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Comparative analysis of Lummer-Gehrcke and Fabry-Perot interferometers

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ABSTRACT. The multiple beam interferometric schemes: Lummer-Gehrcke (LG) and Fabry-Perot, (FP), are analyzed here on the basis of a theoretical scheme which permits an easier way of comparison of the performances of both type of interferometers. We found that LGI as well as the side-illuminated FPI can provide an appreciable gain in the maximum intensity of bright fringes, in fringe contrast and in efficiency in comparison to that for the conventional FPI, even when LGI has a finite number of interfering rays. We also demonstrated that for a given number of reflections inside an interferometric cavity there always exists an optimal reflection coefficient which provides the maximum efficiency (the ratio between a maximum intensity in a bright fringe and the intensity of the incident beam) of an interferometric device.

RESUMEN. Los esquemas interferométricos de haces múltiples, Lummer-Gehrcke (LG) y Fabry-Perot (FP), son analizados sobre la base de un esquema teórico que facilita la comparación del desempeño de estos dos dispositivos. Encontramos que el LG al igual que un FP iluminado lateralmente pueden proporcionar apreciables ganancias en lo tocante a la intensidad pico de las franjas brillantes, en contraste de franjas y en eficiencia en comparación con un FP convencional, aun cuando el LG emplea un número finito de haces. También demostramos que dado un número de reflexiones dentro de la placa, existe un coeficiente de reflectancia óptimo que rinde la mayor eficiencia; definida ésta como la razón entre la intensidad pico y la intensidad incidente en la placa.

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1. INTRODUCTION

The interference of multiple beams of light produced by reflections on dielectric plane parallel plates has contributed significantly to the development of high resolution spectroscopy. The Fabry-Perot etalon [1, 2] and the so called Lummer Plate [3–5] preceded the devices known as the Fabry-Perot and the Lummer Gehrcke interferometers, both developed early in the XX century (see Refs. 6 and 7). The FP is considered one of the most compact high resolution spectroscopes which has found a great many applications in various scientific fields. Besides, it comprises the concepts of resonant cavity and that of interference filter on which the laser is based. The Lummer plate has become obsolete due mainly to the greater flexibility and less difficulty in fabrication of the FP. In addition, the development of the thin film technology has favored decisively the latter. Nevertheless, the interest on this device is alive [8] particularly by its connection with

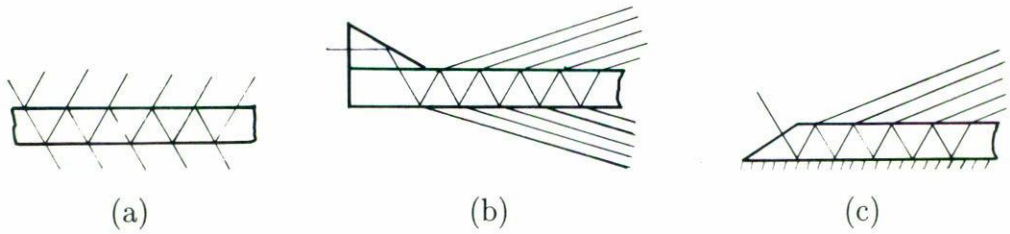


FIGURE 1. Schematic diagram of multiple beam interference in FP (a), and LG (b) and (c) interferometers.

its potential utility in integrated optics systems [9]. Other aspects of the multiple beam interference that have kept current interest can be found in Ref. 10.

