Revista Mexicana de Física **43**, *No. 6 (1997) 906–915*

The local spectral expansion method for the scattering of light from two-dimensional perfectly conducting surfaces.

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Recibido el 22 de octubre de 1996; aceptado el 13 de enero de 1997

Abstract. The local spectral expansion method for electromagnetic scattering from perfectly conducting surfaces rough in one dimension was recently formulated and it was shown in Ref. 13 that it is a more efficient formulation than other available analytical approaches. In this paper the formulation of the local spectral expansion method is extended for the scattering from perfectly conducting surfaces rough in two dimensions. The full vector nature of the problem is accounted for. The original formulation for one dimensional rough surfaces is recovered from the present formulation in the appropriate limits.

Resumen. Recientemente se formuló el método de "expansión espectral local" para la difracción de ondas electromagnéticas por superficies conductoras ideales y rugosas en una dimensión y se mostró, en la Ref. 13, que dicho método es más eficiente que otros métodos actualmente en uso En este artículo se extiende la formulación original para la difracción de ondas electromegnéticas por superficies conductoras ideales pero rugosas en dos direcciones. Se toma encuenta en forma rigurosa la naturaleza vectorial de los campos. La formulación original para superficies rugosas en una dimensión se recupera en los límites correspondientes.

PACS: 42.25.B; 42.25.F; 78.66

1. Introducción

There are two classic approaches to the study of wave scattering from rough surfaces. These are the Rayleigh method or the small height perturbation method (SPM) to first order [1, 2] and the physical optics or Kirchhoff approximation (KA) [3]. For a long time it has been recognized that these two approaches are not entirely compatible with each other and there has been a continued interest to develop and study new analytical approximations to the solution of the problem. Unfortunately, there is still not a simple rule to determine the regions of applicability of an a pproximation to the scattering of waves from an arbitrary surface. Nevertheless, the regions of validity for random Gaussian rough surfaces in one dimension have been studied recently in a quantitative manner [4–9], permitting us to get an approximate idea of the regions of applicability of the different theories.

The main interest so far has been to formulate a theory capable of bridging the gap between perturbation theory and the Kirchhoff approximation. Several analytical theories have been put forward by different authors in recent years [10–12], all of them with some degree of success. In a recent paper we formulated the local spectral expansion method (LSEM) for electromagnetic scattering from perfectly conducting surfaces rough in one dimension [13]. A comparison with other available analytical approaches showed that the results based on the LSEM provide more efficient approximations than the other theories. A systematic series solution was formulated using the LSEM and the first and second order approximations were analyzed to some extent.

The solution of the scattering problem for one-dimensional (1-D) rough surfaces is important in that it gives the understanding of the scattering processes, of the physical important aspects of the problem, and of the approximations that are important. However, most practical problems deal with two-dimensional (2-D) rough surfaces. In this paper we extend the formulation of the LSEM to deal with 2-D perfectly conducting surfaces. In this case we have to take into account the full vector nature of the problem. The case of 1-D rough surfaces is recovered from the present formulation when the surface is made constant along one direction and when the angle of incidence is contained within the plane perpendicular to the surface undulations.

The extension to 2-D rough surfaces is not obvious and needs some consideration. Here we only derive the first order results using the tangent plane approximation in the present formulation. Higher order results analogous to those for the 1-D case may be derived using the present formulation. The present results bridge the gap between first order perturbation theory and the Kirchhoff approximation. The extension of the LSEM for the scattering from dielectric-dielectric rough interfaces is also of importance and has been published elsewhere [14].

2. THE SCATTERING EQUATIONS

The rationale of the method for 2-D surfaces is analogous to that for 1-D surfaces, the main difference being that we now deal with vector equations and their transforms, and these must be used carefully.

The surface profile function is now described by the equation $y = h(x, z)$ and the normal to the surface is given by

$$\hat{\mathbf{n}} = \hat{\mathbf{N}}/\sqrt{1 + h_x^2 + h_z^2} \quad \text{where} \quad \hat{\mathbf{N}} = -\hat{\mathbf{a}}_x h_x + \hat{\mathbf{a}}_y h_y - \hat{\mathbf{a}}_z h_z. \tag{1}$$

We use the same set of local basis functions ψ_o and ψ_e [13] to expand the fields. We follow closely the formulation of the regular full wave theory by Collin [15] to set up the scattering equations and find their solution. The approximation is, however, different and follow closely the concepts introduced in Ref. 13.

