A dissipative Joule-Brayton cycle model

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In this paper we present an irreversible model of the Joule-Brayton cycle. As is well known, the gas-turbines follow approximately this thermal cycle. Our model reproduces several characteristics of a real gas-turbine such as convex curves of power *versus* pressure ratio and efficiency *versus* pressure ratio. Typical loop-shaped curves of power *versus* efficiency for real heat engines are also recovered. Our model is based in a lumped friction-like term and in a parameter arising from the Clausius inequality. We also suggest a procedure for improving the power and the efficiency of the cycle.

Keywords: Thermal cycle; endoreversibility; finite-time

En este artículo presentamos un modelo irreversible del ciclo de Joule-Brayton. Como es bien sabido, las turbinas de gas siguen aproximadamente este ciclo térmico. Nuestro modelo reproduce varias características de turbinas reales, como son curvas convexas de potencia contra razón de presión y eficiencia contra razón de presión. Curvas en forma de rizo, que son típicas de maquinas reales, también son obtenidas. Nuestro modelo está basado en un término tipo fricción global y en un parámetro que surge de la desigualdad de Clausis. También sugerimos un procedimiento para mejorar la potencia y eficiencia del ciclo.

Descriptores: Ciclo térmico; endorreversibilidad; tiempo finito

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1. Introduction

The development of the so-called finite-time thermodynamics (FTT) has been very important for heat-engine analysis [1–5]. As is well known, within the context of classical equilibrium thermodynamics (CET) the models of thermal engines usually lead to numerical values of typical performance variables such as efficiency, far above of the corresponding values for real engines. In fact, the CET-models for thermal engines are the reversible limits (therefore with null power output) of real engines. One of the aims of FTT has been to provide more realistic models for thermal engines (among many other physical systems). By means of FTT-methods has been possible to elaborate models in good agreement with experimental values of several process variables of heat engines [6–9]. Recently, several authors [10–14] have presented FTT-models of the Joule-Brayton cycle, which is a thermal cycle approximately followed by gas turbines. In these models (as in common engineering analysis), the main irreversibility source is the fluid friction of gas against the turbine blades and these losses are quantified by means of a lumped parameter refered to as isentropic efficiency, which measures the degree of departure of the adibatic branches from a true reversible isentropic regime (see Fig. 1). In Fig. 1, a temperature (T)-entropy (S) diagram for the Joule-Brayton cylce is depicted, and in Fig. 2, the corresponding pressure (p)-volume (V) diagram is presented. For a closed gas-turbine, its basic components are showed in Fig. 3. When the internal irreversibilities of the Joule-Brayton cycle are taken into account by means of the isentropic efficiencies of the turbine and the compressor respectively (as in Ref. 10), many realistic features of a gas-turbine are obtained. For example, loop-shaped curves of power output (P) versus efficiency (η) are recovered, such as ocurrs in real gas turbines [10]. In the present paper, we propose an alternative irreversible model for a Joule-Brayton cycle, which also reproduces many realistic features of real gas-turbines. Our model is very simple and pedagogical. In Sect. 2, we present the irreversible model by means of a friction-like lumped parameter and we obtain convex curves of power output and efficiency versus some design parameters as is common in real Joule-Brayton engines [15]. We also obtain P *vs.* η loop-shaped curves. In Sect. 3, we use a recent procedure for taking into account internal losses through the Clausius inequality [16–18], and we obtain an excellent agreement with typical efficiency values for real gas-turbines. Finally we suggest a procedure for improving the power output and the efficiency of the Joule-Brayton cycle.