Figure 1 describes three ways to produce multiple beams in plane parallel plates. The first one corresponds to an etalon which features two interference patterns: by transmission and by reflection. Since the surfaces possess high reflectances, the first external reflection, particularly strong, contributes to form a pattern with fine dark fringes on a bright background; the transmitted pattern, as is well known, is complementary of the reflection one if the losses are negligible. In both cases the maximum intensity is nearly unitary. In Fig. 1b a Lummer plate is used in the conventional fashion, *i.e.* light is introduced by means of a Herschell wedge, eliminating so the first strong reflection and employing most of the useful radiation to form two identical patterns of the transmission type, one above and one below the plate. It can be shown that in this situation, the peak intensity at the center of a bright fringe is always greater than unity. This is perhaps the reason why the LGI is recognized as being more useful on dealing with a weak source or in conditions of economy of light, than the FPI, where the peak intensity is equal to one at most. Under normal use the LGI requires the internal reflections to occur at incidence angles near the critical angle, in order to achieve reflectances sufficiently high, this reduces the number of interfering beams to a number between 10 to 30 typically. A small number of beams is undesirable for it produces fringe broadening and the appearance of secondary maxima near the main peak which might be interpreted as weak adjacent lines. This does not occur with a FPI for there the number of interfering beams is practically infinite and, consecuently, the relative maxima disappear.

Figure 1c depicts a Lummer plate one of whose faces is fully reflecting. This can be achieved, for example, by incidence with an angle greater than the critical angle of this interface. The other interface is partly reflecting because the incidence occurs at an angle smaller than the corresponding critical angle of that surface. In this fashion, most of the luminous flux available is employed to generate only one pattern above the partly reflecting interface. The later method is more efficient than the cases (a) and (b) for in these cases the available flux is used to produce two redundant patterns with essentially the same amount of information.

In this manuscript a comparison is established between the well known FP and LG interferometers on the basis of a treatment common to both devices which permits an easier way of comparison of their intensity parameters like peak intensity, contrast and resolution.

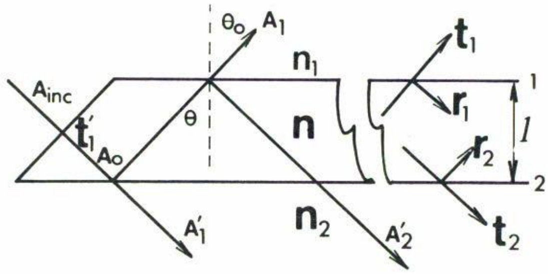


FIGURE 2. Defining the plate parameters.

2. GENERAL THEORETICAL TREATMENT

A theoretical treatment which is common to both LG and FP interferometres is considered in this section. We suppose that a finite size solid plate of width l is positioned between two media. The plate has a refraction index n, while the refraction indices above and below the plate are n_1 and n_2 , correspondingly. We refer to Fig. 2 to define the terms employed. We suppose here that A_{inc} is the amplitude of light in the incident beam before entrance into the plate; A_0 is the amplitude of light just before the first reflection inside the interferometric device; r_1 is the amplitude reflection coefficient inside a device from the upper surface ($n \to n_1$); t_1 is the corresponding amplitude transmission coefficient; t_1' is the amplitude transmission coefficient of the entrance surface; r_2 is the amplitude reflection coefficient inside a device from the lower surface ($n \to n_2$).

From Fig. 2, the interfering beams in the outside plane of surface 1 have the following amplitudes (the amplitudes of the successive rays reflected from the plate are):

$$A_1 = A_0\, r_2\, t_1; \quad A_2 = A_0\, r_2\, (r_1\, r_2)\, t_1 \exp{(i\delta)}\,; \quad A_3 = A_0\, r_2\, (r_1\, r_2)^2\, t_1 \exp{(i2\delta)}\,; \ldots$$
$$A_N = A_0\, r_2\, (r_1\, r_2)^{N-1}\, t_1 \exp{[i\,(N-1)\,\delta]}\,, \tag{1}$$

The transmitted amplitudes are given by

$$A_1' = A_0\, t_2; \quad A_2' = A_0\, (r_2\, r_1)\, t_2 \exp{(i\delta)}\,; \quad A_3' = A_0\, (r_2\, r_1)^2\, t_2 \exp{(i2\delta)}\,; \ldots$$
$$A_N' = A_0\, (r_2\, r_1)^{N-1}\, t_2 \exp{[i\,(N-1)\,\delta]}\,. \tag{2}$$

Here the phase difference between the two consecutive beams is given by

$$\delta = \frac{2\pi}{\lambda}\frac{2l}{\cos\theta} + \Phi, \tag{3}$$

where λ is the wave length of light, θ is an incident angle inside a device, Φ is the change of phase at the reflection (if a change takes place).