For a flat surface we have $E_x = E_z = H_y = 0$ and we will expand these field components in terms of the odd basis functions ψ_o. The other components E_y, H_x, and H_z are expanded using the even set of basis functions ψ_e. Hence we define the vector transforms

$$\tilde{\mathbf{e}} = \hat{\mathbf{a}}_x \tilde{e}_{x\mathrm{o}} + \hat{\mathbf{a}}_y \tilde{e}_{y\mathrm{e}} + \hat{\mathbf{a}}_z \tilde{e}_{z\mathrm{o}} \qquad \text{and} \qquad \tilde{\mathbf{h}} = \hat{\mathbf{a}}_x \tilde{h}_{x\mathrm{e}} + \hat{\mathbf{a}}_y \tilde{h}_{y\mathrm{o}} + \hat{\mathbf{a}}_z \tilde{h}_{z\mathrm{e}}, \tag{2}$$

where the second subscript refers to the ψ_e- or ψ_0-transform. Maxwell's equation for a sourceless $e^{j\omega t}$-field are

$$\nabla\times\mathbf{E}=-j\omega\mu_0\mathbf{H}\quad\text{and}\nabla\times\mathbf{H}=j\omega\varepsilon_0\mathbf{E}. \tag{3}$$

If we operate on both sides of each cartesian-components of these vector equations with either

$$\int_h^\infty dy\,\psi_e^*(k_y)\qquad\text{or}\qquad\int_h^\infty dy\,\psi_o^*(k_y), \tag{4}$$

accordingly with the vector transforms defined in (2), we obtain six coupled linear inhomogeneous differential equations. For example, when considering the z-component of the first of Maxwell's equation in (3) we note that $\partial\psi_e^*/\partial y=jk_y\psi_o^*$ and apply the first operator in (4) to both sides of the equation. Applying the appropriate operator to each component of Maxwell's equation (3) and integrating by parts as indicated in Ref. 15 yields

$$\frac{\partial\tilde{e}_{ye}}{\partial x}+jk_y\tilde{e}_{xo}=-j\omega\mu_0\tilde{h}_{ze}-\left[(E_x+h_xE_y)\,\psi_e^*\right]\Big|_h+\int_h^\infty E_y\frac{\partial\psi_e^*}{\partial x}\,dy.$$

We may define

$$\nabla\equiv\hat{\mathbf{a}}_x\frac{\partial}{\partial x}-jk_y\hat{\mathbf{a}}_y+\hat{\mathbf{a}}_z\frac{\partial}{\partial z} \tag{5}$$

and write the transformed Maxwell's equations as

$$\nabla\times\tilde{\mathbf{e}}=-j\omega\mu_0\tilde{\mathbf{h}}+(2,0,2)\cdot(\hat{\mathbf{N}}\times\mathbf{E})|_h+\mathbf{I}_m, \tag{6a}$$

$$\nabla\times\tilde{\mathbf{h}}=-j\omega\mu_0\tilde{\mathbf{e}}+(0,2,0)\cdot(\hat{\mathbf{N}}\times\mathbf{E})|_\mathbf{h}+\mathbf{I}_\mathbf{e}, \tag{6b}$$

where

$$\mathbf{I}_m=-\hat{\mathbf{a}}_x\int_h^\infty E_y\frac{\partial\psi_e^*}{\partial z}\,dy+\hat{\mathbf{a}}_y\left(\int_h^\infty E_x\frac{\partial\psi_o^*}{\partial z}\,dy-\int_h^\infty E_z\frac{\partial\psi_o}{\partial x}\,dy\right)+\hat{\mathbf{a}}_z\int_h^\infty E_y\frac{\partial\psi_e^*}{\partial x}\,dy$$

