Radiation emission in inhomogeneous media: dipole between two plane-parallel mirrors

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We present a classical method for the calculation of the total power radiated by a dipole between two perfect mirrors. We use the modal theory developed by Glauber and Lewenstein [1] and extend the calculations of Dowling and Bowden [2], to take into account radiation in all directions. This approach simulates spontaneous power emission by an atom. The method is suitable for understanding how the dipole couples to the allowed medium modes and how environment affects the power emission. Results are presented for the power radiated by a dipole placed between two plane-parallel mirrors and in front of a single mirror for all dipole orientations. We also calculate the power radiated by a gas of noninteracting excited atoms in the cavity formed by the mirrors.

Keywords: Electromagnetic modal theory; inhomogeneous media; metallic cavity

Presentamos un método clásico para el cálculo de la potencia total radiada por un dipolo oscilante entre dos espejos metálicos perfectos. Usamos el método modal desarrollado por Glauber y Lewenstein [1] y extendemos los cálculos de Dowling y Bowden [2], tomando en cuenta radiación en todas direcciones. Esta aproximación simula la emisión espontánea de potencia de un átomo excitado. El método es también útil en el entendimiento de cómo el dipolo acopla los modos permitidos en el medio y cómo el ambiente afecta la emisión de potencia. Presentamos resultados para la potencia radiada por un dipolo puesto entre dos espejos paralelos planos y enfrente de un solo espejo, tomando en cuenta todas las posibles orientaciones del dipolo. También calculamos la potencia radiada por un gas de átomos que no interactúan entre ellos dentro de la cavidad formada por los espejos.

Descriptores: Teoría electrónica modal; medios inhomogeneos; cavidades metálicas

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1. Introducción

Since Purcell [3] showed that spontaneous emission can be enhanced or suppressed, there has been a lot of interest in this topic. Some of these works solve the problem using the image charge method [4, 5] and calculate the power emitted by a dipole inside a cavity, considering the interaction of the dipole with the fields reflected by the walls of the cavity. Meschede [5] reviews the main results of spontaneous emission in metallic cavities until 1992. Kleppner [6] and Rippin and Knight [7] present results on the decay of two-level atoms (TLA) in cylindrical metallic cavities. Other works [8] use the method of self energies to calculate de radiation decay rate of an atom taking into account the energy shift of the levels due to the presence of the medium. A recently published work [9] deals with spontaneous emission in cavities with a so-called photonic-well which is a small dent in one of the cavity mirrors. Experiments on spontaneous emission in microcavities have been done by DeMartini *et al.* [10, 11], whose measurements of the radiated power demonstrate a significant decay reduction of up to 25 percent with respect to the power emitted in free space. In those experiments, the spontaneous emission was measured by means of the Eu-dibenzoylmethane emission linewidth in a tunable microcavity.

Glauber and Lewenstein [1] develop a theory for the calculation of the decay constant for an initially excited TLA when it is placed in a medium with inhomogeneities. This calculation basically solves the problem by finding the coupling of a TLA with one vacuum field mode and by performing the summation over all the allowed modes present in the medium. They employ two quantization schemes, and they calculate the decay of the upper level population using the Wigner-Weisskopf approximation. Although spontaneous emission enhancement and reduction seem more likely to be explained in quantum mechanical terms, Dowling and Bowden [2] proved that such modifications are a purely classical effect. In their work they solve the wave equation with the radiated fields expressed as a superposition of the allowed modes given by the Helmholtz equation. They calculate the power radiated by the dipole from the work done on it by the fields present in the medium. The power result is essentially the same as that derived from the Wigner-Weisskopf approximation. The paper of Dowling and Bowden artificially restricts the wave propagation to the direction perpendicular to the mirrors, thus ignoring the fact that the radiation can couple into off-axial modes. It also fails to take into account the independent TE, TM, and TEM modes of the system. Nevertheless, the importance of this work lies in the semiclassical description of the spontaneous emission in inhomogeneous media. The authors also consider radiation in periodic dielectric media. The latter is a very interesting subject these days because of the diverse applications based on photonic crystals [12].

In this paper we complete the calculation of radiation by Dowling and Bowden [2] by considering all the possible directions of propagation for the radiated power. That is, we take into account not only the on-axis propagation, but also the off axis propagation of the electromagnetic waves. The method described shows how the radiation of a point source couples into the normal (or allowed) TE, TM, and TEM modes in a very simple inhomogeneous system. The total power radiated depends directly on how many of these modes are being excited.

In Sect. 2 we summarize the derivation of the expression for the radiated power given in Ref. 2. This result is used in Sect. 3 to calculate the power radiated by a point dipole between two parallel metallic plates. We consider separately the power radiated into the independent mode polarizations of the field namely TE, TM, and TEM. The density of states calculation within the cavity in Sect. 4 is useful for the explanation of the discontinuities presented by the emitted power found in Sect. 5. Plots of the radiated power and its analysis are presented in Sect. 5. The power radiated by the dipole can be handled in terms of two limiting configurations: the dipole placed parallel or perpendicular to plates. We also analyze two dipole positions between the plates: $x_o = d/2$ and $x_o = 3d/5$ where d is the plates' separation. Section 6 presents the case of a dipole in front of a single mirror; here the power is obtained as a limiting case of the results given in Sect. 5. The power emitted by a gas of noninteracting excited atoms is calculated in Sect. 7, also on the basis of the results of Sect. 5. In the last section we present the conclusions of this work.

