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Flavour changing neutral tensor currents contributions to the weak magnetic moment of leptons

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Contributions from nonstandard neutral vector bosons to the weak magnetic moment of leptons are calculated within the context of models where violation of lepton family number is allowed through tensor couplings.

Keywords: Neutral bosons, weak magnetic moment

Se calculan las contribuciones de bosones vectoriales neutros no estandar al momento débil magnético de leptones en modelos con violación del número leptónico a través de acoplamientos tensoriales.

Descriptores: Bosones neutros, momento débil magnético

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Many extensions of the standard model (SM) of electroweak unification, predicts the existence of extra neutral vector bosons, whose couplings to fermions (standard or new ones) may violate family number. In some of these models an admixture of the standard Z boson with an extra $\widetilde{Z}$ can induce, at the tree level or at the one loop, the decay $Z \to e\mu$ [1]. The observation of such decays at LEP [2] would then be an indication of physics beyond SM. Also, in grand unified models it is predicted the existence of very heavy particles bearing both lepton and baryon number, named leptoquarks. Recent results at HERA seems to indicate the possibility of both types of particles.

The existence of $\widetilde{Z}$ can give contributions to some other observable. The purpose of this paper is to report results for their contributions to the weak magnetic moment of leptons. The weak magnetic moment is the analogous to the magnetic moment in QED, and can, in principle, be measured [3] in the reaction $e^+e^- \to Z \to \tau^+\tau^-$. There, a measurement of the polarization and spin correlation of the final state can be performed and information on the weak magnetic moment can be extracted, including an anomalous contribution from radiative corrections. It has been also shown [1] that the transverse and normal components of the polarization of the τ-pair at e^+e^- unpolarized collision are sensitive to the anomalous weak magnetic moment of the τ.

The most general lepton flavour violating (LFV) interaction between leptons ℓ_i and ℓ_j and the $\widetilde{Z}$ is

$$L_{ij} = - ig_{\widetilde{Z}}\overline{\ell}_i\gamma_\mu(a^{ij} + b^{ij}\gamma_5)\ell_j\widetilde{Z}^\mu + g_{\widetilde{Z}}\overline{\ell}_i\sigma_{\mu\nu}\frac{k^\nu}{M_{\widetilde{Z}}}(A^{ij} + B^{ij}\gamma_5)\ell_j\widetilde{Z}^\mu + h.c., \quad (1)$$

where k^ν is the $\widetilde{Z}$ boson 4-momentum. If the model extending SM is gauged the terms a^{ij} and b^{ij} are well defined. The terms A^{ij} and B^{ij} can arise by extra effective operators in the Lagrangian, like $O_e = (\overline{\ell}\sigma_{\mu\nu}e)\phi B^{\mu\nu}$, which respect the SU(3)×SU(2)×U(1)symmetry, where ϕ is the Higss field, and $B^{\mu\nu}$ is the tensor associated to the U(1) group [4]. In general they are unknown, although some limits can be extracted from low energy data. Results for the contributions from the a^{ij} and b^{ij} couplings were reported in Ref. 5. Here we concentrate in the A^{ij} and B^{ij} couplings. In order to be as simple as possible we will assume no tensor $Z\ell_i\ell_j$ couplings, since, as shown in Ref. 4, they can occur if the scale of new interactions is very large ($\Lambda \gg 10^4$ TeV) and we assume no new additional heavy fields are present. New dynamics at the Λ scale is likely associated with heavy fields carrying non-zero lepton number. We will consider $\Lambda \sim 1$ TeV. Then, going one step further we can consider models like the one in reference [6], including this extra term.

The diagram in Fig. 1 gives the contribution from the tensor LFV interactions coming from Eq. (1), represented by a dot. Then, the amplitude is given by

$$\mathcal{A} = \frac{g_Z g_{\widetilde{Z}}^2}{M_{\widetilde{Z}}^2}\varepsilon_\mu(q)\mathcal{M}^\mu, \quad (2)$$

where

$$\mathcal{M}^\mu = \overline{u}(p_2)(A^{ij} + B^{ij}\gamma_5)I^\mu(A^{ij} + B^{ij}\gamma_5)u(p_1) \quad (3)$$

and

$$I^\mu = \int \frac{d^4k}{(2\pi)^4}\sigma^{\rho\beta}(k - \frac{p_+}{2})_\rho S_F(k + \frac{q}{2})\gamma^\mu(a + b\gamma_5) \times S_F(k - \frac{q}{2})(k - \frac{p_+}{2})\sigma^{\alpha\nu}\Delta_{\alpha\beta}(-k + \frac{p_+}{2}). \quad (4)$$

