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Application of the extended first-order chromatic theory to the correction of secondary spectrum.

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ABSTRACT. The extended first-order chromatic theory developed by C.G. Wynne [1] is applied to imaging optical systems to correct their secondary spectrum using only glasses having normal relative partial dispersions. An algorithm based on this theory, which evaluates the secondary spectrum contribution at each surface, is applied to correct this residual chromatic aberration.

RESUMEN. La teoría cromática extendida de primer orden, desarrollada por C.G. Wynne [1], es aplicada a sistemas ópticos formadores de imágenes con el fin de corregir el espectro secundario usando únicamente vidrios con dispersiones parciales relativas normales. Un algoritmo basado en esta teoría que evalúa la contribución de cada superficie al espectro secundario, se aplica para corregir esta aberración cromática residual.

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1. INTRODUCCION

The refractive index of any medium other than vacuum depends on the wavelength of light. This property is called dispersion and produces the effect that the focal position and the magnification in refracting imaging systems are functions of the wavelength. The variation of focal length of an optical system with wavelength is called longitudinal chromatic aberration, whereas the change of magnification is called transverse chromatic aberration.

By combining two lenses made of glasses having different dispersions it is possible to correct the longitudinal chromatic aberration for two wavelengths, that is, bring to a common focus light with wavelengths C and F, say; such a system is called an achromat. There is, however, a residual chromatic aberration known as secondary spectrum, which can be measured as the difference between the common focal position for the two wavelengths C and F, and the focal position for a third wavelength d, where $F < d < C$. The secondary spectrum is often a limiting aberration in refracting imaging systems [2].

If one plots values of reciprocal dispersive power, V, *versus* the relative partial dispersion of the glass, P, *i.e.*, V *vs.* P, a near straight line is obtained for most of the optical glasses. One refers to those glasses as glasses having normal relative partial dispersion.

Assuming that the difference in the incident height of the paraxial rays, in each surface of the system, corresponding to different colours is small, Conrady [3] showed that the only way to reduce the secondary spectrum is by using materials which depart from this "straight line", that is, materials having abnormal relative partial dispersion. In achromats, the order of magnitude of the secondary spectrum for any pair of glasses having normal relative partial dispersion is approximately given by $f/2000$, where f is the focal distance of the achromat and the 2000 is a consequence of the glass chemistry [4].

The reduction of secondary spectrum has become the subject of an extensive literature [5–15]. Most of the papers [5–13], however, are concerned with the problem of selecting glasses which depart from the "straight line", that is, materials having anormal relative partial dispersion, such as fluorite, in order to reduce the secondary spectrum. Unfortunately, there are only a few of these special materials that effectively reduce the secondary spectrum, In addition, they are expensive, unobtainable in large pieces and difficult to work to a good polish [16]. For these reasons they are mostly used in microscope objectives, since the material costs in lenses of small diameters become insignificant.

To the author's knowledge, until 1955 there was no evidence that secondary spectrum could be corrected with glasses having normal relative partial dispersion. This shows that the statement made by Conrady [3], in the sense that the correction of secondary spectrum required the use of abnormal glasses, was generally accepted.

In 1955, however, E.L. McCarthy [14] discovered and published in a patent an imaging optical system which was substantially freed of secondary spectrum by using only normal glasses. Later, in 1977, C.G. Wynne [15] argued that the classical theory of first-order chromatic aberrations, *i.e.*, the primary chromatic aberration formulas, had led to the erroneous conclusion that the correction of the secondary spectrum required the use of special glasses, since this theory neglects terms of paraxial order in the variations of the aperture with wavelength and also powers higher than the first in glass dispersions. In 1978, Wynne extended the theory of first-order chromatic aberrations to include those previously neglected terms [1]. From now on we will refer to the formulas developed in this extended theory of first-order chromatic aberration as Wynne formulas.