The amplitude of a reflected resultant wave, which is a superposition of waves Eq. (1), is readily found to be equal to

$$A = \sum_{m=1}^{N} A_m = A_0\, r_2\, t_1\, \frac{1 - (r_1\, r_2)^N \exp{(iN\delta)}}{1 - (r_1\, r_2) \exp{(i\delta)}}, \tag{4}$$

where N is a number of interfering beams. The intensity distribution, which corresponds to the wave Eq. (4), is as follows:

$$I = |A|^2 = I_0\, r_2^2\, t_1^2\, \frac{\left[1-(r_1\,r_2)^N\right]^2 + 4\,(r_1\,r_2)^N \sin^2(N\delta/2)}{(1-(r_1\,r_2))^2 + 4\,(r_1\,r_2)\sin^2(\delta/2)} \tag{5}$$

For a transmitted wave amplitude and intensity distributions we have from Eq. (2) the following formulae:

$$A' = \sum_{m=1}^{N} A'_m = A_0\, t_2\, \frac{1-(r_1\,r_2)^N \exp(iN\delta)}{1-(r_1\,r_2)\exp(i\delta)}, \tag{6}$$

$$I' = |A'|^2 = I_0\, t_2^2\, \frac{\left[1-(r_1\,r_2)^N\right]^2 + 4\,(r_1\,r_2)^N \sin^2(N\delta/2)}{(1-(r_1\,r_2))^2 + 4\,(r_1\,r_2)\sin^2(\delta/2)}. \tag{7}$$

We note that, up to the moment, the scheme and metodology (even the final formulas) of calculation of multiple beam interference were identical for both LG and FP interferometers. The difference appears only in treating the transmission coefficient t'_1, of the entrance surface which depends on how an incident beam enters the device in each particular case, and in the number of interfering beams N. The N is supposed to be very large ($N \to \infty$) for FP devices, while N is supposed to be finite for LG ones, especially when one wants to exploit the total inner reflection. In addition, a side illuminated FPI can be considered as a LGI with entrance surface having the transmission coefficient $t'_1 = 1$.

3. Lummer-Gehrcke based interferometry

First, we consider a LG device with total internal reflection (TIR) from the lower surface of the plate.[1] Thus, the reflection coefficient $r_2 = 1$, and the number of reflections inside the plate being N. The amplitude of light before the first reflection inside the plate is equal to $A_0 = A_{\text{inc}}\, t'_1$, where A_{inc} is the amplitude of an incident beam, and t'_1 is an amplitude transmission coefficient of the inclined surface, and we observe an interference pattern in reflected light. (We take $r_1 \equiv r$).

[1]The idea of increasing the reflectance of one surface of the LGI is not new. As is known, during the first decades of the century, a number of researchers [11–14], used metallic coatings for this aim. It was then recognized [15], that a substantial improvement in peak intensity and contrast in the interferograms was attainable under this modification. Such coatings in the best of cases produced appreciable losses by absorption and the reflectances attained did not exceeded 96%. Substantially improved multilayer dielectric coatings capable of rendering substantially greater reflection coefficient with minimum losses have never been used for this purpose, to our knowledge. Total internal reflection, on the other hand, guarantees ideal reflectance without any coating whatsoever.

In order to have TIR from the surface 2 we have to satisfy one of the following conditions: (1) $n > n_1 > n_2$ (this is the most realistic condition for LG); (2) $n_1 > n > n_2$. If the first condition is valid, the angle of incidence is restricted to change in the range

$$\arcsin \frac{n_2}{n} = \theta_{2(C)} < \theta < \theta_{1(C)} = \arcsin \frac{n_1}{n}. \tag{8}$$

The conditions required for TIR practically restrict the number $N \leq 40$.