and

$$\mathbf{I}_e=-\hat{\mathbf{a}}_x\int_h^\infty H_y\frac{\partial\psi_o^*}{\partial z}\,dy+\hat{\mathbf{a}}_y\left(\int_h^\infty H_x\frac{\partial\psi_e^*}{\partial z}\,dy-\int_h^\infty H_z\frac{\partial\psi_e^*}{\partial x}\,dy\right)-\hat{\mathbf{a}}_z\int_h^\infty H_y\frac{\partial\psi_o^*}{\partial x}\,dy$$

and we used $\psi_o^*|_h=0$, $\psi_e^*|_h=2/\sqrt{2\pi}$. We can decouple these equations by operating on both sides of the equations with $\nabla\times$ and combining the results. For the electric field transform we get

$$\begin{aligned}\nabla\times\nabla\times\tilde{\mathbf{e}}-k_o^2\tilde{\mathbf{e}}=&-j\omega\mu_0(0,2,0)\cdot(\hat{\mathbf{N}}\times\mathbf{H})|_h\\&+\nabla\times(2,0,2)\cdot(\hat{\mathbf{N}}\times\mathbf{E})|_h-j\omega\mu_0\mathbf{I}_e+\nabla\mathbf{I}_m,\end{aligned} \tag{7}$$

$(\hat{\mathbf{N}} \times \mathbf{E})|_h$ is known from the boundary conditions but the term $(0,2,0) \cdot (\hat{\mathbf{N}} \times \mathbf{H})|_h = 2\hat{\mathbf{a}}_y(h_x H_z - h_z H_x)$ is not. On the other hand, $\mathbf{I}_m$ and $\mathbf{I}_e$ contain the derivative of $\psi_{\rm e}$ or $\psi_{\rm o}$ with respect to x or z, and thus, the integrands involved have the factor h_x or h_z. Hence, if the guess to $\mathbf{H}|_h$ on the right hand side of (7) is correct to order n in the perturbative parameter, the resulting approximation te $\tilde{\mathbf{e}}$ from (7) will be correct to order $n+1$ in the perturbative parameter. This is similar to the case of 1-D rough surfaces in Ref. 13.

We will proceed along the same guide lines we used in the case of 1-D rough surfaces. Let us define the $\mathbf{A}(k_y)$ of the electric field as

$$\mathbf{A}(k_y) = \hat{\mathbf{a}}_x \int_h^\infty E_x e^{jk_y y}\, dy + \hat{\mathbf{a}}_y \int_h^\infty E_y e^{jk_y y}\, dy + \hat{\mathbf{a}}_z \int_h^\infty E_z e^{jk_y y}\, dy. \tag{8}$$

Then the vector-transform $\tilde{\mathbf{e}}$ is given as

$$\hat{\mathbf{e}} = \frac{1}{\sqrt{2\pi}} \left[e^{-jk_y h(x,z)} \mathbf{A}(k_y) - e^{-jk_y(x,z)} \overline{\overline{\mathbf{X}}} \cdot \mathbf{A}(-k_y) \right], \tag{9}$$

where $\overline{\overline{\mathbf{X}}} = \hat{\mathbf{a}}_x\hat{\mathbf{a}}_x - \hat{\mathbf{a}}_y\hat{\mathbf{a}}_y + \hat{\mathbf{a}}_z\hat{\mathbf{a}}_z$. We may find the vector equation satisfied by $\mathbf{A}(k_y)$ by a similar procedure used to derive (7). We find

$$\nabla \times \nabla \times \mathbf{A} - k_0^2 \mathbf{A} = \frac{-j\omega\mu_0}{\sqrt{2\pi}} e^{jk_y h(x,z)} (\hat{\mathbf{N}} \times \mathbf{H})|_h + \frac{1}{\sqrt{2\pi}} e^{jk_y h(x)} \nabla \times (\hat{\mathbf{N}} \times \mathbf{E})|_h. \tag{10}$$

Thus, we may solve for $\mathbf{A}(k_y)$ from this equation and then construct $\tilde{\mathbf{e}}$ and invert the transforms to obtain the scattered field. We may actually apply first the inversion operator to the different terms in (9) and then construct the scattered field. To maintain more compact our expressions we will do it the latter way.

