$ as a function of squark mass for three values of chargino mass, 100 GeV (solid line), 200 GeV (dashed) and 400 GeV (dot-dashed).

We conclude that the branching ratio for $c \rightarrow u\gamma$ calculated in the MSSM gets enhanced by 7 to 10 orders of magnitude with respect to the SM calculations. We want to point out that the $q_i \rightarrow q_j\gamma$ decays were worked out under the assumption that these decays are dominated by short distance penguin diagrams; this assumption is true for B physics but is not conclusive for the decay $c \rightarrow u\gamma$, where the long distance effects are expected to dominate. We have estimated only the short distance contributions for $c \rightarrow u\gamma$ which as we mentioned before are of the order of 10^{-6} in the MSSM. The long distance contributions were calculated in Ref. 16, where they have obtained a value larger than ours. However, we want to stress the similarity between $c \rightarrow u\gamma$ and $b \rightarrow s\gamma$ in the MSSM. Obviously, we have to take into account that up until now the parameter space for the squark mass $\tilde{u}_i - \tilde{d}_j$ and its hierarchy, has not been covered completely,

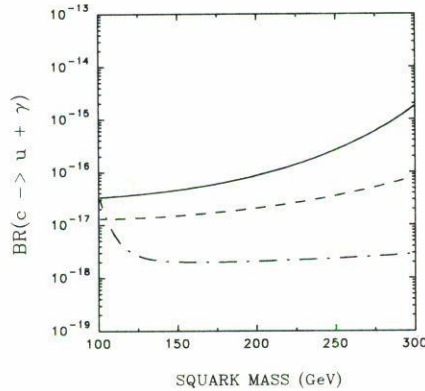


FIGURE 4. The neutralino contribution to the branching ratio $BR(c \rightarrow u\gamma)$ as a function of squark mass for three values of neutralino mass 50 GeV (solid line), 40 GeV (dashed) and 30 GeV (dot dashed).

even though $D \rightarrow X + \gamma$ decays could be a good test for QCD corrections.

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