2. Power emission in an inhomogeneous medium

We define an inhomogeneous medium as one whose dielectric constant depends on the position. In particular, a dielectric body characterized by a position-*independent* dielectric constant, however bounded by vacuum or by a perfect conductor, also gives rise to inhomogeneity. There are several ways to explain why the power emission is altered by the medium inhomogeneities. One is that, in the presence of inhomogeneity, the allowed normal modes, into which the radiation can couple, are different than the ubiquitous plane-waves that propagate in an unlimited, homogeneous medium. Another explanation is that reflections are present due to the inhomogeneities, so that the dipole actually interacts with its own redirected field.

To start with, we have to solve the Maxwell wave equation for an inhomogeneous medium. In the absence of sources, the wave equation for such a medium is

$$\nabla \times \left(\nabla \times \vec{A}\right) + \frac{\epsilon(\vec{r})}{c^2}\frac{\partial^2}{\partial t^2}\vec{A} = 0 \qquad (1)$$

where $\vec{A}(\vec{r},t)$ is the vector potential of the field and the dielectric constant $\epsilon(\vec{r})$ is a function of the position due to the inhomogeneity. The field fulfills the Coulomb or transverse gauge [1, 2]

$$\nabla \cdot \left[\epsilon(\vec{r})\vec{A}\right] = 0 \qquad (2)$$

rather than $\nabla \cdot \vec{A} = 0$ as in an unbounded, homogeneous medium. For a given material geometry, we can describe the field therein as a linear superposition of *normal modes* or *eigenmodes*. Each mode may be labeled according to its wavevector $\vec{k}$ and polarization index p. So, the total monochromatic field present in the medium is given by

$$\vec{A}(\vec{r},t) = \sum_p \sum_{\vec{k}} \vec{a}_{\vec{k}p}(\vec{r}) \exp\left(-i\omega_{\vec{k}p}t\right) \delta(\omega_{\vec{k}p} - \omega). \qquad (3)$$

Here $\omega_{\vec{k}p}$ and $\vec{a}_{\vec{k}p}(\vec{r})$ are the eigenfrequency and eigenvector of the $\vec{k}$, p mode. The Dirac delta function ensures that the fields oscillate only at frequencies ω that the inhomogeneous medium can admit, namely the eigenfrequencies $\omega_{\vec{k}p}$. These normal modes are monochromatic solutions of the Helmholtz equation, that is

$$\nabla \times \left[\nabla \times \vec{a}_{\vec{k}p}(\vec{r})\right] - \frac{\omega^2_{\vec{k}p}}{c^2}\epsilon(\vec{r})\vec{a}_{\vec{k}p}(\vec{r}) = 0. \qquad (4)$$

They also have to fulfill the normalization and closure conditions given by Eqs. (5) and (6), respectively:

$$\int d^3r\, \epsilon(\vec{r})\, \vec{a}^{\,\star}_{\vec{k}'p'}(\vec{r}) \cdot \vec{a}_{\vec{k}p}(\vec{r}) = \delta\left(\vec{k} - \vec{k}'\right)\delta_{pp'}, \qquad (5)$$

$$\sum_{p=1}^{2} \int d^3k\, \vec{a}^{\,\star}_{\vec{k}p}(\vec{r}\,')\, \vec{a}_{\vec{k}p}(\vec{r}) = \overleftrightarrow{\delta}\left(\vec{r} - \vec{r}\,'\right). \qquad (6)$$

Note that both sides of the last equation are dyadics. Equations (5) and (6) ensure that the $\vec{a}_{\vec{k}p}$ are a complete set of orthonormal functions. Dowling and Bowden [2] simplified the modal theory of Glauber and Lewenstein [1] in order to find an expression for the power emission in terms of classical quantities. Basically, they solve the inhomogeneous wave equation for a source localized in space, that is

$$\nabla \times \left(\nabla \times \vec{A}\right) + \frac{\epsilon(\vec{r})}{c^2}\frac{\partial^2}{\partial t^2}\vec{A} = \frac{4\pi}{c^2}\vec{J}, \qquad (7)$$

where $\vec{J}(\vec{r},t)$ is a current density corresponding to a point like dipole, namely,

$$\vec{J}(\vec{r},t) = \omega_o\vec{\mu}\cos(\omega_o t)\,\delta(\vec{r} - \vec{r}_o)\,\Theta(t). \qquad (8)$$

The dipole has a moment $\vec{\mu}$, is located at the point $\vec{r}_o$, is oscillating with frequency ω_o, and is turned on at the time $t = 0$, as evident from the step function $\Theta(t)$. Equation (7) can be solved in terms of the normal modes as given in Eq. (3), and then one can calculate the work done by the dipole current against the ambient electric field to find the radiated power. Following the procedure in Ref. 2, the power radiated by the point dipole in steady state is

$$P = \pi^2 \omega_o^2 \mu^2 \sum_{p=1}^{2} \int d^3k \left| \vec{a}_{\vec{k}p}(\vec{r}_o) \cdot \hat{\mu} \right|^2 \delta\left(\omega_o - \omega_{\vec{k}p}\right), \quad (9)$$

where $\hat{\mu}$ is a unit vector parallel to $\vec{\mu}$. Equation (9) implies that the power emitted by the point radiator depends on the normal modes being excited. The Dirac delta function selects the modes that have the frequency of the radiator and therefore contribute to the radiated power. The total power can be decomposed into independent contributions from each polarization mode. In the following section we study the radiation of a point dipole inside a cavity formed by two plane-parallel mirrors.