Let us consider the scattered fields, $\mathbf{E}^s$ and $\mathbf{H}^s$, only. This is a source-less field above the surface and satisfy the equations above. The boundary conditions yield $(\hat{\mathbf{N}} \times \mathbf{E}^s)|_h = -(\hat{\mathbf{N}} \times \mathbf{E}^i)|_h$. Following our previous work we obtain the first order approximation using the tangent plane approximation on the right hand side of (10). This is $(\hat{\mathbf{N}} \times \mathbf{H}^s)|_h \approx -(\hat{\mathbf{N}} \times \mathbf{H}^i)|_h$.

3. Scattering of a TM wave

Let us consider an incident TM wave given by

$$\mathbf{H}^i = \sqrt{\frac{\mu_0}{\varepsilon_0}} E_0 e^{-jk_x ix + jk_y iy - jk_z iz} \left(-\hat{\mathbf{a}}_x \cos\theta_i \sin\varphi_i + \hat{\mathbf{a}}_y \sin\theta_i + \hat{\mathbf{a}}_z \cos\theta_i \cos\varphi_i \right)$$

and

$$\mathbf{E}^i = E_0 e^{-jk_x ix + jk_y iy - jk_z iz} \left(\hat{\mathbf{a}}_x \cos\theta_i \sin\varphi_i + \hat{\mathbf{a}}_y \sin\theta_i + \hat{\mathbf{a}}_z \cos\theta_i \cos\varphi_i \right). \tag{11}$$

The solution to (10), when the right hand side is known, may be obtained by Fourier transforming both sides. We multiply both sides of the equation by $\exp(jk_xx+jk_zz)$ and integrate on dx and dz from minus infinity to plus infinity. Integrating by parts yields

$$(\mathbf{k}\times\mathbf{k}+k_0^2)\,\mathbf{F}\{\mathbf{A}^s(k_y)\} = \frac{j\omega\mu_0}{\sqrt{2\pi}}\mathbf{F}\{e^{jk_yh}(\hat{\mathbf{N}}\times\mathbf{H}^i)|_h\} - \frac{j}{\sqrt{2\pi}}\mathbf{k}\times\mathbf{F}\{e^{jk_yh}(\hat{\mathbf{N}}\times\mathbf{E}^i)|_h\}.$$

The solution from this equation may be separated in its TM and TE components. The TM component is given by [15]

$$\mathbf{F}\{\mathbf{A}^s(k_y)\}_{\mathrm{TM}} = -\frac{(\mathbf{k}\times\mathbf{k}\times\hat{\mathbf{a}}_y)(\mathbf{k}\times\mathbf{k}\times\hat{\mathbf{a}}_y)}{k^2(k^2-k_0^2)(k_x^2+k_z^2)}\cdot\left(\frac{j\omega\mu_0}{\sqrt{2\pi}}\mathbf{F}\{e^{jk_yh}(\hat{\mathbf{N}}\times\mathbf{H}^i)|_h\}\right.$$
$$\left.+\frac{1}{\sqrt{2\pi}}\mathbf{k}\times\mathbf{F}\{e^{jk_yh}(\hat{\mathbf{N}}\times\mathbf{E}^i)|_h\}\right). \quad (12)$$

The Fourier operator on the right hand side of the equation may be brought outside the whole expression.

The contribution to the scattered TM-field corresponding to the first term in (9) is obtained by inverse-Fourier transforming (12), applying the inversion operator for the ψ_{e}- or ψ_{o}-transform accordingly to the definition of $\tilde{\mathbf{e}}$ in (2), and multiplying by $\exp(-jk_yh)/(2\pi)^{1/2}$. Thus, we should apply the operator

$$\frac{e^{-jk_yh}}{(2\pi)^{3/2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dk_x\,dk_y\,e^{-jk_xx-jk_zz}\int_0^{\infty}dk_y\,\frac{1}{\sqrt{2\pi}}\left(e^{-jk_y(y-h)}\pm e^{jk_y(y-h)}\right)$$

to $\mathbf{F}\{\mathbf{A}^S(x',z')\}_{TM}$ in (12), where the minus sign is used for the x- and z-components of $\mathbf{F}\{\mathbf{A}^S\}_{TM}$ and the plus sign for the y-component. Upon expanding the vector products and rearranging terms we get