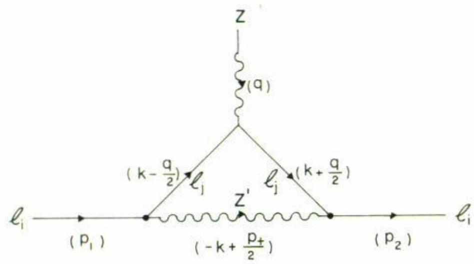

FIGURE 1. $\widetilde{Z}$ exchange diagram contributing to the weak magnetic dipole moment of lepton ℓ_i. The dot represent a lepton flavour violating tensor interaction vertex. Four-momenta are indicated in parentheses.

We have denoted by a and b (A^{ij} and B^{ij}) the vector and axial-vector (tensor and axial-tensor) couplings to the leptons with strength g_Z ($g_{\widetilde{Z}}$). S_F and Δ are the propagators of leptons and vector boson in the loop. $\varepsilon_\mu(q)$ is Z boson polarization vector, with momentum $q = p_2 - p_1$, and p_+ is defined as $p_1 + p_2$. Proceeding as usual we have to extract the terms of the form $\overline{u}(p_2)i\sigma_{\mu\nu}q^\nu u(p_1)\varepsilon^\nu(q)$. After the standard straightforward algebra we obtain

$$\mathcal{A} = \frac{g_Z}{2m_i}\overline{u}(p_2)i\sigma_{\mu\nu}q^\nu u(p_1)\varepsilon^\nu(q)F_{2W}(q^2), \tag{5}$$

where

$$F_{2W}(q^2) = \frac{g_{\widetilde{Z}}^2}{4\pi^2 M_{\widetilde{Z}}^2}\sum_j\left\{\left[a\Big((A^{ij})^2 + (B^{ij})^2\Big)\left(\frac{7}{6}m_i^2 - \frac{m_i m_j}{2}\right) + \frac{7}{3}bm_i^2 A^{ij}B^{ij}\right]\left(\ln\frac{\Lambda^2}{M_{\widetilde{Z}}^2} - \frac{3}{2}\right) + a\Big((A^{ij})^2 + (B^{ij})^2\Big)J_+(q^2) - 2bA^{ij}B^{ij}J_-(q^2)\right\}, \tag{6}$$

and

$$J_+(q^2) = m_i^2\int_0^1 dx\int_0^{1-x}dy\left\{\left[-1 + \frac{7}{2}(x+y) + 2(x+y)^2 + \frac{-8+9(x+y)}{2}\frac{m_j}{m_i}\right]I_2 + \Big[(x+y)^2m_i^2 - xyq^2\Big]\left[(1-x-y)(4-x-y) + 3(x+y)\frac{m_j}{m_i}\right]I_0 + 2xyq^2I_0\right\}, \tag{7}$$

$$J_-(q^2) = m_i^2\int_0^1 dx\int_0^{1-x}dy\left\{\left[-1 - \frac{7}{2}(x+y) + 2(x+y)^2 - \frac{3(x-y)}{2}\frac{m_j}{m_i}\right]I_2 + \left[(x+y)^2m_i^2 - xyq^2\right]\left[(1-x-y)(4-x-y) + 3(x+y)\frac{m_j}{m_i}\right]I_0 + 2\left[xyq^2 - (x+y)^2m_j^2\right]I_0\right\}, \tag{8}$$

with

$$I_0(q^2) = -\frac{1}{2}\left[(x+y)^2m_i^2 - (x+y)(m_i^2 - m_j^2) - xyq^2 + (1-x-y)M_{\widetilde{Z}}^2\right]^{-1}, \tag{9}$$

$$I_2(q^2) = \ln\frac{(x+y)^2m_i^2 - (x+y)(m_i^2 - m_j^2) - xyq^2 + (1-x-y)M_{\widetilde{Z}}^2}{M_{\widetilde{Z}}^2}. \tag{10}$$

Here m_i and m_j are the masses of the leptons ℓ_i and ℓ_j; Λ is an ultraviolet cutoff, introduced to ensure convergence of the Feynman loop integrals, and represents a mass scale.