Unlike the primary chromatic aberration coefficients Wynne coefficients allow the correction of secondary spectrum using only normal glasses. Most of the optical design programs which are commercially available at the present, however, evaluate the primary chromatic aberration coefficients for the longitudinal and the transverse chromatic aberrations, but not the Wynne coefficients. An alternative approach is adopted in the optical design program Eikonal [17], which traces paraxial rays in three wavelengths and then computes the paraxial contribution to chromatic aberration for the actual image, making thus possible to correct the secondary spectrum using normal glasses. A disadvantage of this method is that no individual surface contribution for the chromatic aberration is computed, whereas the advantage of using Wynne coefficients is that every surface contribution to the secondary spectrum may be evaluated.

In other words, until now, Wynne coefficients have not been used in the process of correcting the secondary spectrum of a refracting imaging system. In a previous paper [18] it was shown that the approaches of McCarthy and Wynne are in fact equivalent, and an

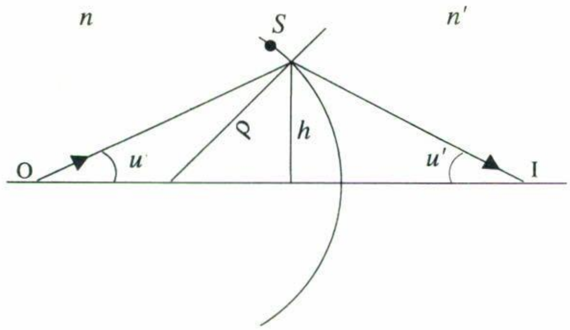

FIGURE 1. Parameters for the evaluation of the Wynne coefficients.

algorithm to evaluate Wynne coefficients for the longitudinal chromatic aberration and the secondary spectrum was presented. The optical systems suggested by McCarthy and Wynne work for an object localized at infinity, but no examples were given by them of systems working with finite conjugates. In this paper the theory developed by Wynne is applied to the correction of secondary spectrum in imaging systems using normal glasses, and an example of an optical system working with an object localized at a finite distance is given.

2. THEORY

In monocromatic aberrations, the first terms in the wavefront aberration expansion depend on the fourth power of the aperture and field and are called primary aberrations. The succeeding terms depend on the sixth, eighth, etc., power and are called secondary, tertiary, etc., aberrations, respectively. The primary and secondary chromatic aberrations that we are going to discuss in this section must not be understood in this sense since these aberrations are evaluated only in the paraxial region. The primary chromatic aberration refers to the longitudinal chromatic aberration between wavelengths C and F whereas the secondary chromatic aberration refers to the secondary spectrum.

2.1. WYNNE COEFFICIENTS: EVALUATION OF PARAXIAL PRIMARY LONGITUDINAL CHROMATIC ABERRATION INCLUDING ALL ORDERS IN THE GLASS DISPERSION

Consider a refracting surface S, of paraxial radius of curvature ρ, bounded by media with refractive indices n and n', for some mean wavelength, d. Let u and u' be the angles of the paraxial marginal ray coming from O, the object point, measured with respect to the optical axis before and after refraction respectively (Fig. 1).

According to C.G. Wynne's theory [1], the contribution of a refracting surface to the paraxial primary longitudinal chromatic aberration, $Lchr$, of a gaussian image of an axial object point, including all orders in the glass dispersion for a single surface is given by

$$Lchr = A\,h\,\Delta\left(\frac{\delta n}{n}\right) + h\,k\,\delta h + h\,\Delta(\delta n\,\delta u), \tag{1}$$

where Δ stands for the difference on refraction of the quantity between the parenthesis, h is the incidence height of the paraxial marginal ray coming from O, A is the refraction invariant given by

$$A = n\left(\frac{h}{\rho} + u\right), \tag{2}$$

and k is the chromatic power given by

$$k = \frac{\Delta(\delta n)}{\rho}. \tag{3}$$

The surface contribution to the paraxial primary longitudinal chromatic aberration for the short wavelength F and the long wavelength C would be then

$$\begin{aligned} Lchr_{(C-F)} &= n_F\left(\frac{h_F}{\rho} + u_F\right) h_F \left(\frac{n'_C - n'_F}{n'_F} - \frac{n_C - n_F}{n_F}\right) \\ &+ h_F\left[\frac{(n'_C - n'_F) - (n_C - n_F)}{\rho}\right](h_C - h_F) \\ &+ h_F\left\{\left[(n'_C - n'_F) - (n_C - n_F)\right] \times \left[(u'_C - u'_F) - (u_C - u_F)\right]\right\}. \end{aligned} \tag{4}$$

Equation (4) gives the individual surface contribution to paraxial longitudinal chromatic aberration as a wavefront aberration including all orders in the glass dispersion. For an optical system, $Lchr$ is given by the sum of the individual surface contributions.

