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Reflection of evanescent waves

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ABSTRACT. In the attenuated total reflection (ATR) experiment using Otto configuration, at the base of the coupling prism an evanescent wave is produced by the frustrated total reflection. This evanescent wave passes through an air gap of the order of the light wavelength and arrives at the surface of a metal adjacent to the prism. Under the phase-matching condition, surface plasmon resonance (SPR) will be excited at the interface between the air and the metal. This in turn determines the coupling angles at which the power of the incident beam is tunneled into the SPR. It is found that at the coupling angles the amplitude of the "reflection coefficient" at the air/metal exceeds unity. This physically meaningless "reflection coefficient" of the evanescent wave is actually necessary for fulfilling the dispersion relation in the excitement of SPR.

RESUMEN. En el experimento de reflexión atenuada (ATR) usando la configuración Otto, en la base del prisma acoplador se produce una onda evanescente por la reflexión total frustada. Esta onda evanescente atraviesa una brecha de aire con dimensiones comparables a la longitud de onda de la luz y llega a la superficie de un metal adyacente al prisma. Bajo condiciones de acoplamiento de fase, una resonancia de plasmon superficial (SPR) se excitará en la interfaz entre aire y metal. Esto a su vez determinará los ángulos de acoplamiento a los cuales la potencia del haz incidente se canaliza al SPR. Se ha encontrado que a los ángulos de acoplamiento, la amplitud del "coeficiente de reflexión" de la onda evanescente en la interfaz aire/metal excede la unidad. Este "coeficiente de reflexión" de la onda evanescente, carente fisicamente de sentido, es necesario para cumplir con las relaciones de dispersión en la excitación de un SPR.

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1. INTRODUCTION

At the first glance it seems only a conceptual game: evanescent waves only can propagate through a distance of the order of the light wavelength. What does their "reflection" mean? However, a thorough investigation shows that this problem indeed has a very practical experimental background. In characterizing optical properties of materials, particularly when metallic media are involved, the measurement of surface plasmon resonance (SPR) is extensively used. SPR is a kind of electromagnetic fields that only exist at a boundary between metal and dielectric media. A convenient way to excite SPR is the attenuated total reflection (ATR) method. In Otto configuration [1], a light beam is launched into a half cylinder "prism", as shown in Fig. 1. At the base of the prism, the angle θ_p exceeds the critical angle $\theta_c = \arcsin(n_0/n_p)$, and total reflection is established.

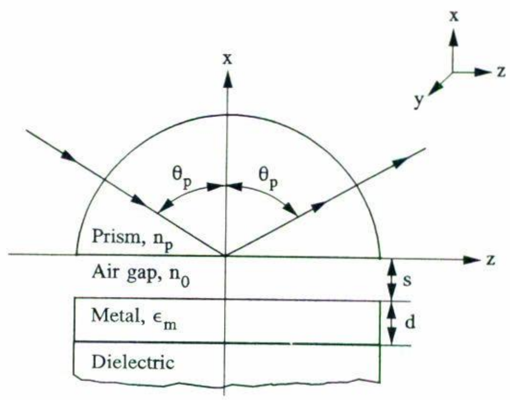

FIGURE 1. Schematic of Otto configuration.

When a metal is brought close to the prism with the spacing of the order of the light wavelength, $s \sim \lambda$, the evanescent wave at the prism base can arrive at the surface of the metal. In this way, since the attenuation of the evanescent wave is not too strong, at the metal surface there is still significant light power. This in turn gives rise to the physically meaningful reflection of the evanescent wave.

The excitation of SPR requires the so called "phase-machting" condition, the tangential component of the evanescent wave vector equals to that of the plasmon. Under this condition, the light power of the incident beam is tunneled into the SPR and one can observe a drop in the reflectance. In another similar approach, the Kretschmann configuration [2, 3], a very thin metal layer is directly deposited onto the prism base and SPR is excited at another side of the metal, the interface between the metal and the air. The reader may consult Refs. 4 and 5 to obtain a detailed overview about the ATR technique and the excitation of SPR. Here we will concentrate on the reflection of evanescent waves in Otto configuration. It is found that the "reflection coefficient" at the air/metal interface has an unusual behavior. Its amplitude may exceed unity.