The complex amplitude of the resultant wave[2] in the plane of surface 1 we obtain from Eq. (4), taking $r_2 = 1$

$$A = \sum_{m=1}^{N} A_m = A_0\, t_1 \frac{1 - r_1^N \exp(iN\delta)}{1 - r_1 \exp(i\delta)}. \tag{9}$$

The intensity distribution is given by

$$I = |A|^2 = I_0\, t_1^2 \frac{\left(1 - r_1^N\right)^2 + 4r_1^N \sin^2(N\delta/2)}{\left(1 - r_1\right)^2 + 4r_1 \sin^2(\delta/2)}. \tag{10}$$

In order to find the extrema of the intensity distribution we differentiate Eq. (10) with respect to δ, and (supposing that t_1 does not depend on the angle of incidence θ or δ, we discard this small dependence) making the result equal to zero:

$$I'(\delta) = I_0\, t_1^2 \left[2r_1^N N \sin N\delta \left(1 + r_1^2 - r_1 2\cos\delta\right) - \left(1 + r_1^{2N} - r_1^N 2\cos N\delta\right) 2r_1 \sin\delta\right] \times \left(1 + r_1^2 - r_1 2\cos\delta\right)^{-2} = 0.$$

From this equation we find that at $\delta_{\max} = \pi 2k$ $(k = 0, \pm1, \pm2, \dots)$ the intensity takes on its (absolute) maximum values

$$I_{\max(\text{abs})}(N, r_1) = I_0\, t_1^2 \frac{\left(1 - r_1^N\right)^2}{\left(1 - r_1\right)^2} = I_{\text{inc}}\, t_1'^2\, t_1^2 \frac{\left(1 - r_1^N\right)^2}{\left(1 - r_1\right)^2} = I_{\text{inc}}\, t_1'^2 f(N, r_1), \tag{11}$$

and at $\delta_{\min} = \pi(2k+1)$ $(k = 0, \pm1, \pm2, \dots)$ the intensity takes on its (absolute) minimum values

$$I_{\min(\text{abs})}(N, r_1) = I_0\, t_1^2 \frac{\left(1 + r_1^N\right)^2}{\left(1 + r_1\right)^2} = I_{\text{inc}}\, t_1'^2\, t_1^2 \frac{\left(1 + r_1^N\right)^2}{\left(1 + r_1\right)^2}. \tag{12}$$

[2] A rigorous analysis of the fringe profile function in a LGI must take into consideration diffraction effects. Kolacek [16], in a quite thorough study of this interferometer pointed out that the light leaving the plate is better described as diffracted rather than refracted. Light leaving the plate does not obey strictly Snell's law since it is a wave propagating over an angular interval centered precisely in this particular direction. However, he demonstrated that if the external angle of refraction θ in Fig. 2 is defined as the diffraction angle, the path of light inside the plate is close enough to that predicted by Snell's law so that the error is negligible and thus justifies the simplified treatment provided by geometrical optics.

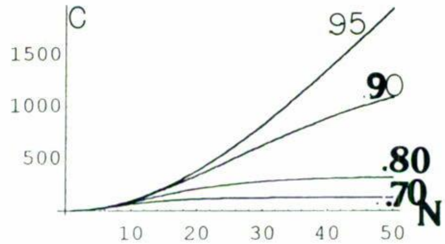


FIGURE 3. The contrast function $C(N, R)$ of LGI *vs.* N at a given R (= 0.7, 0.8, 0.9, 0.95).

In Eq. (11) we introduced the function

$$f(N, r_1) = (1 + r_1) \frac{\left(1 - r_1^N\right)^2}{1 - r_1}, \tag{13}$$

where the relation $t_1^2 + r_1^2 = 1$ has been used.