2.2. WYNNE COEFFICIENTS: EVALUATION OF PARAXIAL SECONDARY LONGITUDINAL CHROMATIC ABERRATION, *i.e.*, SECONDARY SPECTRUM, INCLUDING ALL ORDERS IN THE GLASS DISPERSION

If an optical system is corrected for primary longitudinal chromatic aberration, that is, if $Lchr = 0$, then the focus for the long and short wavelengths will be the same. A measure of the secondary spectrum can then be given by the difference between the focal positions for the short and mean wavelengths or between the focal positions for the long and mean wavelengths. We shall arbitrarily choose the last difference as a measure of the secondary spectrum, $SSpec$.

The individual surface contribution to the secondary spectrum, given as a wavefront aberration, is given by Eq. (4) substituting F by d, that is

$$\begin{aligned} SSpec_{(C-d)} &= n_d\left(\frac{h_d}{\rho} + u_d\right) h_d \left(\frac{n'_C - n'_d}{n'_d} - \frac{n_C - n_d}{n_d}\right) \\ &+ h_d\left[\frac{(n'_C - n'_d) - (n_C - n_d)}{\rho}\right](h_C - h_d) \\ &+ h_d\left\{\left[(n'_C - n'_d) - (n_C - n_d)\right] \times \left[(u'_C - u'_d) - (u_C - u_d)\right]\right\}, \end{aligned} \tag{5}$$

where the subscripts C and d refer to the long and the mean wavelength. The first term in Eq. (5) gives the secondary spectrum according to the classical, first-order chromatic aberration theory. Although the other two terms in this equation do not appear explicitly elevated to the second and higher order terms in the glass dispersion, δn, they are of higher order than the first in the glass dispersion since δh and δu are functions of the glass dispersion. For a group of thin lenses in contact these last two terms are equal to zero.

3. ALGORITHM

For completeness a brief description of the algorithm presented in Ref. 18 will be included here. Let $h_F(i)$, $h_d(i)$, $h_C(i)$ be the heights of the paraxial marginal rays evaluated at the i-th surface for the short, mean and long wavelengths, respectively. And let $u_F(i)$, $u_d(i)$, $u_C(i)$ be the convergence angles of the paraxial marginal rays evaluated at the i-th surface for the same wavelengths. By tracing paraxial marginal rays with these wavelengths (F, d and C), the heights and the angles at each surface and for each wavelength can be evaluated. The individual surface contribution to the paraxial primary longitudinal chromatic aberration, *Lchr*, is evaluated from Eq. (4); and the contribution to secondary spectrum, *SSpec*, is evaluated from Eq. (5). The total value of *Lchr* and *SSpec*, including all orders in the glass dispersion can be obtained by adding the individual surface contributions.

4. EXAMPLES

To correct secondary spectrum using normal glasses it is necessary to have an optical system composed of both chromatically overcorrected and chromatically undercorrected optical components. Taking the radius of curvature of the surface as positive if its centre of curvature is to the right of the surface and the ray slopes as positive if the ray must be moved clockwise to reach the optical axis, the lens is said to be undercorrected if the value of *Lchr* given by Eq. (4) is negative. Otherwise the lens is said to be overcorrected. Two examples are presented in this section where the Wynne's theory described in Sect. 2 has been applied to correct the secondary spectrum in two different optical systems. The first example is an aspheric achromatic doublet working for an object at infinity. Its performance on axis is limited by secondary spectrum which will be reduced by means of a corrector. The second example is a system composed of two lenses; one overcorrected and the other undercorrected. To correct the secondary spectrum a field lens is introduced in this system to control the height of the rays at the second lens.

