The optimum weight of a baseball bat

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By using a well known relation of muscle biomechanics called the Hill's force-velocity relation, and modelling the ball and bat collision as a frontal inelastic collision, we find the optimum weight of a bat for driving the ball as far as possible, given the batter anthropometric conditions and the bat and ball mechanical properties. A discussion of the results is also presented.

Keywords: Hill's force-velocity relation

Usando un resultado experimental generalmente aceptado acerca de la biomecánica del músculo, conocido como la relación fuerza-velocidad de Hill, y modelando el choque de un bat de beisbol con la bola como una colisión frontal inelástica, encontramos la masa óptima del bat para enviar la bola tan lejos como sea posible, dadas las condiciones antropométricas del bateador. Se presenta también una discusión de los resultados obtenidos.

Descriptores: Relación fuerza-velocidad de Hill

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1. Introduction

Doubtlessly baseball has something that the rest of the sports do not have, and makes it so special. That is, the optimality reached with time in many of its aspects. For example the distance between the bases, the distance from the hill to the plate, the length of the bat, the size an weight of the ball, etc., have values such that the game reaches a very fine equilibrium which makes it as spectacular as it is. Altering the value of any one of them, would make the batters predominate over the pitchers, or the runners over the fielders, or viceversa.

In 1995 there appeared in *Physics Today* an excellent review article (of many papers published mainly in the *Am. J. Phys.*) by Robert K. Adair, about the physics of baseball [1]. In that paper, Adair analyses among other things how the roughness of the ball affects its flight, the movements of the curveball and the knukleball in therms of hydrodynamics, the biomechanics of batting and throwing, and finally, the mechanics of the ball's hit by the bat. In respect to this last point, very interesting facts are elucidated, such as that the collision between the bat and the ball lasts about 1 millisecond with most of the momentum transfer taking place in about 0.6 milliseconds. But also that the collision impulse signal takes about 8 milliseconds to go the 5 feet from the point of impact to the batter hands. So when batted, the ball never knows wether or not hands were holding the bat, and the momentum of inertia of the bat is not relevant in any simple way to the collision kinematics. On the other hand, it is also stated that when the ball hits a wooden bat, the bat compresses about 2% as much as the ball, and thus stores about 2% of the collision energy. The ball with a coefficient of restitution at high velocities of about 0.45 returns about 20% of its 98% of the stored energy, while the bat, which is as elastic as the ball, returns about the same proportion. By contrast, a hollow aluminum bat is distorted about as 10% as the ball by the collision and so stores about 10% of the collision energy; after the hit it returns that energy efficiently, probably at a level of 80%. Adding the ball and bat contributions, about 26% of the collision energy is returned, explaining why aluminum bats drive the ball much faster than wooden bats do.

In 1938, A.V. Hill [2] made mechanical experiments with frog muscles and obtained the following relation between the muscle's contraction velocity V and the load the muscle had to rise F:

$$V = V_{\max}\,\alpha\,\frac{F^o - F}{F + \alpha F^o}, \tag{1}$$

where $V_{\max}$ is the maximum contraction velocity, which is reached at $F = 0$, F^o is the maximum load the muscle can rise, and α is an adjusting parameter. These three quantities are characteristic of every muscle. When plotted, Eq. (1) renders a hyperbola that cuts the V axis at $V_{\max}$, the F axis at F^o, and whose curvature is determined by α; the bigger α is, the smaller the curvature of the hyperbola. In a recent work [3], an interpretation to α was given as a parameter that measures the compromise between power and efficiency reached by a muscle. The relation expressed by Eq. (1) is usually known as the Hill's force-velocity relation and has been tested many times from 1938 up to now. A recent review paper [4] shows how this relation is valid for muscles of all kinds of animals. Indeed, De Koning [5] has found that the muscles of the human arm obey the same relation. From this, it is reasonable to assume that a similar relation must be obeyed between the velocity of a bat while batting, and the bat's weight.