3. Power emission in a metallic cavity

Consider a pair of perfectly conducting, plane-parallel and infinite metallic plates and a dipole between them, as shown in the inset in Fig. 1. By the modal theory of the previous section, the emitted radiation must couple to the allowed modes. Indeed, the radiated power can be decomposed into a superposition over all the allowed and independent modes. In order to calculate these modes we distinguish between three independent polarizations, TE, TM and TEM†. In these polarization modes, the electric (magnetic) field $\vec{E}$ ($\vec{B}$) is parallel to the mirror planes for TE (TM, TEM) modes. The independent polarization modes are defined in the following subsections. Now we can suppress, without ambiguity, the mode index p.

3.1. TE modes

For the TE mode, the parallel components of $\vec{E}$ must vanish at the surfaces of the perfectly conducting mirrors, and hence this polarization field fulfills the following boundary condition:

$$\vec{E}(x = 0, y, z) = \vec{E}(x = d, y, z) = 0. \quad (10)$$

Our power calculation in Eq. (9) requires the vector potential; we relate it to the electric field using the gauge that the scalar potential vanishes. Then

$$\vec{E}_{\vec{k}} = i\frac{\omega}{c}\vec{a}_{\vec{k}}. \quad (11)$$

Inside the cavity there is vacuum so we can take the sinusoidal solution of the Helmholtz equation and adjust it to the boundary conditions. Then the solutions for the TE modes

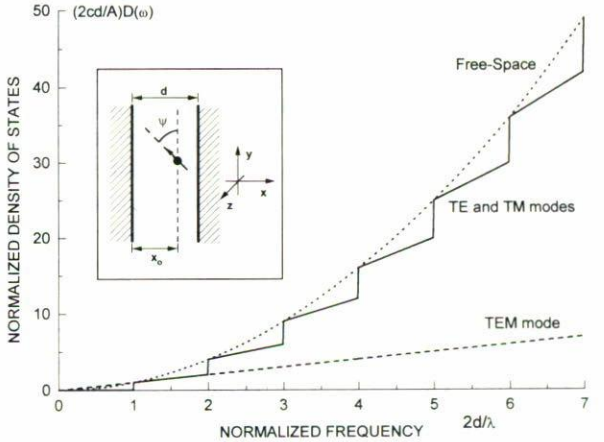


FIGURE 1. The inset depicts the general case of a dipole between two perfect mirrors. In the body of the figure we plot the cavity density of states (DOS) for the TE, TM, TEM modes, and for one polarization mode in free-space. The normalized frequency is defined as $\omega/(\pi c/d) = 2d/\lambda$, that is, it is the number of half-wavelengths that fit between the mirrors. The discontinuities are caused by successive, resonant excitation of standing waves for an integer ($n = 1, 2, \ldots$) number of half-wavelengths between the mirrors. For the DOS function normalization we divide it by $A/2cd$.

are given by

$$\vec{a}_{\vec{k}}(x, y, z) = C_1 \sin\left(\frac{n\pi}{d}x\right) \exp\left(ik_y y + ik_z z\right) \hat{e}_{\vec{k}}, \quad (12)$$

where $\hat{e}_{\vec{k}}$ is any vector lying in the yz plane (see Fig. 1), n is a positive integer, and C_1 is a normalization constant. Equation (12) represents the allowed eigenvectors in the case of an electric field parallel to the plates. The field is perpendicular to the wavevector, so the polarization unit vector is related to the wavevector as

$$\hat{e}_{\vec{k}} = \frac{k_z}{k_\rho}\hat{y} - \frac{k_y}{k_\rho}\hat{z} \quad (13)$$

where $k_\rho^2 = k_y^2 + k_z^2$.

3.2. TM modes

For the TM modes, the parallel components of the first derivative of the magnetic field must vanish at the surfaces, so the boundary conditions are given by

$$\frac{\partial}{\partial x}\vec{B}\left(x = 0, y, z\right) = \frac{\partial}{\partial x}\vec{B}\left(x = d, y, z\right) = 0. \quad (14)$$

To calculate the TM power contribution, we also assume a harmonic solution for the magnetic field and modify it to fulfill de boundary conditions (14). Then the magnetic field is given by

$$\vec{B}_{\mathbf{k}} = C_2 \cos\left(\frac{n\pi}{d}x\right) \exp\left(ik_y y + ik_z z\right) \hat{e}_{\vec{k}}, \quad (15)$$

which is related to the vector potential by

$$\vec{B}_{\vec{k}} = \nabla \times \vec{a}_{\vec{k}}. \quad (16)$$

Using Eqs. (16) and (4) we find that (for $\epsilon = 1$)

$$\vec{a}_{\vec{k}} = \left(\frac{c}{\omega_{\vec{k}}}\right)^2 \nabla \times \vec{B}_{\vec{k}}. \quad (17)$$

Hence, the TM normal mode expression results in

$$\vec{a}_{\vec{k}} = \frac{c^2}{\omega_{\vec{k}}^2} C_2 \left[-ik_\rho \cos\left(\frac{n\pi}{d}x\right)\hat{x} + \frac{n\pi}{d}\sin\left(\frac{n\pi}{d}x\right) \times \left(\frac{k_y}{k_\rho}\hat{y} + \frac{k_z}{k_\rho}\hat{z}\right)\right] \exp\left(ik_y y + ik_z z\right). \quad (18)$$

3.3. TEM modes

The TEM modes are just the special case $n = 0$ of the TM modes. Then, using Eq. (18) with $n = 0$ we get

$$\vec{a}_{\vec{k}} = \frac{-ic}{\omega_{\vec{k}}} C_3 \exp\left(ik_y y + ik_z z\right)\hat{x}. \quad (19)$$

The reason to consider the TEM modes separately is because they differ from the TM modes by the normalization constant. For this mode, the electric field is perpendicular to the plates and the magnetic field (as for the TM modes) is parallel to the plates.