2. THEORY

Figure 1 shows the schematic of Otto configuration. The half cylinder prism has a refractive index n_p. The dielectric constant of the metal $\epsilon_m = \epsilon'_m + i\epsilon''_m$ fulfills relations $\epsilon'_m < 0$, $\epsilon''_m > 0$, and $-\epsilon'_m \gg \epsilon''_m$. The air gap is of the order of the light wavelength that allows the damped evanescent wave to excite SPR at the air/metal interface. The metal layer is thick enough to insolate the influence of the dielectric (either a buffer layer or the substrate) on the SPR. When a p-polarized light beam is launched into the prism with an angle θ_p at the base of the prism exceeding the critical angle, an evanescent wave is produced at the prism base. This evanescent wave is reflected at the interface between the prism and the air. The Fresnel reflection coefficient reads

$$r_{p0} = \frac{(k_{xp}/n_p^2) - (k_{x0}/n_0^2)}{(k_{xp}/n_p^2) + (k_{x0}/n_0^2)}\,, \tag{1}$$

where k_{xp} and k_{x0} are the normal components of the wave vector in the prism and the air, respectively. As the air gap is of the order of the light wavelength, the damped evanescent wave still has considerable intensity. It arrives at the interface between the air and the metal and is reflected there. The "reflection coefficient" at this interface is defined by the normal component of the wave vector k_{xm} in the metal and that in the air:

$$r_{0m} = \frac{(k_{x0}/n_0^2) - (k_{xm}/\epsilon_m)}{(k_{x0}/n_0^2) - (k_{xm}/\epsilon_m)}. \tag{2}$$

The boundary conditions require the continuity of the tangential components of the wave vector in all media, $k_z = k_{zp} = k_{z0} = k_{zm} =$ constant. This gives

$$\begin{aligned} &k_z = k_{zp} = k_0\, n_p \sin\theta_p, && k_{xp} = k_0\, n_p \cos\theta_p, \qquad k_0 = 2\pi/\lambda, \\ &k_{x0} = k_0\, n_0 \cos\theta_0 = ik_0\, q_0'', && q_0'' = \sqrt{n_p^2 \sin^2\theta_p - n_0^2}, \\ &k_{xm} = k_0\sqrt{\epsilon_m - (k_z/k_0)^2} = k_0 q_m\,, && q_m = \sqrt{\epsilon_m' + i\epsilon_m'' - n_p^2 \sin^2\theta_p} = q_m' + iq_m''. \end{aligned} \tag{3}$$

Introducing the expressions for k_{xp} and k_{x0} in Eq. (3) to Eq. (1), one obtains the reflection coefficient at the prism/air interface

$$r_{p0} = \frac{n_0^2 \cos\theta_p - i\, n_p\, q_0''}{n_0^2 \cos\theta_p + i\, n_p\, q_0''} = \exp\left(-i\, 2\Phi_{p0}\right). \tag{4}$$

Its unity amplitude shows that total reflection occurs at this interface. If the metal is absent, one will have hundred percent of the power of the incident beam reflected back. The phase shift on the total reflection is defined by

$$\cos 2\Phi_{p0} = \frac{n_0^4 \cos^2\theta_p - in_p^2\, q_0''^2}{n_0^4 \cos^2\theta_p + n_p^2\, q_0''^2}, \qquad \sin 2\Phi_{p0} = \frac{2n_0^2\, n_p\, q_0'' \cos\theta_p}{n_0^4 \cos^2\theta_p + n_p^2\, q_0''^2}, \tag{5}$$