From Eqs. (11) and (12) we find the contrast function as follows:

$$C(N, r_1) = \frac{I_{\max(\text{abs})}(N)}{I_{\min(\text{abs})}(N)} = \frac{(1 + r_1)^2}{(1 - r_1)^2} \frac{\left(1 - r_1^N\right)^2}{\left(1 + r_1^N\right)^2} = C(\infty, r_1) \frac{\left(1 - r_1^N\right)^2}{\left(1 + r_1^N\right)^2}, \tag{14}$$

where we introduce the contrast function $C(\infty, r_1)$ for an infinite number of interfering rays, when $r_1 < 1$ and $N \to \infty$.

The other characteristic of an interference pattern, which is of practical interest, is the ratio of $I_{\max(\text{abs})}(N, r_1)$ to the intensity of incident light I_{inc}. We call it the efficiency function $E(N, r_1)$. In the considered case, we have the following relations between the amplitudes and coefficients of reflection and transmission: $A_0 = A_{\text{inc}}\, t_1'$, $t_1^2 + r_1^2 = 1$, $I_0 = |A_0|^2 = I_{\text{inc}}\, t_1'^2$. Thus, the efficiency of LGI with TIR from the lower surface is equal to

$$E(N, r_1) = \frac{I_{\max(\text{abs})}(N)}{I_{\text{inc}}} = t_1'^2 (1 + r_1) \frac{\left(1 - r_1^N\right)^2}{1 - r_1} \equiv t_1'^2 f(N, r_1). \tag{15}$$

We note that the contrast of an interference pattern depends only on the number of interfering beams N and on the reflection coefficient r_1 of the surface 1 from inside an interferometric device, for both LG and FP schemes [see Eq. (14)]. It does not depend on the conditions at the entrance of a device, in other words, of $I_{\text{inc}}\, t_1'^2$. But, both the brightness $[I_{\max(\text{abs})}(N, r_1)$ and $I_{\min(\text{abs})}(N, r_1)]$ and the efficiency $E(N, r_1)$ in fringes depend also on the conditions at the entrance of the device.

In the considered case of LG device with a total internal reflection, we suppose that the number of interfering beams does not exceed 50. The plots of the contrast $C(N, r_1)$ as a function of N at given values of $R = r_1^2 = (0.7, 0.8, 0.9, 0.95)$ have clearly demonstrated that $C(N, r_1)$ is a monotonically increasing function of N (see Fig. 3).

The plots of $f(N, r_1)$ in Fig. 4 demonstrate that $E(N, r_1)$ is also a monotonically increasing function of N at a given value of $R = r_1^2 = (0.7, 0.8, 0.9, 0.95, 0.99, 0.999)$,

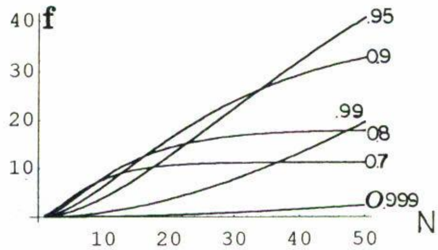


FIGURE 4. The function $f(N, R)$, Eq. (13), of LGI *vs.* the number reflections $N < 50$ at different values of the reflection coefficient $R = 0.7, 0.8, 0.9, 0.95, 0.99, 0.999$.

but when $R \to 1$ the value of $E(N, r_1)$ at a given N tends to 0. The etalon becomes a waveguide in the considered limit case and does not permit light to go out of the gap. It is easy to show that

$$\lim_{r\to 1} f\left(N, r\right) = \lim_{r\to 1} \frac{\left(1 - r^N\right)^2}{1 - r} = 0, \tag{16}$$

when N is a given number. This leads to an interesting behavior of $E(N, r_1)$ as a function of R at a given number N. We see that each plot has a maximum (see Fig. 5a for $N = 30$, 40, 50, 100), and a value of the maximum increases with N. For $N = 50$ the maximum value is about 40. This behavior $E(N, r_1)$ gives a possibility of optimization of LG for a given number N. The optimal value of the reflection coefficient is readily obtained from the equation