To get an estimation of $g_{\widetilde{Z}}^2((A^{ij})^2 + (B^{ij})^2)$ we will consider its contribution to the muon anomaly [6], where $q^2 = 0$ and integrals in Eqs. (7) and (8) can be easily done. For $\Lambda^2 \gg M_{\widetilde{Z}}^2 \gg m_i^2, m_j^2$ we obtain

$$J_+(0) = \frac{1}{2}m_i m_j\left[\frac{M_{\widetilde{Z}}^4 - 9M_{\widetilde{Z}}^2m_j^2 + 2m_j^4}{2(M_{\widetilde{Z}}^2 - m_j^2)^2} - \ln\frac{\Lambda^2}{M_{\widetilde{Z}}^2} + \frac{3}{2} - \frac{M_{\widetilde{Z}}^4 + 3M_{\widetilde{Z}}^2m_j^2 - m_j^4}{(M_{\widetilde{Z}}^2 - m_j^2)^3}m_j^2\ln\frac{m_j^2}{M_{\widetilde{Z}}^2}\right]. \tag{11}$$

We note that our result in Eq. (11) agrees with the result in Ref. 6 for the logarithmic terms, but disagrees in the other. This is due to the different regularization method to render the integrals finite. However, since the main contribution comes from the $\ln(\Lambda^2/M_{\widetilde{Z}}^2)$ both results agree, as should be. Since here $b = 0$ we get that the contribution to the anomalous magnetic moment of leptons is given by

$$F_2(0) = \frac{g_{\widetilde{Z}}^2}{4\pi^2 M_{\widetilde{Z}}^2}\sum_j\left[(A^{ij})^2 + (B^{ij})^2\right]J_+(0). \tag{12}$$

For Λ =1 TeV the values of $J_+(0)/(4\pi^2 M_{\widetilde{Z}}^2)$ are in the interval $(-7.5$ to $+1.17) \times 10^{-7}$ when $100 \leq M_{\widetilde{Z}} \leq 450$ in GeV.

Then comparing with the experimental result we obtain

$$\left|g_{\bar{Z}}^2\left[(A^{ij})^2+(B^{ij})^2\right]\right|\leq(0.26\text{ to }1.7)\times10^{-2} \quad (13)$$

(for $\Lambda=10$ TeV the above result is to be multiplied by a factor of 3). The largest contribution coming from the heaviest fermion in the loop. This is the τ-lepton in the three family version of SM.

Since we cannot obtain the separate bounds for $A^{\mu j}$ and $B^{\mu j}$, we will assume they contribute for equal. The evaluation of $J_{\pm}(M_Z^2)$ shows that they are complex function of $R=M_Z^2/M_{\bar{Z}}^2$. The imaginary part is denoted as the absorptive contribution [7], which in this case is one or two orders of magnitude smaller than the real part. So we will neglect it. (The appearance of this imaginary part, absent in the $q^2=0$ case, is due to the fact that we are evaluating on the Z mass shell). Furthermore, within the same interval of values, $\mathrm{Re}\,J_-(M_Z^2)$ is greater than $\mathrm{Re}\,J_+(M_Z^2)$, leading us to the result

$$F_{2W}(M_Z^2)\simeq2g_{\bar{Z}}^2(A^{\mu\tau})^2\left[\frac{m_\mu m_\tau}{4\pi^2M_{\bar{Z}}^2}\left(\ln\frac{\Lambda^2}{M_{\bar{Z}}^2}-\frac{3}{2}\right)+\mathrm{Re}\,J_-(M_Z^2)\right]. \quad (14)$$

Finally, the expression in brackets in this last equations turn out to be $\sim 10^{-7}$, and then we obtain for the weak magnetic nomaly of the muon

$$\left|F_{2W}(M_Z^2)\right|\leq(0.26\text{ to }1.7)\times10^{-9}. \quad (15)$$

Having in mind the possibility of measuring the analogous result for the τ lepton at LEP, we only have to interchange the role of m_μ and m_τ, showing that they are of the same size. The result in Eq. (15) is two or three orders of magnitude smaller than the radiative corrections, to one loop, in SM. Thus to see effects of the class of interactions considered in this work, it is required a better experimental precision.

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