$$\frac{-jE_0}{(2\pi)^3}e^{-jk_yh}\int\frac{(\mathbf{k}\times\mathbf{k}\times\hat{\mathbf{a}}_y)}{k^2(k^2-k_0^2)(k_x^2+k_z^2)}(S+h_{x'}P+h_{z'}Q)e^{-jk_xx-jk_zz}$$
$$(e^{-jk_y(y-h)}\pm e^{jk_y(y-h)})e^{-j(k_x^i-k_x)x'+j(k_y^i+k_y)h'-j(k_z^i-k_z)z'}\,dk_x\,dk_y\,dk_z\,dx'\,dz',$$

where $\int$ means five integrals over $(-\infty,\infty)$, one for each variable, and

$$S = k_0k_xk_y\sin\varphi_i + k_0k_zk_y\cos\varphi_i + k^2k_z\cos\theta_i\cos\varphi_i + k^2k_x\cos\theta_i\sin\varphi_i,$$
$$P = k^2k_x\sin\theta_i - k_0(k_x^2+k_z^2)\sin\varphi_i,$$
$$Q = k^2k_z\sin\theta_i - k_0(k_x^2+k_z^2)\cos\varphi_i,$$

where $k^2 = k_x^2+k_y^2+k_z^2$. Noting that $1/(k^2-k_0^2) = 1/[k_z^2-(k_0^2-k_x^2-k_y^2)]$ has two poles in the k_z-plane at $k_z = \pm(k_0^2-k_x^2-k_y^2)^{1/2}$ we may evaluate the integral over k_z using residue theory. The result is $-2\pi jf(k_z=-\sqrt{\cdot})/2\sqrt{\cdot}$ if $z'-z>0$, and $-2\pi jf(k_z=+\sqrt{\cdot})/2\sqrt{\cdot}$ if $z'-z<0$, where $\sqrt{\cdot} = \sqrt{k_0^2-k_x^2-k_y^2}$ and $f(\cdot)$ is the remainder of the expression.

The integrals over k_x and k_y may then be evaluated using the method of stationary phase by assuming than only a finite patch of the surface of area $2L \times 2L$ is illuminated and assuming that the observation point $r = (x^2 + y^2 + z^2)^{1/2}$ is very large compared to L. The term with the factor $\pm e^{jk_y(y-h)}$ does not have a stationary phase and does not contribute in the far-zone. The term with the factor $e^{-jk_y(y-h)}$ has one stationary phase point. The evaluation of the integrals with the method of stationary phase gives a factor $2\pi j|k_z^s|e^{-jk_0 r}/r$ which multiplies the remainder of the integrand with k_x, k_y, and k_z replaced by $k_x^s = k_0 \sin\theta_i \sin\varphi_i$, $k_y^s = k_0 \cos\theta_i$, and $k_z^s = k_0 \sin\theta_i \cos\varphi_i$ [16]. We get

$$\frac{-jE_0 e^{-jk_0 r}}{2\pi r} \frac{(\mathbf{k}^s \times \mathbf{k}^s \times \hat{\mathbf{a}}_y)}{2k_0^2(k_x^{s\,2} + k_z^{s\,2})} \int_{-L}^{L}\int_{-L}^{L} (S + h_{x'}P + h_{z'}Q) e^{-jv_x x' + jp_y h' - jv_z z'}\, dx'\, dz',$$

where $v_x = k_x^i - k_x^s$, $v_z = k_z^i - k_z^s$, $p_y = k_y^i + k_y^s$. Noting that $h_{x'} e^{jp_y h'} = (1/jp_y) d(e^{jp_y h'} - 1)/dx'$ and $h_{z'} e^{jp_y h'} = (1/jp_y) d(e^{jp_y h'} - 1)/dz'$, we may integrate by parts the last two terms from the integral. The edge terms are zero since we have assumed that $h(x,z) = 0$ for $|x|, |z| > L$. Upon expressing $(Sp_y + v_x P + v_z Q)/p_y$ in terms of the angles of incidence and scatter and rearranging terms we may write $(Sp_y + v_x P + v_z Q)/p_y = 2k_0^3 \sin\theta_s f_{vv}^{(1)}(\Omega_i, \Omega_s)$, where