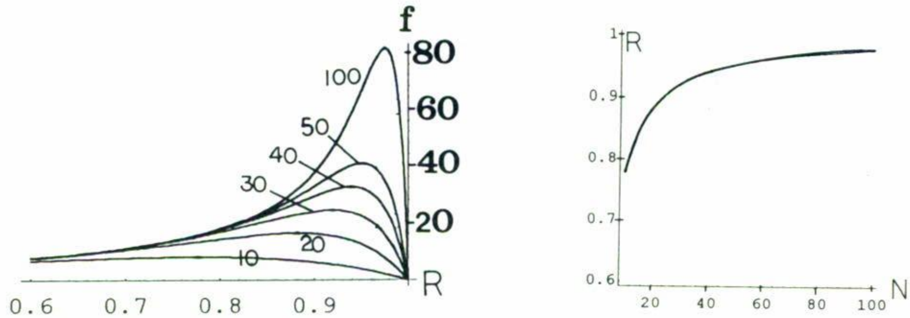
$$\frac{\partial f\left(N, r\right)}{\partial r} = 2\frac{1 - r^N}{1 - r}\left[\frac{1 - r^N}{1 - r} - N r^{N-1}\left(1 + r\right)\right] = 0. \tag{17}$$

Figure 5b shows the optimal reflection coefficient as a function of N.

One of the fundamental characteristics of interferometric devices is the finesse of fringes which finally determines the resolution of spectrometers. Using the above results we can estimate the finesse of LGI as a function of N and R, and compare it with that for the conventional FPI. There are few definitions of finesse [6], and we use here the following one: $F = (\pi/\Delta)$, where Δ is a half-height half-width of a bright fringe. In other words the finesse is defined as the ratio of a separation between two consecutive maxima (which is equal to 2π) to a half height width of a bright fringe. From Eqs. (10) and (11) we find the following implicit equation for the finesse of LGI with one perfectly reflecting surface:

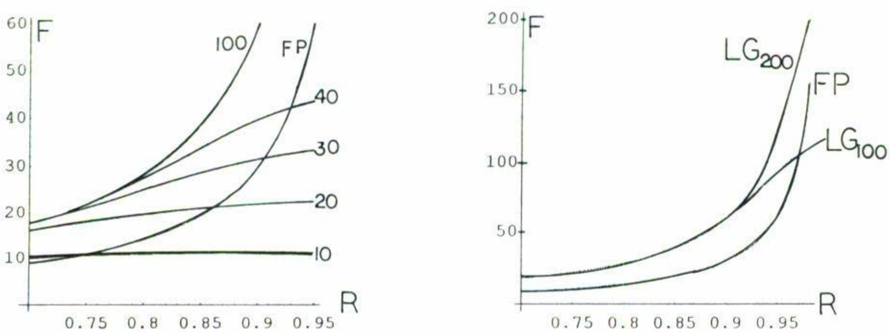
$$\left(1 - R^{N/2}\right)^2 \left[\left(1 - R^{1/2}\right)^2 + 4R^{1/2}\sin^2\left(\frac{\pi}{2F}\right)\right] = 2\left(1 - R^{1/2}\right)^2 \left[\left(1 - R^{N/2}\right)^2 + 4R^{N/2}\sin^2\left(\frac{N\pi}{2F}\right)\right], \tag{18}$$

where $r_1 \equiv r$, $r^2 = R$. The last equation can be also used to calculate the finesse of a side illuminated FPI.



(a) (b)

FIGURE 5. (a) The function $f(N, R)$ of LGI *vs.* the reflection coefficient R at a given number of reflections $N(= 10, 20, 30, 40, 50, 100)$. (b) The optimal value of the reflection coefficient R for different numbers of reflection N inside LGI.