In the present work we will use the above mentioned facts to find the optimum weight of a bat to drive the ball as fast as possible, given the weight of the ball, the ball's approaching velocity, and the force-velocity relation for a player's batting. Finally, we discuss the results obtained.

2. The ball and the bat

Two are the objectives of the batter when he stands over the plate. First, he has to hit the ball. And second, after batted, the ball must fly as far a possible. A very important fact for hitting the ball is the weight of the bat. The slighter the bat is, the more rapid the swing can be and so, the bigger the possibility of hitting rapid balls. In this sense it is better to have a bat as slight as possible. Nevertheless a very slight bat could not have an important amount of momentum no matter how fast if moves and so, even when it can hit the ball, this one will not fly too far. This reasoning suggests that there is an optimum bat's weight for every batter, as all baseball players know. In the following paragraphs we will develop under some reasonable assumptions, a way of calculating the optimum bat's weight for a given batter, if the objective is to send the ball as far as possible.

According to Adair [1], the momentum transfer from the bat to the ball during the hit is so fast, that the ball never knows wether the batter's hands are holding the bat or not. Moreover, the torque the batter's hands can exert over the bat at the hit's moment is irrelevant. This fact permits us to model the hit as the collision of two simple objects, the bat and the ball, without considering the batter's body. If at this point we assume that the collision if frontal (as must be for sending the ball far enough) so, the collision can be considered as happening in one dimension. With all these assumptions the velocity of the ball after the collision can be calculated as a function of the velocities of the bat and the ball before the collision, and the weights of both the bat and the ball, given that we know the fraction of energy restored (about 20% with a wooden bat and 26% with an aluminum bat, in the system of the center of mass [1]).

As explained above, let's assume that the bat hits the ball in a frontal collision, that the bat with a mass m_b approaches with a velocity v_b, that the ball with a mass m_p does it with a velocity $-v_p$, and that in the reference system of the center of mass η percent of the energy is restored. The velocities of the bat V_b and ball V_p after the collision can be calculated from the equations of momentum's conservation and of energy balance. Let $v_{\rm cm}$ the velocity of the center of mass which is given by

$$v_{\rm cm} = \frac{m_b v_b - m_p v_p}{m_b + m_p}. \tag{2}$$

Since in the center of mass system the momentum is zero before the collision, the equation which stands for the momentum's conservation is

$$m_b (V_b - v_{\rm cm}) - m_p (V_p - v_{\rm cm}) = 0. \tag{3}$$

On its own, the equation for the energy balance in the same system is given by

$$\eta\left[m_b (v_b - v_{\rm cm})^2 + m_p (v_p + v_{\rm cm})^2\right] = m_b (V_b - v_{\rm cm})^2 + m_p (V_p - v_{\rm cm})^2. \tag{4}$$

By solving for $V_b - v_{\rm cm}$ in Eq. (3) and substituting into Eq. (4) we obtain an equation that can be solved for V_p giving

$$V_p = v_{\rm cm} + \left\{\frac{m_b}{m_p}\frac{1}{m_b + m_p}\eta \left[m_b (v_b - v_{\rm cm})^2 + m_p (v_p + v_{\rm cm})^2\right]\right\}^{\frac{1}{2}}. \tag{5}$$

Equation (5) is nothing else than the formula which permits us calculating V_p in terms of v_b, v_p, m_b, and m_p.

Up to now we have four apparently independent variables which are v_b, v_p, m_b, and m_p. Nevertheless every batter knows that the bat's velocity depends on their own strength and velocity, but also on the bat's weight. In other words, there must be possible to write v_b in terms of m_b among other parameters. This last relation would give us the maximum possible velocity that can be impinged to the bat by the set of muscles of the batter's body, as a function of the bat's weight or load. Hill's force-velocity relation [Eq. (1)] gives the velocity of contraction of a given muscle V as a function of the load F. Very important parameters in this relation are the maximum velocity of contraction $V_{\rm max}$, the maximum load that can be raised by the muscle F^o, and α, which measures the compromise between power output and efficiency reached by the muscle. $V_{\rm max}$, F^o, and α are characteristic for every muscle. Hill's force-velocity relation is obeyed for muscles of very different species, as shown in Ref. 4, but also for sets of muscles as those of the human arm [5], in this case α measures the global compromise between power and efficiency reached by the set of muscles as a whole. From what has been told above, it's reasonable to assume that the relation between v_b and m_b is governed by Hill's equation:

$$v_b = v_b^{\rm max}\alpha\frac{m_b^{\rm max} - m_b}{\alpha m_b^{\rm max} + m}, \tag{6}$$