2. The irreversible model

As it is depicted in Figs. 1 and 2, the Joule-Brayton cycle have four branches: two isentropic and two isobaric ones. In our model we take the heating process ($2 \rightarrow 3$) and the cooling process ($4 \rightarrow 1$) proceeding at temporal constant ratios, given by [19],

$$\frac{dT}{dt} = k_1, \qquad \text{(for process } 2 \rightarrow 3)$$

and

$$\frac{dT}{dt} = k_2, \qquad \text{(for process } 4 \rightarrow 1), \tag{1}$$

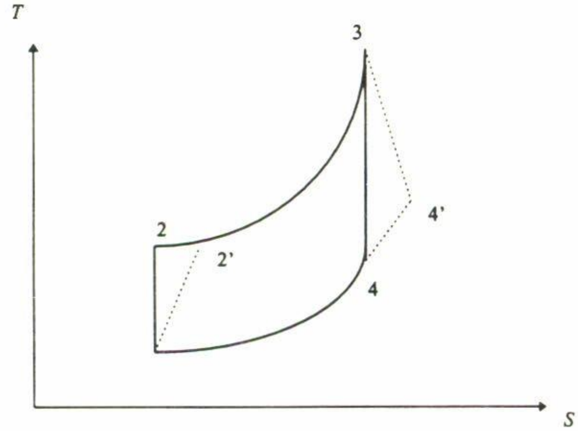


FIGURE 1. Temperature-entropy diagram of the cycle.

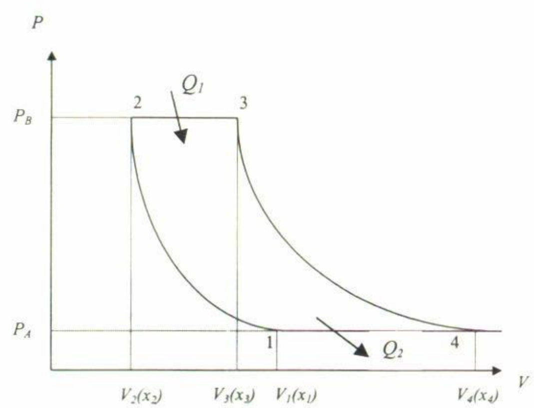


FIGURE 2. Pressure-volume diagram of the Joule-Brayton cycle. V_i $(i = 1, 2, 3, 4)$ correspond to displacements x_i $(i = 1, 2, 3, 4)$. The relation between V_i and x_i is $V_i = Ax_i$ where A is the cross section.

where T is the absolute temperature, t the time and k_1, k_2 are constants. Eqs. (1) may be taken as mean heating and cooling rates respectively. By integration of Eqs. (1), we obtain the heating and cooling times t_1 and t_2 respectively,

$$t_1 = K_1 (T_3 - T_2),$$

and

$$t_2 = K_2 (T_4 - T_1), \quad (2)$$

where $K_1 = 1/k_1$ and $K_2 = 1/k_2$ are new constants and T_j $(j = 1, 2, 3, 4)$ are the temperatures of the states 1 to 4 respectively. As it is usual in FTT-models, we take the adiabats $1 \to 2$ and $3 \to 4$ as instantaneous processes, *i.e.* the internal relaxation times in the adiabats are considered to be negligibly short compared to the duration of the process [20]. Thus, the period of the cycle is dominated by the nonadiabatic times and is given by

$$\tau \cong t_1 + t_2 = K_1 (T_3 - T_2) + K_2 (T_4 - T_1). \quad (3)$$

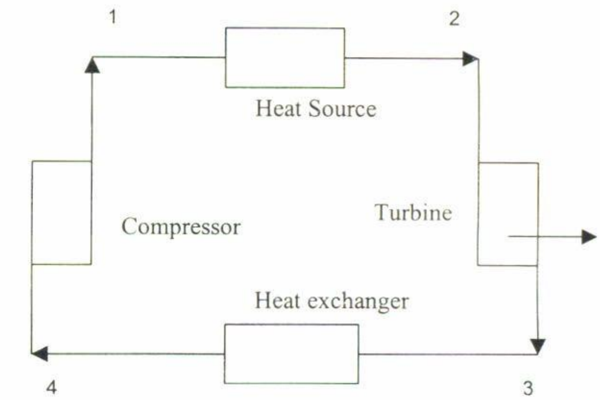


FIGURE 3. Basic components of the closed gas-turbine.