Analogously, the Fresnel coefficient at the air/metat interface reads

$$r_{0m} = -\frac{n_0^2\, q_m' + \epsilon_m''\, q_0'' + i(n_0^2\, q_m'' - \epsilon_m'\, q_0'')}{n_0^2\, q_m' - \epsilon_m''\, q_0'' + i(n_0^2\, q_m'' + \epsilon_m'\, q_0'')} = \rho_{0m} \exp\left(-i\, 2\Phi_{0m}\right). \tag{6}$$

Its amplitude is given by

$$\rho_{0m} = \left[\frac{(n_0^2\, q_m' + \epsilon_m''\, q_0'')^2 + (n_0^2\, q_m'' - \epsilon_m'\, q_0'')^2}{(n_0^2\, q_m' - \epsilon_m''\, q_0'')^2 + (n_0^2\, q_m'' + \epsilon_m'\, q_0'')^2}\right]^{1/2} = \sqrt{\frac{1 - G_{0m}}{1 + G_{0m}}}, \tag{7}$$

where

$$G_{0m} = \frac{2n_0^2\, q_0''(\epsilon_m'\, q_m'' - \epsilon_m''\, q_m')}{n_0^4\, ({q_m'}^2 + {q_m''}^2) + ({\epsilon_m'}^2 + {\epsilon_m''}^2)\, q_0''^2}. \tag{8}$$

In the following paragraphs we will show that this "reflection coefficient" has an unusual behavior, its amplitude expressed in Eq. (8) will exceed unity. The phase term in Eq. (6) is detemined by the relations

$$\cos 2\Phi_{0m} = \frac{1}{\sqrt{1-G_{0m}^2}} \frac{-n_0^4\,({q'_m}^2 + {q''_m}^2) + ({\epsilon'_m}^2 + {\epsilon''_m}^2)\, q_0''^2}{n_0^4\,({q'_m}^2 + {q''_m}^2) + ({\epsilon'_m}^2 + {\epsilon''_m}^2)\, q_0''^2},$$
$$\sin 2\Phi_{0m} = -\frac{1}{\sqrt{1-G_{0m}^2}} \frac{2\, n_0^2\, q_0''(\epsilon'_m\, q''_m + \epsilon''_m\, q'_m)}{n_0^4\,({q'_m}^2 + {q''_m}^2) + ({\epsilon'_m}^2 + {\epsilon''_m}^2)\, q_0''^2}. \tag{9}$$

In practice one can only detect the power of the light combining the beam reflected at the prism/air interface and that reflected at the air/metal interface. The reflection coefficient taking into account both interfaces is described by the Airy formula [6]:

$$r_{p0m} = \frac{r_{p0} + r_{0m} \exp{(i\, 2k_{x0}\, s)}}{1 + r_{p0}\, r_{0m} \exp{(i\, 2k_{x0}\, s)}}. \tag{10}$$

Accordingly, the reflectance reads:

$$R_{p0m} = |r_{p0m}|^2 = \frac{1 + \rho_{0m}^2 \exp{(-4\, k_0\, q_0'' s)} + 2\rho_{0m} \exp{(-2\, k_0\, q_0''\, s)} \cos 2(\Phi_{p0} - \Phi_{0m})}{1 + \rho_{0m}^2 \exp{(-4,\, k_0\, q_0''\, s)} + 2\rho_{0m} \exp{(-2\, k_0\, q_0''\, s)} \cos 2(\Phi_{p0} + \Phi_{0m})}. \tag{11}$$