3.4. Power radiation contributions

The boundary conditions of the TE, TM, and TEM modes lead to the discreteness of the x-component of the wavevector, so because now $\vec{k} = (n\pi/d, k_y, k_z)$ the dispersion relation is

$$\frac{\omega_{\vec{k}p}^2}{c^2} = \frac{n^2\pi^2}{d^2} + k_y^2 + k_z^2. \quad (20)$$

Considering the discreteness of k_x, the normalization condition Eq. (5) has to be rewritten as

$$\int d^3r \vec{a}^{\star}_{\vec{k}'}(\vec{r}) \cdot \vec{a}_{\vec{k}}(\vec{r}) = \delta_{nn'}\delta\left(k_y - k_y'\right)\delta\left(k_z - k_z'\right). \quad (21)$$

Using this normalization condition, we get the normalization constants C_1, C_2, and C_3 stated in the previous sections as

$$C_1 = \sqrt{\frac{2}{d}}\frac{1}{2\pi},$$
$$C_2 = \frac{\omega_{\vec{k}}^2}{c^2\pi\sqrt{2d}} \frac{1}{\sqrt{k_y^2 + k_z^2 + n^2\pi^2/d^2}},$$
$$C_3 = \frac{\omega_{\vec{k}}}{2\pi c\sqrt{d}}. \quad (22)$$

According to Eq. (9), we separate the power due to the independent field polarizations and add them up to find the total power emitted. In order to calculate the power contributions of each polarization mode, it is important to note that Eq. (9) is intended for a wavevector with the three continuous components. This is not the case in our problem in which the x-component is discrete. To take this into account, Eq. (9) is rewritten as

$$P = \pi^2\omega_o^2\mu^2 \sum_{p=1}^{2} \int d^3k \left|\vec{a}_{\vec{k}p}(\vec{r}_o) \cdot \hat{\mu}\right|^2 \times \delta\left(\omega_o - \omega_{\vec{k}p}\right)\delta\left(k_x - \frac{n\pi}{d}\right). \quad (23)$$

Without limitation of the generality, we assume that the dipole lies on the xy plane and that it forms an angle ψ with the y axis. Then, using the mode expressions given in Eqs. (12), (18), and (19), the normalization constants (22), and the power expression in Eq. (23), we have that the power contributions of each mode are

$$P_{\text{TE}}\left(\psi, \frac{x_o}{d}, N(\omega_o)\right) = \frac{\pi\omega_o^3\mu^2}{2dc^2}\cos^2\psi \sum_{n=1}^{N(\omega_o)} \sin^2\left(\frac{n\pi}{d}x_o\right), \quad (24)$$

$$P_{\text{TM}}\left(\psi, \frac{x_o}{d}, N(\omega_o)\right) = \frac{\omega_o\mu^2\pi}{d}\sin^2\psi \sum_{n=1}^{N(\omega_o)} \left(\frac{\omega_o^2}{c^2} - \frac{n^2\pi^2}{d^2}\right)\cos^2\left(\frac{n\pi}{d}x_o\right) + \frac{\omega_o\mu^2\pi}{2d}\cos^2\psi \sum_{n=1}^{N(\omega_o)} \frac{n^2\pi^2}{d^2}\sin^2\left(\frac{n\pi}{d}x_o\right), \quad (25)$$

$$P_{\text{TEM}}\left(\psi, N(\omega_o)\right) = \frac{\pi\omega_o^3\mu^2}{2dc^2}\sin^2\psi. \quad (26)$$

Here $N(\omega_o) = [d\omega_o/\pi c] = [2d/\lambda]$. The square brackets imply that we take the integer part of the number therein. The number N corresponds to radiation with $k_y = k_z = 0$, and thus to the maximum possible value of the component, which is $(k_x)_{\max} = \omega_o/c = N\pi/d$. In other words, whenever $k_y = k_z = 0$, an integer number of half-wavelengths $N = 2d/\lambda$ fits between the mirrors.

4. Density of states

We calculate the photon density of states (DOS) in the space between the mirrors. It is important to distinguish between the polarization modes because the TEM mode has a continuous dispersion relation while for the TE and TM modes ($n \neq 0$) the dispersion relation is discontinuous. In the case of the TEM mode the wavevector is restricted to the yz plane [see Eq. (19)] and hence the dispersion relation is given by Eq. (20) with $n = 0$. We assume periodic boundary conditions for the vector potential, Eq. (19). This means that the area in $\vec{k}$ space occupied by one mode will be $(2\pi)^2/A$ for an area A in the yz plane. To calculate the DOS, we take the area in the k_y, k_z plane of the $\vec{k}$ space corresponding to the frequency interval $(\omega, \omega + d\omega)$, and divide it by the area occupied by one mode. This gives the number of modes in the interval $(\omega, \omega + d\omega)$, or, alternatively, in the interval $(k_\rho, k_\rho + dk_\rho)$ between two concentric circles,

$$\frac{A}{(2\pi)^2} 2\pi k_\rho dk_\rho = \frac{A\omega}{2\pi c^2} d\omega \equiv D(\omega) d\omega. \tag{27}$$