$$f_{vv}^{(1)}(\Omega_i, \Omega_s) = \frac{(1 + \cos\theta_i \cos\theta_s)\cos(\varphi_s - \varphi_i) - \sin\theta_i \sin\theta_s}{\cos\theta_i + \cos\theta_s},$$

where Ω_i and Ω_s stand for θ_i, φ_i and θ_s, φ_s. Thus, the latter integral becomes

$$\frac{-jE_0 e^{-jk_0 r}}{2\pi k_0 r} \frac{(\mathbf{k}^s \times \mathbf{k}^s \times \hat{\mathbf{a}}_y)}{\sin\theta_s} \Bigg[f_{vv}^{(1)}(\Omega_i - \Omega_s) \int_{-L}^{L}\int_{-L}^{L} e^{-jv_x x - jv_z z}(e^{jp_y h} - 1)\, dx\, dz$$
$$\frac{1}{2}(\cos\theta_i + \cos\theta_s)\cos(\varphi_s - \varphi_i) \int_{-L}^{L}\int_{-L}^{L} e^{-jv_x x - jv_z z}\, dx\, dz \Bigg]. \quad (13)$$

The first term is the non-specular component and the second term is the specular component. If at this point we let L increase to infinity the second integral becomes $(2\pi)^2 \delta(k_x^i - k_x^s)\,\delta(k_z^i - k_z^s)$ and we may replace θ_i and φ_i by θ_s and φ_s respectively in the scattering coefficient multiplying the integral. Because $f_{vv}^{(1)}(\Omega_i, \Omega_s)$ in the specular direction coincides with the latter result, we may include the specular term in the first term in (13) by dropping the -1 inside the parenthesis.

The scattered TM field predicted by the first order approximation consists of (13) plus the term arising from the second term in (9), $-e^{jk_y h(x)}\overline{\overline{\mathbf{X}}} \cdot \mathbf{A}(-k_y)$. We may obtain its contribution in the far-zone from the result in (13), simply by changing the sign of k_y. Thus, we should change (k_x^s, k_y^s, k_z^s) to $(k_x^s, -k_y^s, k_z^s)$ in $\mathbf{k}^s \times \mathbf{k}^s \times \hat{\mathbf{a}}_y$ and $\cos\theta_s$ to $-\cos\theta_s$ ($\cos\theta_s$ came from $k_y = k_0 \cos\theta_s$) in $f_{vv}^{(1)}(\Omega_i, \Omega_s)$ in (13), and dot-multiply the result with $-\overline{\overline{\mathbf{X}}}$ (the factor $e^{jk_y h}$ is unity when considering the far-zone). Note that $-\overline{\overline{\mathbf{X}}} \cdot (\mathbf{k}^s \times \mathbf{k}^s \times \hat{\mathbf{a}}_y)|_{k_y \to -k_y} = (\mathbf{k}^s \times \mathbf{k}^s \times \hat{\mathbf{a}}_y)$. Thus we obtain the scattered TM field as

$$\mathbf{E}_{VV} = \frac{-jE_0 e^{-jk_0 r}}{2\pi k_0 r} \frac{(\mathbf{k}^s \times \mathbf{k}^s \times \hat{\mathbf{a}}_y)}{\sin\theta_s} \left[f_{vv}^{(1)}(\Omega_i, \Omega_s) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-jv_x x - jv_z z} e^{jp_y h} \, dx \, dz \right.$$
$$\left. + f_{vv}^{(2)}(\Omega_i, \Omega_s) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-jv_x x - jv_z z} e^{jq_y h} \, dx \, dz \right], \quad (14)$$

where $q_y = k_y i - k_y s$,

$$f_{vv}^{(2)}(\Omega_i, \Omega_s) = \frac{(1 - \cos\theta_i \cos\theta_s)\cos(\varphi_s - \varphi_i) - \sin\theta_i \sin\theta_s}{\cos\theta_i + \cos\theta_s},$$

and the notation $\mathbf{E}_{VV}$ stands for the TM (vertical polarization) scattered wave arising from a TM incident wave.