The coupling of incident beam into SPRs means that the intensity of reflected light is dissipated. So the condition that the reflectance approaches minimum indicates the coupling to SPRs. Obviously $R_{p0m} \to$ minimum when

$$\Phi_{p0} - \Phi_{0m} = \frac{\pi}{2}. \tag{12}$$

Particularly, $R_{p0m} \to 0$, when

$$\exp{(2\, k_0\, q_0'' s)} = \rho_{0m}. \tag{13}$$

Now let us have some discussions about these results. First, since $\epsilon'_m < 0$ and $\epsilon''_m > 0$, one knows that the phase angle of the quantity $\epsilon'_m - n_p^2 \sin^2\theta_p + i\epsilon''_m$ ranges from 90° to 180°. It turns out that the phase angle of the quantity q_m ranges over 45° to 90°, meaning $q'_m > 0$ and $q'_m > 0$. Moreover, since $-\epsilon'_m \gg \epsilon''_m$, one has $q'_m \ll q''_m$, giving

$$q'_m = \frac{1}{\sqrt{2}}\Big[|q_m|^2 - (n_p^2 \sin^2\theta_p - \epsilon'_m)\Big]^{1/2},$$
$$q''_m = \frac{1}{\sqrt{2}}\Big[|q_m|^2 + (n_p^2 \sin^2\theta_p - \epsilon'_m)\Big]^{1/2}, \tag{14}$$
$$|q_m|^2 = \Big[(n_p^2 \sin^2\theta_p - \epsilon'_m)^2 + {\epsilon''_m}^2\Big]^{1/2}$$

Introducing Eq. (14) into Eq. (8), one obtains $G_{0m} < 0$, leading to $\rho_{0m} > 1$. *This means that the "reflection coefficient" of the evanescent wave at the air/metal exceeds unity. This result, of course, violates the energy conservation law and has no physical* *significance.* The point is that here this so called "reflection coefficient" only mathematically exists. One cannot insert a pair of photodetectors in the air gap and measure the reflectance of the evanescent waves at the air/metal interface. Under these circumstances, I suggest that we would call this particular "reflection coefficient" of the evanescent wave "the Fresnel coefficient on reflection". This implies that this coefficient does not bear the meaning related to the reflectance of a plane wave.

On the other hand, the tangential component of the wave vector in the SPR is determined by a dispersion relation [5]

$$\beta^2 = \frac{k_0^2\, n_0^2\, \epsilon_m}{n_0^2 + \epsilon_m}. \tag{15}$$

Defining "effective index" $N = \beta/k_0$, one obtains a relation

$$N^2 = n_0^2\, \frac{\epsilon'_m(n_0^2 + \epsilon'_m) + {\epsilon''_m}^2 + i n_0^2\, \epsilon''_m}{(n_0^2 + \epsilon'_m)^2 + {\epsilon''_m}^2}. \tag{16}$$

This equation shows that the phase angle of N^2 ranges from 0° to 90°. In addition, since $-\epsilon'_m \gg \epsilon''_m$, one knows that the phase angle of N^2 is close to 0°. In consequence, the phase angle of N ranges from 0° to 45° and is close to 0°, resulting in $N' > 0$, $N'' > 0$, and $N' \gg N''$. The effective index is then expressed by

$$N' = \sqrt{\tfrac{1}{2}\left(|N|^2 + \mathrm{Re}\,[N^2]\right)}, \qquad N'' = \sqrt{\tfrac{1}{2}\left(|N|^2 - \mathrm{Re}\,[N^2]\right)}, \tag{17}$$

where $\mathrm{Re}\,[N^2]$ is the real part of the variable N^2 expressed in Eq. (16), and where

$$|N|^2 = \frac{n_0^2\sqrt{{\epsilon'_m}^2 + {\epsilon''_m}^2}}{\sqrt{(n_0^2 + \epsilon'_m)^2 + {\epsilon''_m}^2}}. \tag{18}$$

Equation (17) shows that the tangential component of the wave vector β in the SPR is complex. The tangential component of the incident wave vector k_z, however, is real, as it is shown in Eq. (3). This means that the phase-matching only can be approximately fulfilled, $k_z = \mathrm{Re}\,[\beta] = k_0 N'$. This approximation will determine a coupling angle $\theta_p = \arcsin\,(N'/n_p)$, at which the power of the incident beam is tunneled into the SPR, provided $N'' \ll N'$. In the following computational results we can find that at this coupling angle the reflectance R_{p0m} reaches a minimum, because at this angle the requirement of Eq. (12) is approximately met.