The cases of TE and TM polarizations both lead to the same DOS. By Eqs. (12) and (15) the wavevector is $\vec{k} = (n\pi/d)\hat{x} + k_y\hat{y} + k_z\hat{z}$ and the dispersion relation is as given in Eq. (20). Then the dispersion relation depends on the quantum number $n\ (= 1, 2, \ldots)$. In order to calculate the DOS, we have to realize that, for a given frequency ω only the quantum numbers $n = 1, 2, \ldots, n_{\max}(\omega)$ can be excited. By Eq. (20) $n_{\max}$ is obtained for $k_\rho = 0$: it is the largest integer that is smaller than $\omega d/\pi c$, namely $n_{\max} = [\omega d/\pi c] = [2d/\lambda] = N(\omega)$. Then the total area in the k_y, k_z plane of $\vec{k}$ space corresponding to the frequency interval $(\omega, \omega + d\omega)$ will be composed of all the contributions $n \leq N(\omega)$. Apparently, if $\lambda/2 > d$ then $[2d/\lambda] = 0$ and no TE and TM modes can be excited: for these modes the DOS vanishes in this low-frequency range. If $1 \leq 2d/\lambda < 2$ then only the $n = 1$ mode can be excited; clearly, in this frequency range the DOS's of the TE and TM modes are the same as that of the TEM mode—Eq. (29). Next, let's assume that for a given frequency ω only the states $n = 1$ and $n = 2$ are excited ($N = 2$). Then, the total area covered divided by the area of one mode will be given by

$$\frac{A}{(2\pi)^2}(2\pi k_{\rho 1}\, dk_{\rho 1} + 2\pi k_{\rho 2}\, dk_{\rho 2}) = \frac{A\omega}{\pi c^2} d\omega \equiv D(\omega) d\omega, \tag{28}$$

valid for the frequency range $2\pi c/d \leq \omega < 3\pi c/d$. [Here the lower (upper) limit is determined by substituting in Eq. (20) $k_\rho = 0$ and $n = 2$ ($n = 3$).] For the general case, we just sum over the $k_{\rho n} dk_{\rho n}$ area elements covered by the frequency interval $(\omega, \omega + d\omega)$. Then the DOS is given by

$$\frac{A}{2\pi} \sum_{n=1}^{N(\omega)} k_{\rho n} dk_{\rho n} = \frac{A\omega}{2\pi c^2} N(\omega) d\omega \equiv D(\omega) d\omega; \tag{29}$$

now the frequency interval is $N\pi c/d \leq \omega < (N + 1)\pi c/d$. To summarize, the DOS for the photons inside the cavity is

$$D(\omega) = \begin{cases} A\omega/2\pi c^2 = A/c\lambda, & \text{TEM polarization} \\ A\omega N(\omega)/2\pi c^2 = (A/c\lambda)\,[2d/\lambda], \quad [2d/\lambda] \leq 2d/\lambda \leq [2d/\lambda] + 1, & \text{TE and TM polarizations} \end{cases} \tag{30}$$

We plot the Eqs. (30) and the free-space DOS (for one polarization) in Fig. 1. Notice the discontinuities for any integral value of the normalized frequency for the TE and the TM modes.

These discontinuities further arise as the frequency increases, for every integer number value of $2d/\lambda\ (= n)$ a new cavity mode is excited which starts contributing abruptly to the DOS. It is interesting to note that both slopes and the discontinuities of the normalized TE and TM densities of states are given by $[2d/\lambda]$. Also, for $2d/\lambda = [2d/\lambda]$ (just after the jumps), the DOS's of the TE and TM modes are equal to the DOS in free space (corresponding to one polarization mode). The free-space DOS $(Ad)\omega^2/\pi^2 c^3$ can be recovered from Eq. (30) in the limit $d \to \infty$ by replacing $N(\omega)$ by $d\omega/\pi c$ and multiplying it by two, to consider the TE and TM propagations. In this limit the contribution of the TEM mode to the DOS is negligible.

5. Analysis of the emitted power

As we mentioned before, the total radiated power is the sum of all the polarization contributions. That is

$$P_{\mathrm{TOT}} = P_{\mathrm{TE}} + P_{\mathrm{TM}} + P_{\mathrm{TEM}}, \tag{31}$$

where the formulas for each power contribution are given by the Eqs. (24), (25), and (26). In terms of the dipole's inclina-