The net work of the cycle is given by the first law of thermodynamics

$$W_{TOT} = |Q_1| - |Q_2|, \quad (4)$$

the heat fluxes Q_1 and Q_2 are indicated in Fig. 2, and are given by

$$Q_1 = C_{P_B} (T_3 - T_2)$$

and

$$Q_2 = C_{P_A} (T_4 - T_1), \quad (5)$$

where C_{P_B} and C_{P_A} are the heat capacities of the working substance at the isobaric processes P_B and P_A respectively. By using Eq. (5) into Eq. (4), we obtain

$$W_{\mathrm{TOT}} = C_{P_B} (T_3 - T_2) - C_{P_A} (T_4 - T_1). \quad (6)$$

Dividing Eq. (6) by Eq. (3), we obtain a "reversible power", that is

$$P_R = \frac{W_{TOT}}{\tau} = \frac{C_{P_B} - C_{P_A} \dfrac{T_4 - T_1}{T_3 - T_2}}{K_1 + K_2 \dfrac{T_4 - T_1}{T_3 - T_2}}. \quad (7)$$

If we use one of the adiabatic relations given by [15],

$$T p^{(1-\gamma)/\gamma} = \mathrm{const.}$$

with $\gamma = C_P/C_V$ (C_P and C_V are the heat capacities at pressure and volume constants respectively), we can rewrite Eq. (7) in the following form:

$$P_R = \frac{C_{P_B} - C_{P_A} r_p^{(1-\gamma)/\gamma}}{K_1 + K_2 r_p^{(1-\gamma)/\gamma}}, \quad (8)$$

where $r_p = P_B/P_A$ is the so-called pressure ratio [15]. If we plot Eq. (8), we see that power output P_R is a monotonically increasing function of r_p and this contradicts the experimental fact that P_R is a convex function of r_p [15]. The function $P = P(r_p)$ can be converted in a convex function if we take into account dissipative effects. This is accomplished by means of considering a generalized friction that lumps all losses. We propose this friction force proportional to the velocity of the power stroke from state 2 up to state 4 (see Fig. 2). That is,

$$f_\mu = -\mu v = -\mu \frac{dx}{dt}, \tag{9}$$

where μ is a generalized friction coefficient that takes into account the global losses [6, 9], and x is the instantaneous displacement of the working fluid. Thus, the loss power by this mechanism is

$$P_\mu = \frac{dW_\mu}{dt} = f_\mu \frac{dx}{dt}, \tag{10}$$

or

$$P_\mu = -\mu \left(\frac{dx}{dt}\right)^2 = -\mu v^2. \tag{11}$$

If we take as an approximation the mean velocity between states 2 and 4, we get

$$\bar{v} = \frac{x_4 - x_2}{\Delta t_{42}} = \frac{x_2}{\Delta t_{42}}(r-1), \tag{12}$$

where $\Delta t_{42} \simeq \tau/2$ (since we take adiabats as instantaneous) and $r = V_4/V_2$ (see Fig. 2) is the expansion ratio, which can be rewritten in terms of the pressure ratio r_p, by means of the adiabatic relation PV^γ = const. and the equation of state of the working fluid (taken as a perfect gas, $PV = nRT$), obtaining

$$r = \theta r_p^{\left(2-\gamma/\gamma\right)}, \tag{13}$$

where $\theta = T_3/T_1$ is the quotient between the maximun temperature T_3 and the minimun one T_1. By means of Eqs. (13) and (12), Eq. (11) becomes

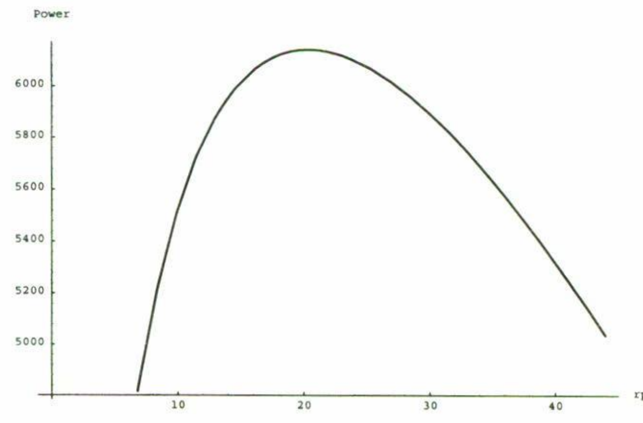
$$P_\mu = -b\left[\theta r_p^{(2-\gamma)/\gamma} - 1\right]^2, \tag{14}$$