3. RESULTS AND DISCUSSIONS

In order to illustrate the above results, let us take some examples in the literature. Examples No. 1 and 2 are taken from Ref. 7, and the third example is taken from Ref. 8, as shown in Table I. In Ref. 7 a rutile coupler with $n_0 = 2.585$ and $n_e = 2.873$ at $\lambda = 0.6328\,\mu$m is used to excite SPR. When the prism is positioned with its optical axis

TABLE I. Dielectric constants of the silver films $\epsilon'_m + i\epsilon''_m$, and refractive index n_p of the prism couplers.

Example No.	1	2	3
n_p	2.585	2.585	1.457
ϵ'_m	-15.54	-15.04	-14.436
ϵ''_m	1.00	1.16	0.456
References	[7], sample #3	[7], sample #4	[8], Ag @ 0.6328 μm

perpendicular to the incident plane, one has $n_p = n_0 = 2.585$. Silver films are deposited onto silicon dioxide buffer layers. When the silver films are thick enough (samples No. 3 and No. 4 in Ref. 7), the anisotropic effects of the buffer layers are negligible. The silver films of samples No. 3 and 4 have dielectric constant $\epsilon_m = -15.94 + i\,1.00$ and $\epsilon_m = -15.04 + i\,1.16$, respectively. The imaginary part of the dielectric constant in Ref. 7 shows a different sign. This is related to the different expressions for plane waves. When a plane wave is expressed as $\exp[i(\omega t - \mathbf{k}\cdot\mathbf{r})]$, one has a complex refractive index $n - i\kappa$. The negative sign in front of the extinction coefficient κ gives rise to the same sign in front of the imaginary part of the dielectric constant: $\epsilon = \epsilon' - i\epsilon''$. Here the plane waves are expressed in the form of $\exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)]$, leading to a positive sign. In Ref. 8 a fused silica prism with $n_p = 1.45708$ at $\lambda = 0.6328\,\mu$m is used as the coupler. The silver film with $\epsilon' = -14.436$ and $\epsilon'' = 0.456$ is deposited onto a glass substrate.

Figures 2a, 2b and 2c plot the calculated reflectance R_{p0m} curves correspond to these examples. First, it is shown that the coupling angle, *i. e.*, the position of minimum in the reflectance curves, is independent of the air gap spacing s. This coupling angle is determined by the relation $\theta_p = \arcsin(N'/n_p)$. For Example No. 1, the coupling angle is determined $\theta_p = 23^\circ\,33'$. At this angle, one has $\cos 2(\Phi_{p0} - \Phi_{0m}) = -0.961$ and $\cos 2(\Phi_{p0} + \Phi_{0m}) = 0.941$, giving the minimum in the reflectance curves, as shown in Fig. 2a. Similar results hold for examples No. 2 and 3. In the case of example No. 2, the coupling angle reads $23^\circ\,34'$, giving rise to $\cos 2(\Phi_{p0} - \Phi_{0m}) = -0.971$ and $\cos 2(\Phi_{p0} + \Phi_{0m}) = 0.948$. At this angle again the minimum in the reflectance curve is found in Fig. 2b. For Example No. 3, one has $\theta_p = 45^\circ\,21'$, corresponding to $\cos 2(\Phi_{p0} - \Phi_{0m}) = -0.868$ and $\cos 2(\Phi_{p0} + \Phi_{0m}) = 0.848$. This coupling angle again is related to the minimum in the reflectance curves, as shown in Fig. 2c.