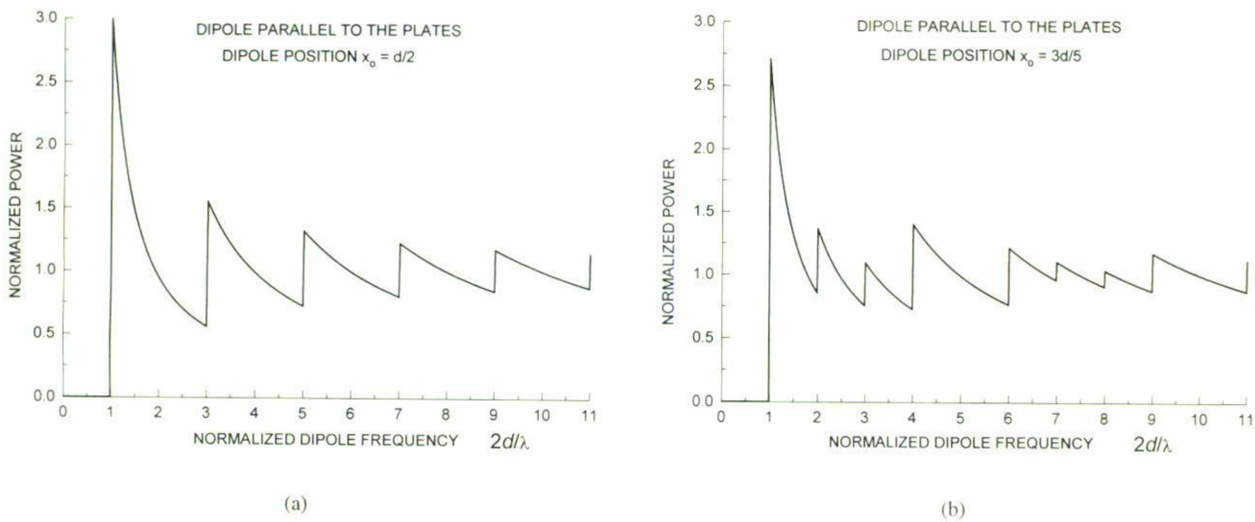


FIGURE 2. Normalized power emitted by a dipole parallel to the mirrors ($\psi = 0$) at a position (a) $x_o = d/2$ and (b) $x_o = 3d/5$. The frequency has been normalized as in Fig. 1, and the power is normalized by the free-space expression $\mu^2\omega_o^4/3c^3$. The radiation is composed of the TE and TM components. The discontinuities arising for an odd number of half-wavelenghts within the mirrors (and their absence for an even number) are explained in the text.

tion, this may be expressed as

$$P_{\rm TOT} = P_{\|}\cos^2\psi + P_{\perp}\sin^2\psi, \tag{32}$$

where $P_{\|}$ and $P_{\perp}$ are the power expressions for a dipole placed parallel and perpendicular to mirrors, respectively. So, no matter what direction the dipole moment may have, we can express the total radiated power in terms of these two basic configurations. It is convenient to normalize by means of the power emitted by the same dipole in free space, *i.e.* dividing by $\omega_o^4\mu^2/3c^3$. Then the normalized powers radiated by the parallel and perpendicular dipoles are

$$P_{\|}\left(\frac{x_o}{d}, N(\lambda)\right) = \frac{3}{2}\left(\frac{\lambda}{2d}\right)\sum_{n=1}^{N(\lambda)}\left[1 + \left(\frac{\lambda}{2d}\right)^2 n^2\right] \times \sin^2\left(n\pi\frac{x_o}{d}\right), \tag{33}$$

$$P_{\perp}\left(\frac{x_o}{d}, N(\lambda)\right) = 3\sum_{n=1}^{N(\lambda)}\left[\left(\frac{\lambda}{2d}\right) - \left(\frac{\lambda}{2d}\right)^3 n^2\right] \times \cos^2\left(n\pi\frac{x_o}{d}\right) + \frac{3}{2}\left(\frac{\lambda}{2d}\right). \tag{34}$$

Here, λ is the wavelength of the dipole radiation and $N = [2d/\lambda]$ is the maximum number of half-wavelengths that is possible to fit between the mirrors for a given ω_o. The power radiated by the dipole depends strongly on $N(\lambda)$ as we see from Eqs. (33) and (34). These equations can be summed up exactly, but it is simpler to present them this way. Nevertheless, we will have to use the exact summation in Sect. 6. The same results were obtained in [4] and [8] using a semiclassical treatment.

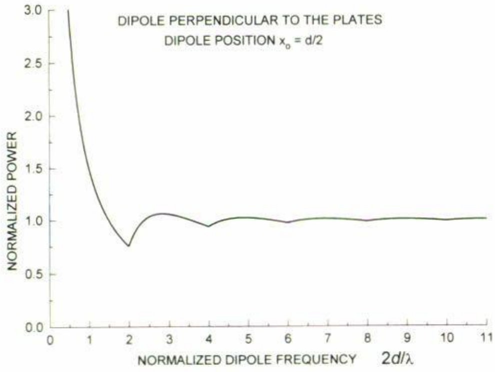
In Figs. 2 and 3, we plot Eqs. (33) and (34) as functions of the normalized dipole frequency $(2d/\lambda)$ for two different dipole positions, namely $x_o = d/2$ and $x_o = 3d/5$. Graphic results are also given in [5] for a dipole placed at $x_o = d/2$.

For the parallel dipole ($\psi = 0$) the allowed radiation consists of TE and TM modes; the TEM mode does not contribute, as is obvious from Eq. (26). However, for $\lambda/2 > d$, $N(\omega_o) = 0$. This is because the summation in Eq. (34) is void of terms. Thus, there is a region of no power emission for the parallel dipole that corresponds to frequencies below the so-called waveguide cutoff. Indeed, in Fig. 2 the radiated power vanishes for $2d/\lambda < 1$. A dipole radiating at the corresponding frequencies has no allowed mode to radiate into.

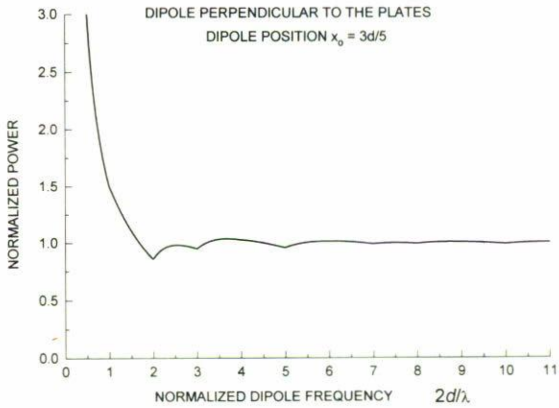
In case of the perpendicular dipole ($\psi = 90^o$) we see from Eq. (24) that the TE mode does not contribute. The radiation consists of the TM and TEM modes. For $\lambda/2 > d$, again, $N(\omega_o) = 0$; thus the right side of Eq. (25) is zero. Therefore the dipole cannot radiate into TM modes below this cutoff. Hence the radiated power excites only TEM modes below the waveguide cutoff and is proportional to ω_o^3 (see Fig. 3).