$v_b^{\rm max}$ being the maximum swing velocity and $m_b^{\rm max}$ the maximum weight for a bat to be held by the batter. Both of them along with α are characteristic for every batter and can be determined experimentally by asking the batter to swing bats of different weights, measuring the swings velocity in each case, and adjusting to Eq. (6).

Of all the parameters that have been mentioned, the ball's mass is given by baseball rules to be $m_p \approx 0.14$ kg, while α, $m_b^{\rm max}$, $v_b^{\rm max}$, and η can measured. In fact $\eta \approx 0.2$ for wooden bats and $\eta \approx 0.26$ for aluminum bats [1]. The values of α,

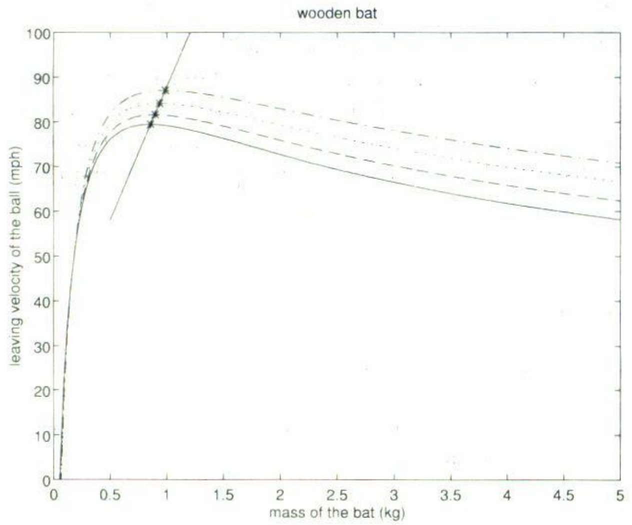

FIGURE 1. Leaving velocity of the ball after a frontal collision with a wooden bat $\eta = 0.2$, *vs.* the mass of the bat, for different approaching velocities of the ball v_p. The solid-line plot (—) corresponds to $v_p = 80$ mph, the dashed-line plot (−−) to $v_p = 90$ mph, the dotted-line (···) plot to $v_p = 100$ mph, and the dashed-dotted-line plot (− · −) to $v_p = 110$ mph. The maximum v_p points are labeled by an asterix in all the plots; the line that best fits these last ones is also shown.

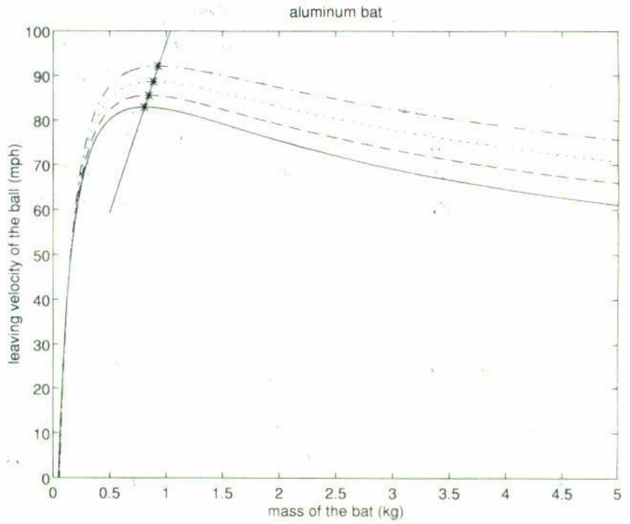

FIGURE 2. Leaving velocity of the ball after a frontal collision with an aluminum bat $\eta = 0.26$, *vs.* the mass of the bat, for different approaching velocities of the ball v_p. The solid-line plot (—) corresponds to $v_p = 80$ mph, the dashed-line plot (−−) to $v_p = 90$ mph, the dotted-line plot (···) to $v_p = 100$ mph, and the dashed-dotted-line plot (− · −) to $v_p = 110$ mph. The maximum v_p points are labeled by an asterix in all the plots; the line that best fits these last ones is also shown.