The spacing has a strong effect on the magnitude of the minimum reflectance and the width of the region where the reflectance drops. As the spacing reduces, the width of the drop in the reflectance curves broadens; meanwhile, the magnitude in the minimum reflectance increases. This is caused by the fact that as the spacings narrows, the absorption $\exp(-2k_0 q''_0 s)$ becomes weaker. In addition, the calculated results here agree quite well with the experimental results shown in Refs. 7 and 8. For Examples No. 1 and 2, the calculated curves with $s = 1.2\,\lambda$ fit the experimental results, except that the width is a little narrower. This may be induced by some random processes in the experiments such as scattering that cause loss of more light power. The calculated reflectance curve with $s = 1.2\,\lambda$ in Fig. 2c also fits the experimental result in Ref. 8. The deviation in the

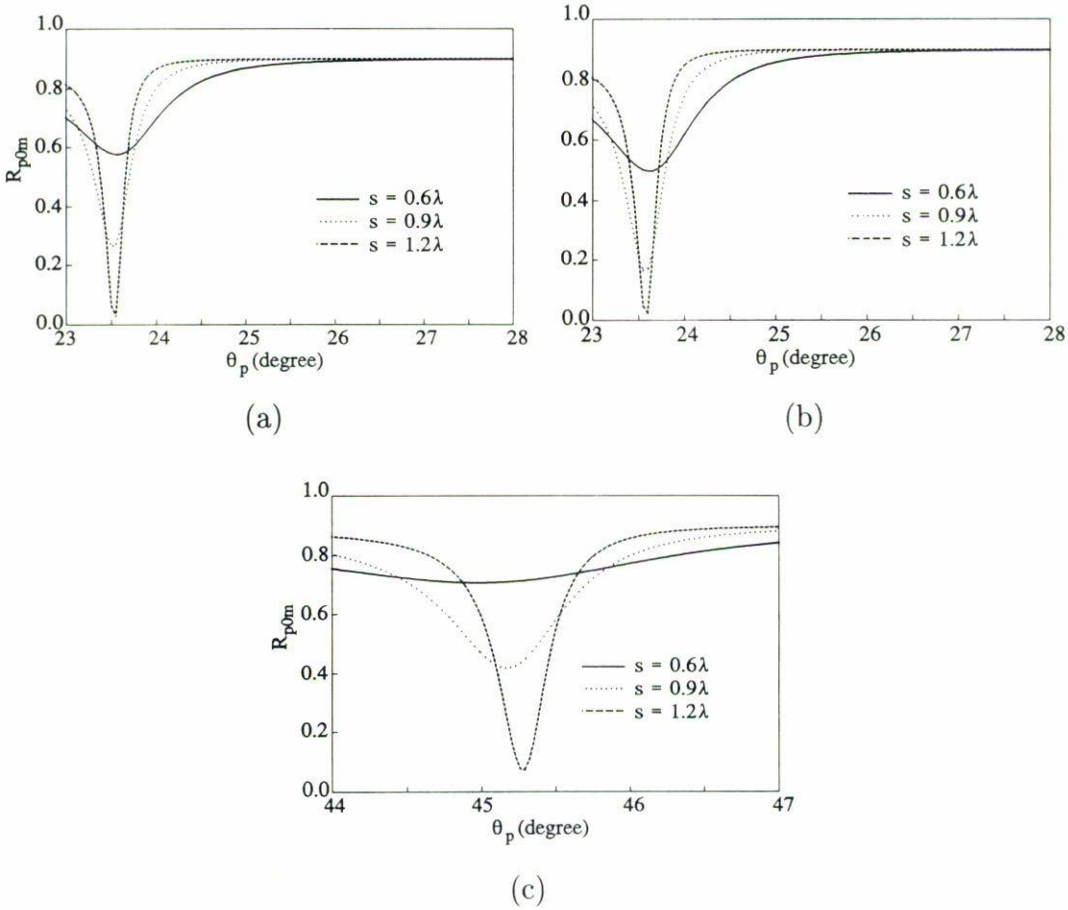

FIGURE 2. (a) The reflectance R_{p0m} curves *versus* the incident angles θ_p at the base of the prism for varios air gap spacing s. (a) Example No. 1. (b) Example No. 2. (c) Example No. 3.

magnitude in the minimum reflectance can be caused by the difference in the spacing here and the actual spacing in experiments in Ref. 8, which is usually unknown.