The discontinuities in Fig. 2 derive from the successive excitation of modes $n = 1, 2, \ldots$ as the frequency increases. Here it is interesting that for $x_o = d/2$ there are no discontinuities for even integral values of the normalized frequency. Similarly, for $x_o = 3d/5$ there are no discontinuities for integral multiples of five. This arises because the mode functions (12) and (18) to be excited at these frequencies have zero value at the position of the dipole. That is, we are placing the dipole at the nodes of the normal mode, so that it will have no interaction with the dipole.

Generally speaking, if the normalized position can be expressed as a fraction p/q, p and q integers, then there won't be mode excitation if the mode index n is an integral multiple of q. This property can also be verified from Eq. (33) in



(a)



(b)

FIGURE 3. As in Fig. 2 for a dipole perpendicular to the mirrors ($\psi = 90^o$). The radiation is composed of TM and TEM components.

which the sine function vanishes if we substitute $x_o/d = p/q$ and n/q is an integer.

For the case of the perpendicular dipole (Fig. 3) we have a strong singularity for $\omega \to 0$ as a consequence of the TEM mode contribution given by the last term of Eq. (34). Here no discontinuities in the function exist at all because the expression inside the curly brackets in Eq. (34) vanishes for the new mode excited $(2d/\lambda = n)$, leaving the summation unchanged at the excitation frequency. There are only discontinuities in the first derivative as a consequence of the new mode excitation.

If $\lambda << d$ then the radiation does not "see" the mirrors, and $P_{\|}$ and $P_{\perp}$ both tend to the free space value as $2d/\lambda \to \infty$.

6. Dipole radiating in front of a mirror

In order to find the expressions for the power radiation of a dipole in front of a single mirror. We shall show that the formulas for $P_{\|}$ and $P_{\perp}$ can be obtained from Eqs. (33) and (34) by taking carefully the limit $d \to \infty$. First it is convenient to perform the summations in Eqs. (33) and (34). The squared sine summation can be expressed as [14]

$$\sum_{n=1}^{N} \sin^2 n\theta = \frac{N}{2} - \frac{\cos(N+1)\theta \sin N\theta}{2\sin\theta}. \tag{35}$$

By the use of trigonometric identities, it is easy to find a similar expression for the squared cosine summation. Taking twice the derivative of Eq. (35) with respect to θ, we find that

$$\sum_{n=1}^{N} n^2 \sin^2 n\theta = \frac{1}{2}\sum_{n=1}^{N} n^2 - \frac{1}{4}\frac{d^2}{d\theta^2}\left(\frac{N}{2} - \frac{\cos(N+1)\theta \sin N\theta}{2\sin\theta}\right), \tag{36}$$

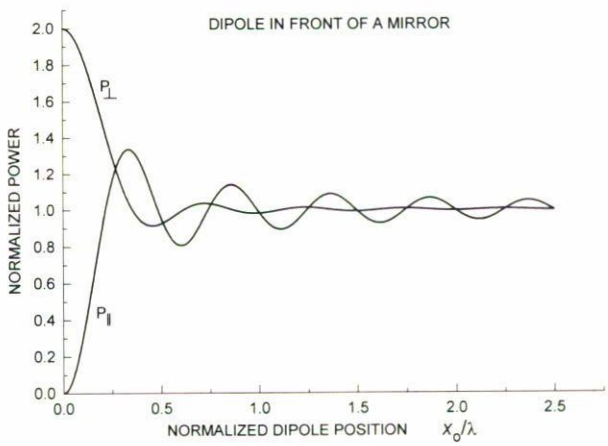


FIGURE 4. Power radiated by a dipole in front of a single mirror. The position is also normalized dividing by the emitted wavelength. Here we can see that the powers for the parallel and perpendicular dipole inclinations are equal whenever $x_o \simeq n\lambda/4$ $(n = 1, 2, \dots)$, a result which becomes exact in the limit $x_o/\lambda \to \infty$.

where [14]

$$\sum_{n=1}^{N} n^2 = \frac{N(N+1)(2N+1)}{6}. \tag{37}$$

A similar procedure can be carried out for the squared cosine in Eq. (33). To obtain the power radiated by a dipole in front of a mirror, we substitute the Eqs. (35)–(37) in Eqs. (33) and (34) and apply the limit $d \to \infty$. Then we find that the free-space normalized power expressions are

$$P_{\|} = 1 - \frac{3\sin(\xi)}{2(\xi)} - \frac{3\cos(\xi)}{2(\xi)^2} + \frac{3\sin(\xi)}{2(\xi)^3}, \tag{38}$$

$$P_{\perp} = 1 - \frac{3\cos(\xi)}{(\xi)^2} + \frac{3\sin(\xi)}{(\xi)^3}, \tag{39}$$

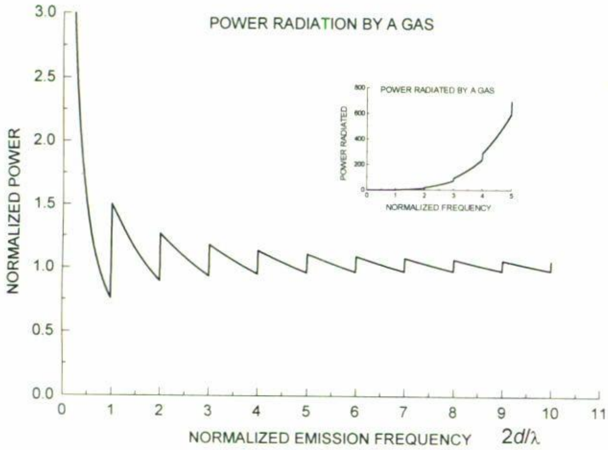


FIGURE 5. Power radiation due to an evenly distributed gas. The frequency and power normalizations are as in Fig. 2. Here we have discontinuities for all integer values of $2d/\lambda$, as for the DOS (Fig. 1), however unlike the case of the parallel dipole, Fig. 2. The greatest reduction of the power is 25 percent of the free space value, and occurs for the fundamental waveguide resonance $\lambda/2 = d$. In the inset the radiated power has been normalized by dividing it by the constant $\mu^2\pi^4 c/(2d^4)$, so we can see the true dependence on the frequency.

where $\xi = 4\pi x_o/\lambda$. We plot these equations in Fig. 4. We can see that, for both polarizations, there are positions for which the dipole radiates as if the mirror were absent, namely the normalized power is equal to one. This occurs because of the interference of the radiated fields with the fields reflected from the mirror. The same result, using the image method, can be found in references [13] and [5].