4.1. OPTICAL SYSTEM WORKING WITH AN OBJECT LOCALIZED AT INFINITY

This example consists of an achromatic doublet with two aspheric surfaces, Fig. 2a, that allow for the correction of spherochromatism, *i.e.*, variation of spherical aberration with

TABLE I. Data of an $f/5$, aspherized achromatic cemented doublet corrected for spherochromatism and zonal spherical aberration. Effective focal length 48.37 mm.

Radius (mm)	Axial separation (mm)	Material	Clear diameter (mm)
28.255		Air	10.160
−18.992*	4.5	BK7	10.160
−67.549**	2.4	EDF648338	10.160
Image plane	44.911	Air	10.160

*Axial radius of curvature. Asphericity $z = 4.455 \times 10^{-5}\rho^4 + 6.415 \times 10^{-8}\rho^6$.
**Axial radius of curvature. Asphericity $z = 9.152 \times 10^{-6}\rho^4 + 8.452 \times 10^{-9}\rho^6$.

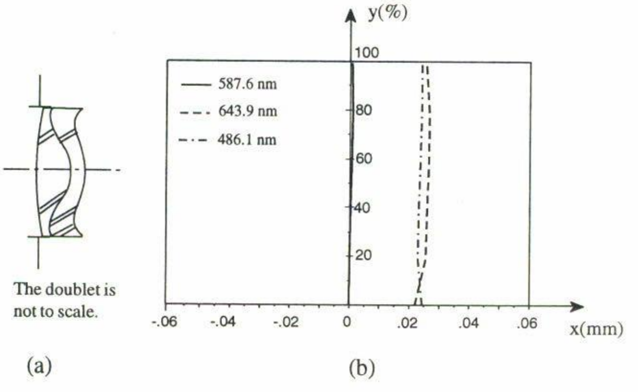

FIGURE 2. (a) Aspheric achromatic doublet, with the parameters of Table I. (b) Longitudinal ray aberrations: 587.6 nm solid line, 643.9 nm dashed line and 486.1 nm dashed line with dots.

wavelength, and of zonal spherical aberration [19], so that the only remaining aberration on axis is secondary spectrum. The doublet is meant to work with an axial object point localized at infinity. Table I shows the data for this doublet which is designed to operate at zero magnification, with a focal ratio of $f/5$ and a focal length of 48 mm. In Fig. 2b the longitudinal ray aberrations, which are given by the distance between point of intersection of the ray with the optical axis and the paraxial focus, are plotted against the height of the ray at the entrance pupil. The y-axis gives the entrance pupil coordinate, the x-axis the optical axis coordinate with the origin at the paraxial image point. Longitudinal aberrations are plotted for three wavelengths: 587.6 nm, 656.3 nm and 486.1 nm (the d, C and F lines of the spectrum, respectively). From Fig. 2b we can see that the axial performance of the system is only limited by the longitudinal secondary spectrum.

Our goal in this example is to design an optical system that in conjunction with the doublet, reduces the secondary spectrum between the lines of 587.6 nm and 656.3

TABLE II. Data of an optical system consisting of corrector + doublet (in Table I), with reduced secondary spectrum by using normal glasses only. Effective focal length 48.37 mm.

Radius (mm)	Axial separation (mm)	Material	Clear diameter (mm)
2620.9078		Air	10.112
17.5620	3.073291	LAK9	10.112
5583.4729	6.146582	SF8	10.112
−6574.6220	66.560926	Air	10.002
17.3008	3.073291	SF8	10.002
−17.3008	6.146000	LAK9	10.002
3687.3156	3.073300	SF8	10.002
−32679.7386	66.560906	Air	10.148
−17.5620	6.146600	SF8	10.148
−3985.6517	3.073300	LAK9	10.148
28.2550	0.000000	Air	10.160
−18.992*	4.500000	BK7	10.160
−67.549**	2.400000	EDF648338	10.160
Image plane	44.918541	Air	1.690

*Axial radius of curvature. Asphericity $z = 4.455 \times 10^{-5}\rho^4 + 6.415 \times 10^{-8}\rho^6$.
**Axial radius of curvature. Asphericity $z = 9.152 \times 10^{-6}\rho^4 + 8.452 \times 10^{-9}\rho^6$.

nm, without modifying any of the parameters of the doublet, that is, its curvatures, asphericities, thicknesses or refractive indices. The additional optical system, which may be called a "corrector", should be designed with glasses having only normal relative partial dispersion.