Figure 3 shows the "Fresnel coefficient on reflection" for all three examples. The magnitude of these curves is far more larger than unity, particularly at the coupling angles. It turns out that for evanescent waves, Fresnel coeffcient on reflection at an air/metal interface exceeds unity and does not bear the meaning of the reflection coefficient any more. This can be more clearly shown by the G-factor curves plotted in Fig. 4. It is shown that G_{0m} curves are invariably lower than zero, meaning the Fresnel coefficient calculated in Eq. (7) is always larger than unity. Particularly at the coupling angles, G_{0m} values are very close to -1, leading to very high ρ_{0m} values in the order of 60 to 140, as shown in Fig. 3.

Finally, Fig. 5 shows the cosine terms in the calculation of the reflectance R_{p0m}. Suppose the condition shown in Eq. (13) is fulfilled: $\exp(2k_0\, q_0''\, s) = \rho_{0m}$. The reflectance in Eq. (11) now reads

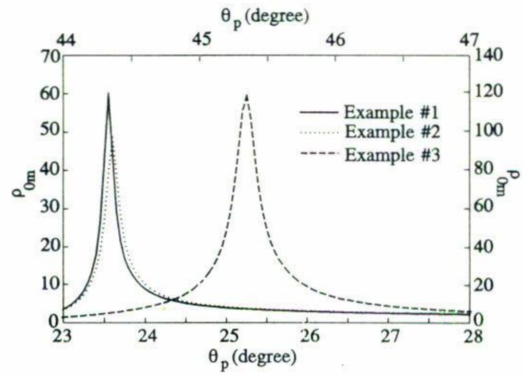

FIGURE 3. The "Fresnel coefficient on reflection" ρ_{0m} at the interface between the air and the metal *versus* the incident angles θ_p. In the results for examples No. 1 and 2, the angle θ_p is measured by the bottom axis, while the "Fresnel coefficient on reflection" ρ_{0m} is measured by the left axis. In the result for example No. 3, the angle θ_p is measured by the top axis, while the "Fresnel coefficient on reflection" ρ_{0m} is measured by the right axis.

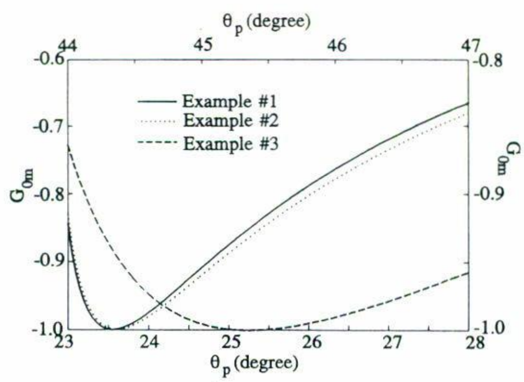

FIGURE 4. The G-factor G_{0m} *versus* the incident angles θ_p. The axes are indicated in the same way as those shown in Fig. 3.

$$R_{p0m} = \frac{1 + \cos 2(\Phi_{p0} - \Phi_{0m})}{1 + \cos 2(\Phi_{p0} + \Phi_{0m})}. \tag{19}$$

As shown in Fig. 5, at the coupling angle the $\cos 2(\Phi_{p0} + \Phi_{0m})$ term is very close to 1, while the $\cos 2(\Phi_{p0} - \Phi_{0m})$ term is very close to -1. This of course gives rise to a minimum in the reflectance. Nevertheless, at the coupling angle even theoretically $\cos 2(\Phi_{p0} - \Phi_{0m})$ does not exactly equal to negative unity, meaning the phase-matching condition only can be approximately fulfilled. This is due to the fact that the tangential component of the wave vector in SPR is complex. This cannot be exactly matched by a real tangential component of the wave vector in the evanescent wave at the base of the prism.