(a) (b)

FIGURE 6. (a) Finesse of LG (N = 10, 20, 30, 40, 100) and FP interferometers (LGI or side illuminated FPI compared to the conventional FPI). (b) Finesse of LG (N = 100, 200) and FP interferometers.

The finesse of a conventional FPI is readily found from Eqs. (7) and (24) (taking the limit $N \to \infty$ and $r_1 = r_2 = r$, $r^2 = R$) to satisfy the well known relation

$$(1 - R)^2 = 4R \sin^2\left(\frac{\pi}{2F}\right). \tag{19}$$

Figure 6a shows the finesse as a function of the reflection coefficient R in the case of a conventional FPI and in the case of LGI with one totally reflecting surface(or the equivalent side illuminated FPI) for different number of reflections N = 10, 20, 30, 40, 100. Comparing the plots we find that for a given number of reflections $N > 10$ there always exists a value of the reflection coefficient such that below this value a finesse of LGI is higher than that of a conventional FPI. The larger is N, the wider is a range of R

where the finesse of a LG is greater than that of the FP. Fig. 6b shows that the finesse of LGI with 100 reflections is higher than that of the conventional FPI in the range of $R < 0.97$.

It is possible that the number of interfering beams be very large when the angle of incidence is small. This condition can be met when the lower surface obtains the desired reflectance by use of multilayer dielectric coating to achive a reflectance approaching unity, $r_2 = 1$. When, at a given $R < 1$, the number $N \to \infty$, the Eqs. (11), (12), (14) and (15) provide us the following results:

$$I_{\max(\mathrm{abs})}(\infty, r_1) = I_0\, t_1^2 \frac{1}{(1-r_1)^2} = I_{\mathrm{inc}}\, t_1'^2 \frac{1-r_1^2}{(1-r_1)^2} = I_{\mathrm{inc}}\, t_1'^2 \frac{1+r_1}{1-r_1} > I_{\max(\mathrm{abs})}(N, r_1), \tag{20}$$

$$I_{\min(\mathrm{abs})}(\infty, r_1) = I_0\, t_1^2 \frac{1}{(1+r_1)^2} = I_{\mathrm{inc}}\, t_1'^2 \frac{1-r_1}{1+r_1} < I_{\min(\mathrm{abs})}(N, r_1), \tag{21}$$

$$C(\infty, r_1) = \frac{I_{\max(\mathrm{abs})}(\infty, r_1)}{I_{\min(\mathrm{abs})}(\infty, r_1)} = \frac{(1+r_1)^2}{(1-r_1)^2} > C(N, r_1), \tag{22}$$

$$E(\infty, r_1) = \frac{I_{\max(\mathrm{abs})}(\infty, r_1)}{I_{\mathrm{inc}}} = t_1'^2 \frac{1+r_1}{1-r_1} > E(N, r_1). \tag{23}$$

These set of equations shows us a better performace of LGI with $N \to \infty$ as compared to the case of LGI with the finite number of reflections.

4. Fabry-Perot based interferometry

There are three cases we will consider: (a) the conventional FPI, (b) a side illuminated FPI, (c) a side illuminated FPI with one surface totally reflecting.

The conventional FPI is specified by the following parameters: $r_1 = r_2 \equiv r < 1$ and $N \to \infty$, $A_0 = A_{\mathrm{inc}}\, t_1'$, here $t_1 = t = t_2$ and $t_1' = t_2' (= t)$, and $t^2 + r^2 = 1$, $R = r^2$. Using Eq. (7) at $N \to \infty$ we can easily find, for an interference pattern in transmitted light, the well known relations [6]

$$I'_{\max(\mathrm{abs})}(\infty, R) = I_0\, t_2^2 \frac{1}{(1-R)^2} = I_{\mathrm{inc}}\, t_1'^2\, t^2 \frac{1}{(1-R)^2} = I_{\mathrm{inc}}\, t_1'^2 \frac{1}{1-R} = I_{\mathrm{inc}}, \tag{24}$$