The simplest configuration of an optical system corrected of secondary spectrum by using only normal glasses is the configuration patented by McCarthy [14]. This configuration consists of two separated doublets; one doublet has zero power at the mean wavelength and is overcorrected, the other gives the power to the system and is undercorrected. In our example the doublet is achromatic so that a single group of lenses in contact in front of this doublet will not correct the secondary spectrum.

On the other hand, a corrector with no power and composed of two separated groups of lenses, so that one group is overcorrected and the other is undercorrected, could reduce the secondary spectrum of the doublet. A problem with this type of corrector, however, is that a large separation between the groups is needed in order to keep the level of axial performance that has the doublet alone. Otherwise, there is an increment in the higher-orders of spherical aberration. In addition, the large separation between the components of the corrector introduces a large amount of transverse chromatic aberration for off-axis object points.

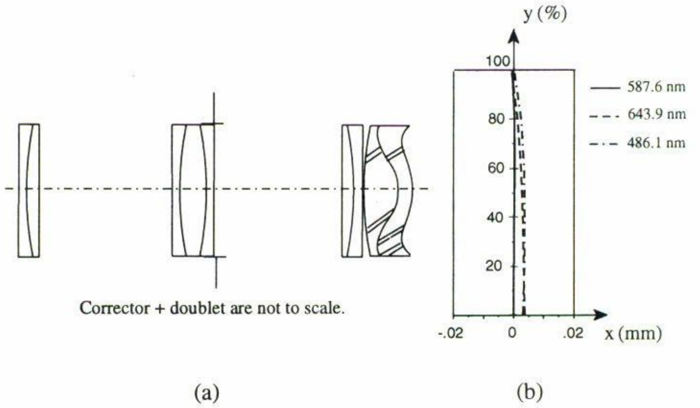

FIGURE 3. (a) Optical system consisting of corrector + aspheric achromatic doublet, with the parameters of Table II. (b) Longitudinal ray aberrations: 587.6 nm solid line, 643.9 nm dashed line and 486.1 nm dashed line with dots.

To overcome these problems, a corrector with no power and composed of three separated groups of lenses was designed, Fig. 3a. This corrector is of the type designed by Wynne [15]. The data for the entire optical system, corrector + aspheric achromatic doublet, are given in Table II. The configuration of the corrector was chosen almost symmetric, in order to reduce the transverse chromatic aberration for off-axis objects points, although the analysis of this aberration is not the subject of this paper. In Fig. 3b the longitudinal ray aberrations are plotted for the three wavelengths: 587.6 nm, 656.3 nm and 486.1 nm. By comparing Fig. 3b with Fig. 2b we can see how the corrector reduces the secondary spectrum in the range covering these wavelengths; however, the secondary spectrum was not completely corrected since this would have increased the amount of higher order of spherical aberration.

4.2. OPTICAL SYSTEM WORKING WITH FINITE CONJUGATES

An optical system composed of a positive undercorrected lens and a positive overcorrected lens has been designed using only normal glasses, Fig. 4. The long separation between the components, however, has the effect of increasing the difference in incident height among the rays of different colours. One form of reducing or controlling this effect is to introduce a field lens between the components.

To reduce this effect, the field lens should be placed at the image conjugate of the overcorrected component. That is, the power of this lens must be such that the lens images the undercorrected component onto the overcorrected component. The differences in incident heights of the rays of different colours at the second component would then be reduced, making negligible the last two terms in Eq. (5).