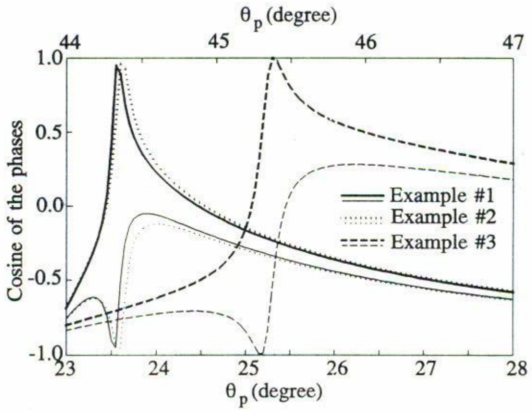

FIGURE 5. The cosine terms in the calculation of the reflectance, Eq. (11). Thick lines: $\cos 2(\Phi_{p0} + \Phi_{0m})$. Thin lines: $\cos 2(\Phi_{p0} - \Phi_{0m})$. The axes are indicated in the same way as those shown in Fig. 3.

4. CONCLUSIONS

Combining the above results and discussions, we have the following conclusions:

First, in the ATR experiments using Otto configuration, total reflection is established when the angle of the incident beam at the base of the prism exceeds the critical angle. Under this condition, evanescent waves are formed near the base at the prism.

Next, when a metal is brought close to the prism base, the evanescent waves can travel across the air gap between the prism and the metal. When the air gap is of the order of the light wavelength, the evanescent waves at the metal surface still have sufficient intensity and will be reflected back to the prism by the metal surface. This reflection, however, has an unusual behavior. Even though the reflection itselt physically exists, the Fresnel coefficient on reflection at the air/metal interface, however does not have any physical meaning, In fact this Fresnel coefficient exceeds unity and does not bear the significance as the "reflection coefficient". Otherwise the energy conservation law will be violated.

The existence of SPR at the air/metal interface requires a special dispersion relation. This relation defines the tangential component of the wave vector in the SPR as a function of the optical constants of both air and the metal. As the complex dielectric constant of the metal is concerned, this tangential component is complex, too. To excite SPR by a p-polarized light, a phase-matching condition must be met. This means the tangential component of the evanescent wave vector near the base of the prism matches that of the SPR. However, since the tangential component of the evanescent wave vector is real, this phase-matching only can be approximately fulfilled. This phase-matching condition in turn determines the coupling angles. It is found that at these coupling angles the reflectance of the incident beam reflected at both the prism/air and air/metal interfaces reaches minimum. Physically, this means that the power of the incident beam is tunneled into SPR at these coupling angles.

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REFERENCES

1. A. Otto, *Z. Physik* **216** (1968) 398.
2. E. Kretschmann and H. Raether, *Z. Naturforschung* **23a** (1968) 2135.
3. E. Kretschmann, *Z. Physik* **241** (1971) 313.
4. H. Raether, in *Physics of Thin Films* **9**, edited by G. Hass, M.H. Francombe, and R.W. Hoffman, (Academic, New York, 1977), p. 145.
5. G.J. Sprokel and J.D. Swalen, in *Handbook of Optical Constants of Solids II*, edited by E.D. Palik, (Academic, Boston, 1991), p. 75.
6. M. Born and E. Wolf, *Principles of Optics*, (Pergamon, Oxford, 1975), p. 62.
7. Haiming Wang, *Optical Materials* **4** (1995) 651.
8. M. Ownner-Petersen, B.-S. Zhu, and E. Dalgaard, *J. Opt. Soc. Am.* **4** (1987) 1741.