7. Radiation by a gas

Here we consider an experimentally realizable situation, namely a gas of non-interacting, identical atoms occupying the space between the mirrors. The positions x_o of the atoms and the directions of their dipole moments, given by the angle ψ, are both random. Thus we can average the total power emitted by an atom, Eq. (32), over ψ and x_o. Both variables appear only in the arguments of squared sines and cosines; these we now simply replace by 1/2. Then using Eqs. (32)–(34) the power emitted per atom is

$$\begin{aligned} P &= \frac{1}{2}\left(\langle P_{\|}\rangle + \langle P_{\perp}\rangle\right) \\ &= \frac{3}{8}\left(\frac{\lambda}{2d}\right)(3N+2) \\ &\quad - \frac{3}{8}\left(\frac{\lambda}{2d}\right)^3 \frac{N(N+1)(2N+1)}{6} \end{aligned} \tag{40}$$

where we have used Eq. (37). We plot the power per atom in Fig. 5. The normalized power approaches the limit 1, as it should. The real power (apart from a trivial numerical factor) is sketched in the inset so we can see that, as the frequency increases, the power is also growing. We also observe that there are discontinuities for all integer values of the normalized frequency, as can be expected from the behavior of the density of states in Fig. 1. Because there are no preferences for the dipoles to have a certain position, none of the discontinuities is missing as in Fig. 2. No complete power suppression exists, and the greatest power reduction is 25 percent of the free power emission for normalized frequencies nearly but less than the unity. Experimental results [10] reveal that there is a reduction of around 25 percent, indeed.

8. Conclusions

We have presented a classical method for calculating the power radiated by a dipole in an inhomogeneous system. It is important to note that Eq. (9) cannot be applied in the case of a dipole immersed in a dielectric, for the radiated power would be also modified by the local field. This method is based on the dipole field excitation of the normal modes of the medium, as described by the Helmholtz equation with a position-dependent dielectric constant. Results on power radiation for a dipole placed between two plane parallel mirrors, considering two dipole orientations were shown. We also explored the cases of a dipole in front of a single mirror and that of a gas of noninteracting excited atoms.

All the results exhibit both enhancement and reduction of the power radiated with respect to the free space power, depending on the emission frequency and the dipole-cavity arrangement. There is only one case in which a complete inhibition of the power emission is achieved; this is the case of a dipole parallel to the mirrors whose frequency of emission is such that $\omega_o < \pi c/d$. This arises because, for a frequency lower than the cavity cut-off, there are no normal mode solutions of Maxwell's equations for the given system.

The method presented in this paper can be applied to other cavity geometries, provided that their normal mode solutions are known. Moreover, it can also be used for the calculation of the power radiated in the presence of a dielectric medium, with the restriction that the dipole be placed in a vacuum cavity. For example, we can think of a set of dielectric layers with the same refractive index and separated by vacuum spaces. In this case, our radiator should be placed in one of those empty spaces, so that we may use Eq. (9) to calculate the radiated power. Therefore, this method can also be applied for power emission calculations in photonic crystals.

As previously mentioned, some of our results have already been demonstrated experimentally. However, several experimental limitations and other effects involved, like superradiance and collective phenomena, are always very difficult to eliminate.

†. The TM and TEM modes in reality correspond to a single polarization of the light, but they are considered separately only for convenience for this particulary simple configuration.

1. R.J. Glauber and M. Lewenstein, *Phys. Rev. A* **43** (1991) 467.
2. J.P. Dowling and C.M. Bowden, *Phys. Rev. A* **46** (1992) 612.
3. E.M. Purcell, *Phys. Rev.* **69** (1946) 618.
4. P.W. Milloni and P.L. Knight, *Opt. Commun.* **9** (1973) 119.
5. D. Meschede, *Physics Reports* **5** (1992) 201, see Fig. (2.4).
6. D. Kleppner, *Phys. Rev. Lett.* **47** (1981) 233.
7. M.A. Rippin and P.L. Knight, *J. Mod. Optics* **43** (1996) 807.
8. A.O. Barut and J.P. Dowling, *Phys. Rev. A* **36** (1987) 649.
9. I. Abram and G. Bourdon, *Phys. Rev. A* **54** (1996) 3476.
10. F. de Martini, G. Innocenti, G. Jacobovitz and P. Mataloni, *Phys. Rev. Lett.* **59** (1987) 2955.
11. F. de Martini, M. Marrocco, P. Mataloni, L. Crescentini, and R. Loudon, *Phys. Rev. A* **43** (1991) 2480.
12. J.D. Joannopoulos, R.D. Meade, and J.N. Winn, *Photonic Crystals*, (Princeton University Press, 1995).
13. J.M. Wylie and J.E. Sipe, *Phys. Rev. A* **30** (1984) 1185.
14. B.W. Jolley, *Summation of series*, (Dover, 1961).