Next we need to find the cross polarized scattered field, *i.e.*, the TE or horizontally polarized scattered wave $\mathbf{E}_{VH}$. The procedure follows very closely that used to obtain $\mathbf{E}_{VV}$. The difference is that we must replace the operator for the TM component

$$\frac{(\mathbf{k} \times \mathbf{k} \times \hat{\mathbf{a}}_y)(\mathbf{k} \times \mathbf{k} \times \hat{\mathbf{a}}_y)}{k^2(k_x^2 + k_z^2)},$$

with the operator for the TE component [15]:

$$\frac{(\mathbf{k} \times \hat{\mathbf{a}}_y)(\mathbf{k} \times \hat{\mathbf{a}}_y)}{(k_x^2 + k_z^2)}.$$

Upon doing this change in (12) and follow an otherwise identical analysis we get

$$\mathbf{E}_{VH} = \frac{-jE_0 e^{-jk_0 r}}{2\pi r} \frac{\mathbf{k}^s \times \hat{\mathbf{a}}_y}{\sin\theta_s} f_{VH}(\Omega_i, \Omega_s) \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-jv_x x - jv_z z} e^{jp_y h} \, dx \, dz \right.$$
$$\left. - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-jv_x x - jv_z z} e^{jq_y h} \, dx \, dz \right], \quad (15)$$

where $f_{VH}(\Omega_i, \Omega_s) = \sin(\varphi_s - \varphi_i)$.

4. THE SCATTERING OF A TE WAVE

In this case the incident field is given by

$$\mathbf{E}^i = E_0 e^{-jk_x ix + jk_y iy - jk_z iz}(-\hat{\mathbf{a}}_x \cos\varphi_i + \hat{\mathbf{a}}_z \sin\varphi_i)$$

and

$$\mathbf{H}^i = -\sqrt{\frac{\mu_0}{\varepsilon_0}} E_0 e^{-jk_x ix + jk_y iy - jk_z iz}(-\hat{\mathbf{a}}_x \cos\theta_i \sin\varphi_i + \hat{\mathbf{a}}_y \sin\theta_i + \hat{\mathbf{a}}_z \cos\theta_i \cos\varphi_i). \quad (16)$$

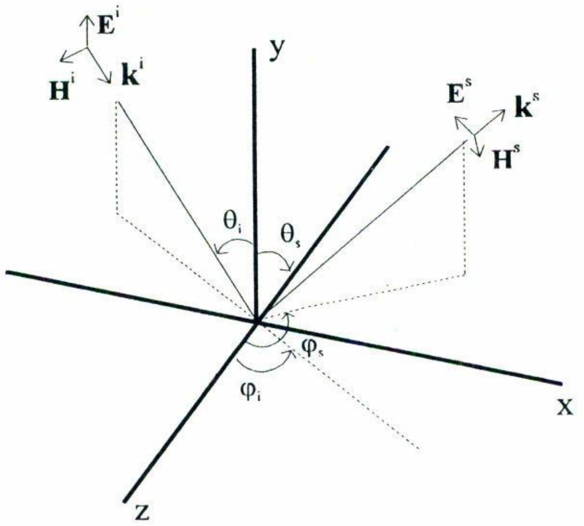

FIGURE 1. Coordinate system and definition of the angles of incidence and scatter. The figure also shows the fields for a TM incident wave and a TM scattered waves.

The analysis is nearly identical to that in the previous Section and the only difference is in the surface terms, $(\hat{\mathbf{N}} \times \mathbf{E}^s)|_h = -(\hat{\mathbf{N}} \times \mathbf{E}^i)|_h$ and $(\hat{\mathbf{N}} \times \mathbf{H}^s)|_h \approx (\hat{\mathbf{N}} \times \mathbf{H}^i)|_h$, which must be evaluated with the TE wave (16). We give here only the final results for the far-fields. The scattered TE wave is obtained as

$$\mathbf{E}_{VH} = \frac{-jE_0 e^{-jk_0 r}}{2\pi r} \frac{(\mathbf{k}^s \times \hat{\mathbf{a}}_y)}{\sin\theta_s} \Bigg[f_{HH}^{(1)}(\Omega_i,\Omega_s) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-jv_x x - jv_z z} e^{jp_y h}\, dx\, dz - f_{HH}^{(2)}(\Omega_i,\Omega_s) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-jv_x x - jv_z z} e^{jq_y h}\, dx\, dz \Bigg], \quad (17)$$

where $f_{HH}^{(1)}(\Omega_i,\Omega_s) = f_{VV}^{(1)}(\Omega_i,\Omega_s)$ and $f_{HH}^{(2)}(\Omega_i,\Omega_s) = f_{VV}^{(2)}(\Omega_i,\Omega_s)$.