$$I'_{\min(\mathrm{abs})}(\infty, R) = I_0\, t_2^2 \frac{1}{(1+R)^2} = I_{\mathrm{inc}}\, t_1'^2 \frac{1-R}{(1+R)^2} = I_{\mathrm{inc}} \left(\frac{1-R}{1+R}\right)^2, \tag{25}$$

$$C(\infty, R) = \frac{I'_{\max(\mathrm{abs})}(\infty, R)}{I'_{\min(\mathrm{abs})}(\infty, R)} = \left(\frac{1+R}{1-R}\right)^2, \tag{26}$$

$$E(\infty, R) = \frac{I'_{\max(\mathrm{abs})}(\infty, R)}{I_{\mathrm{inc}}} = t_1'^2 \frac{1}{1-R} = 1. \tag{27}$$

A comparison of Eqs. (11), (20), (24), Eqs. (14), (22), (26), and Eqs. (15), (23), (27) leads to the conclusion that LGI can provide an appreciable gain in maximum intensity of the bright fringes, in contrast and in efficiency as compared to that for a conventional FPI, even when a LGI has a finite number of interfering rays.

As is well known, the FPI interference pattern in transmitted light consists of bright fine fringes in a dark background. Reflected light produces the complementary pattern, namely, fine dark fringes on a bright background, this is particularly due to the first external reflection suffered by the incident beam which is relatively strong. For practical reasons the transmitted pattern is preferred over the reflection one because it is less noisier with regard to the detection system.

The patterns in a conventional FPI are substantially different when the first external reflection is avoided. The incoming light can be admited into the interferometer with so called side illumination. A side illuminated FPI is conceptually equivalent to a LG, the only difference between them is that a medium between reflecting planes is a solid in a LGI while it is usually air in the case of FPI. Thus, an interference pattern of a side illuminated FPI observed in reflected light (which is very similar to that in a transmitted light) is described by Eqs. (4) and (5), where we have to take $A_0 = A_{\mathrm{inc}}$ and $I_0 = I_{\mathrm{inc}}$. In the case of transmitted light pattern we can use Eqs. (6) and (7), substituting $A_0 = A_{\mathrm{inc}}$ and $I_0 = I_{\mathrm{inc}}$ in it.

In the case of a side iluminated FPI with $r_1 < 1$, $r_2 = 1$ and $N \to \infty$, an interference pattern observed in reflected light is described by Eqs. (20)–(23), where we have to take $t_1'^2 = 1$, *i.e.*, $A_0 = A_{\mathrm{inc}}$ and $I_0 = I_{\mathrm{inc}}$.

We note that a large reflection angle implies necessarily a small number of reflections, wheras a small angle permits us to use a larger number of interfering beams. Now, in a side illuminated FPI or LGI, the angle needs to be small to have N as large as possible but this means that the overlapping of the beams is of small cross section. Therefore, a side illuminated interferometer, in the case $N \to \infty$, be it a LGI or a FPI, have an inefficient employment of the incident beam. The reason is that because of a small angle of incidence, needed to provide multiple (infinite) reflections inside an interferometer, one permits only a small part of the incident beam to enter inside it, and only this part further contributes to form an interference pattern. The major part of the incident beam is lost.

5. CONCLUSIONS

The two basic interferometric multiple beam schemes of optics, Lummer-Gehrcke and Fabry-Perot, have been analyzed here on the basis of theoretical scheme which permits a similar consideration for both type of interferometric schemes. We demonstrated that a LGI with one totally reflecting surface as well as a side-illuminated FPI with one totally reflecting surface can provide an appreciable gain in maximum intensity of bright fringes, in contrast and in efficiency, in comparison to that for the conventional FPI, even when a LGI has a finite number of interfering rays. We also demonstrated that for a given finite number of reflections inside an interferometric cavity there always exists an optimal reflection coefficient which provides the maximum efficiency (the ratio between a maximum intensity in a bright fringe and an intensity of incident beam) of an interferometric device.

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