$m_b^{\max}$, and $v_b^{\max}$ depend on each individual. De Koning [5] has measured α for muscles of the human arm. He found that $\alpha \approx 0.2$ for common men and women, as well as for arm trained athletes. For the purposes of this work we will assume that $\alpha \approx 0.2$ for the set of all the muscles involved in the swing. $m_b^{\max}$, and $v_b^{\max}$ are not as regular as α, but variate very much from individual to individual. Nevertheless some reasonable values can be given. For example, Adair mentions that a typical swing velocity is around 60 mph. Thus, taking into account that common bats weight about 1 kg and that the swing velocity diminishes hyperbolically as the bat's weight augments, the maximum swing velocity $v_b^{\max}$ can be expected to be around 90 mph. Finally we will assume the maximum bat's weight to be $m_b^{\max} \approx 30$ kg. With all this quantities determined, the only variable quantities remaining are the ball's velocity before the collision and the bat's weight. The ball's velocity before the collision depend on the pitcher and range from 80 to 110 mph. In Fig. 1, plots of V_p *vs.* m_b are presented for different values of v_p, with $\eta = 0.2$ (the value for a wooden bat). These plots are done by means of Eq. (5), with v_{cm} given by Eq. (2) and v_b given by Eq. (6). In Fig. 2, similar plots are presented, but with $\eta = 0.26$ (the value for an aluminum bat). In those plots it can be observed that as expected, there is an optimum value for the bat's weight if the ball is required to leave as fast as possible after the collision, and thus to fly as far as possible. In the following section we discuss some interesting facts about this optimum weight.

3. Discussion and conclusions

First of all we must remark that the values of $v_b^{\max}$, $m_b^{\max}$, and α variate from individual to individual, and that in this work we have used reasonable values for such parameters which define a hypothetical batter. All the plots v_p *vs.* m_b shown in Figs. 1 and 2 are convex with a single maximum that determines the optimum bat's weight and the maximum velocity with which the ball leaves the bat after the collision, both for our hypothetical batter. It can be observed that the optimum bat's weight is around 1 kg, and that the maximum ball's velocity is between 80 and 100 mph. These values resemble very much the real ones [1], which indeed confirms that the values of $v_b^{\max}$, $m_b^{\max}$, and α we used, are in fact reasonable. Another immediate observation is that a comparison of Figs. 1 and 2 demonstrates that our hypothetical batter would send the ball farther with an aluminum bat than with a wooden bat, as it is well known for every baseball player. An interesting question arises when one observe that the optimum bat's weight augments with the approaching ball's velocity, and that is, should our hypothetical batter use heavier bats when he confronts faster pitchers? Of course not, since as we mentioned above the ball's velocity after the hit is not the only variable to be optimized, but also the swing velocity, and for that it's preferable a slighter bat. Fortunately, our plots show that the optimum bat's weight does not variate to much with the approaching ball's velocity, and that for bats with a mass close to the optimum one, the leaving ball's velocity is not much smaller than the maximum. These facts permit us conclude that any batter would choose a bat with a weight equal to the optimum weight at 80 mph for or even smaller for having a good chance to hit fast balls, but not to much that it would be in the part where the leaving ball's velocity diminishes rapidly as the bat becomes slighter. Of course, what should never be done is to choose a bat with a weight bigger that the optimum since then both the batting power and the swing velocity would diminish.

Finally, we would like to emphasize that the analysis presented in this work could help with very simple experiments (those necessary for finding v_b^{max}, m_b^{max}, and α), to the determination of the adequate bat for every player with not so much trials.

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