The cross polarized scattered wave is obtained as

$$\mathbf{E}_{VH} = \frac{jE_0 e^{-jk_0 r}}{2\pi k_0 r} \frac{(\mathbf{k}^s \times \mathbf{k}^s \times \hat{\mathbf{a}}_y)}{\sin\theta_s} f_{VH}(\Omega_i,\Omega_s) \Bigg[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-jv_x x - jv_z z} e^{jp_y h}\, dx\, dz + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-jv_x x - jv_z z} e^{jq_y h}\, dx\, dz, \Bigg] \quad (18)$$

5. PROPERTIES OF THE FIRST ORDER SOLUTIONS

The far-zone solutions in (14), (15), (17), and (18) satisfy analogous properties to their counterparts in the case of perfectly-conducting 1-D rough surfaces. The first terms in each case coincide with the Kirchhoff approximation (KA). The second term in E_{vv} and E_{HH} is zero in the specular direction, and the cross polarized fields are zero in the specular direction. Thus, the present results coincides with the KA in the specular direction. The scattered TE fields (15) and (17) are zero for grazing emergence, and thus, the boundary conditions are satisfied. The TM scattered fields in (14) and (18) are not zero for grazing emergence and they coincide with the regular full wave results in this case. If we expand the exponentials in power series and keep only the first two terms (*e.g.*, $e^{jp_y h} \cong 1 + jph(x,z)$) we obtain the first order results from perturbation theory. On the other hand, since our only approximation was the tangent plane approximation, we expect a priori that the present results will coincide with the KA whenever this is accurate.

If we assume that $h(x,z)$ is constant along the z-direction, so that $h = h(x)$ and take $\varphi_i = 0$, we recover the results for 1-D rough surfaces [13].

As in the case for 1-D rough surfaces, we should use a reciprocity rule when dealing with randomly rough surfaces, since the results are not reciprocal. The rule is very simple and follows from the same arguments used in the case of 1-D rough surfaces. We have that we should use the result for which the angle of incidence is smallest. Invoking the reciprocity theorem it follows that

$$\mathbf{M}_{LSEM}^{AB}(\Omega_i,\Omega_s) = (1-\xi)\mathbf{M}_{DLSEM}^{AB}(\theta_i,\varphi_i,\theta_s,\varphi_s) + (\xi)\mathbf{M}_{DLSEM}^{BA}(-\theta_s,\varphi_s,-\theta_i,-\varphi_i), \tag{19}$$

where $\mathbf{M}$ may be $\mathbf{E}$ or $\mathbf{H}$, $\mathbf{M}_{DLSEM}^{AB}$ is any of the direct results given above where A and B may be H or V (for vertical and horizontal polarizations), and

$$\xi = \begin{cases} 1 & \text{for} - \theta_i < \theta_s < \theta_i \\ 0 & \text{otherwise.} \end{cases}$$

$\mathbf{M}_{DLSEM}^{BA}(-\theta_s,\varphi_s,-\theta_i,-\varphi_i)$ used in (19) may be called the reveresed or reciprocal results. Note that the incidence and scatter angles are interchanged and the sign of the polar angle is changed. Note also that the order of the subscript was changed to BA.

6. Conclusions

The formulation of the LSEM for electromagnetic scattering from perfectly conducting surfaces rough in two dimensions was presented and the first order results using the tangent plane approximation were derived. The properties of the first order solutions are analogous to those in the case of 1-D rough surfaces. In particular, it was indicated that the present first order results reduce to first order perturbation theory in the appropriate limit, and on the other hand, the approximations will coincide with the Kirchhoff approximation when this is valid. If the surface is assumed constant along one direction and the incident field is assumed perpendicular to such direction we recover the formulation for 1-D rough surfaces in Ref. 13. A systematic series solution is in principle possible by applying perturbation theory to correct the tangent plane approximation and find higher order approximation as it was done for 1-D rough surfaces.

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