In our example, however, the field lens is not imaging the undercorrected component onto the overcorrected component, instead the lens is controlling the difference of the

TABLE III. Data of an optical system working with an object point at 150 mm from the first lens. Effective focal length 32.3 mm.

Radius (mm)	Axial separation (mm)	Material	Clear diameter (mm)
Plano	150.0	Air	65
100.0000	0.0	F8	65
−65.6314	0.0	PSK3	65
44.2377	750.0001	Air	5
Plano	0.0	BK7	5
200.7808	508.4901	Air	26
−56.2744	0.0	PSK3	26
56.2744	0.0	F8	26
−200.7808	0.0	PSK3	26
Image plane	508.4901	Air	0

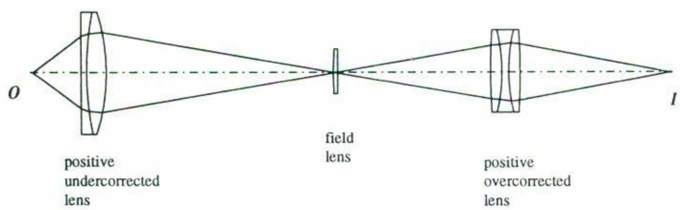

FIGURE 4. Optical system working with an object point at a finite distance. Its parameters are given in Table III. Note that the drawing is not to scale.

heights of the rays at the second component. In this case, the correction of secondary spectrum is only possible when the last two terms of Eq. (5) are included in its evaluation.

The design is for thin lenses, with the parameters given in Table III. The object is localized at 150 mm to the left of the first lens. The optical system has an effective focal length of 32.3 mm and a lateral magnification of 5. The clear diameters are evaluated for a full entrance angular field of 10 degrees. A graph of longitudinal ray aberrations would not clearly show the correction of secondary spectrum since this is a thin lens design and aberrations for non-paraxial apertures and fields are too large. Instead a graph of paraxial focal position versus wavelength is plotted in Fig. 5 to show that this system is corrected for three wavelengths, namely 587.6 nm, 656.3 nm and 486.1 nm.

It is important to point out that a long configuration is required to reduce the secondary spectrum, if only normal glasses are allowed. Therefore, the technique for correcting the secondary spectrum described here would be of practical importance in systems like periscopes, telescopes and flat field anastigmatic objectives, which necessarily require widely separated components.

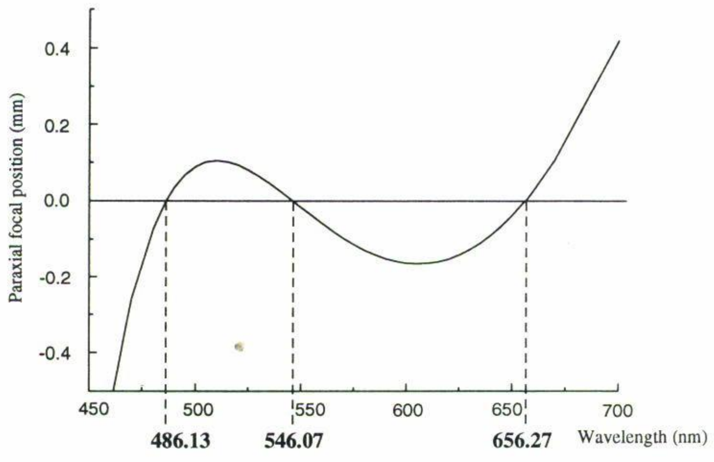

FIGURE 5. Paraxial focal positions plotted against wavelength for the optical system with the parameters of Table III. This configuration is corrected of secondary spectrum by using normal glasses only.

5. CONCLUSIONS

The examples given here show that two facts were important to achieve the correction of secondary spectrum exclusively with normal glasses. The first is the presence of new terms in the expression for the paraxial longitudinal chromatic aberration, these terms include higher order in the glass dispersion which are not considered in the classical first-order chromatic theory. The second is that of producing an appreciable difference of paraxial incident heights of the rays for different colours so that these new terms are significant. This requires that the system has a long configuration and also to be composed of overcorrected and undercorrected components.

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