where $b = \mu(x_2/\Delta t_{42})^2$, x_2 is given by the minimum volume V_2. Once we obtain the dissipative power, we can get the net power by means of Eqs. (14) and (8),

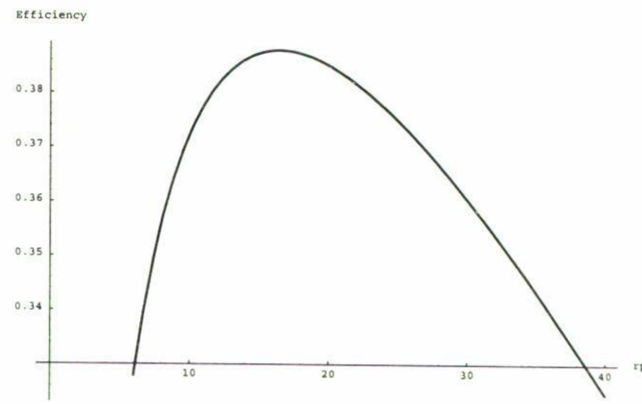
$$P = \frac{C_{P_B} - C_{P_A} r_p^{(1-\gamma)/\gamma}}{K_1 + K_2 r_p^{(1-\gamma)/\gamma}} - b\left\{\theta r_p^{\left[(2-\gamma)/\gamma\right]} - 1\right\}^2. \tag{15}$$

The temperature ratio θ is determined by technological constraints [10]. The thermal efficiency of the cycle can be immediately obtained as $\eta = P/(Q_1/\tau)$, with $Q_1 = C_{P_B}(T_3 - T_2)$. Then, by using Eqs. (15) and (3) we have

$$\eta = 1 - \frac{C_{P_A}}{C_{P_B}} r_p^{(1-\gamma)/\gamma} - \frac{b\left(\theta r_p^{(2-\gamma)/\gamma} - 1\right)^2}{C_{P_B}}\left(K_1 + K_2 r_p^{(1-\gamma)/\gamma}\right). \tag{16}$$



(a)



(b)

FIGURE 4. (a) Power and (b) efficiency *versus* pressure ratio curve.

If in Eq. (16), $C_{P_A} = C_{P_B}$ and $\mu = b = 0$, we immediately obtain the thermal efficiency of the ideal Joule-Brayton cycle, which is $\eta = 1 - r_p^{(1-\gamma)/\gamma}$ [21]. For the analysis of the behavior of functions $P = P(r_p)$ [Eq. (15)] and $\eta = \eta(r_p)$ [Eq. (16)], we take numerical values for the involved constants from Refs. 6, 8, 9, and 15, which are: $b = 32.5$ W, $\gamma = 1.4$, $\theta = 1073/288 = 3.7$, $C_{P_A} = C_{P_B} = 0.418464$ JK^{-1} and $K_1 = 8.128 \times 10^{-6}$ sK^{-1}, $K_2 = 18.67 \times 10^{-6}$ sK^{-1}. With these values, we obtain the Figs. 4a and 4b which show a convex behavior for both functions such as ocurrs in reality. The values $r_p^* = 19.5$ and $r_p^+ = 16.4$ wich maximize the power output and the efficiency respectively are typical pressure-ratio values for real gas-turbines [15].

By means of Eqs. (15) and (16), we obtain the loop-shaped curve depicted in Fig. 5, which also is a charecteristic of real gas turbines [12].

3. The nonendoreversible model

Since the FTT-pioneering work of Curzon and Ahlborn [22] the so-called endoreversibility hypothesis has played a very important role in FTT-analysis. This approximation consists

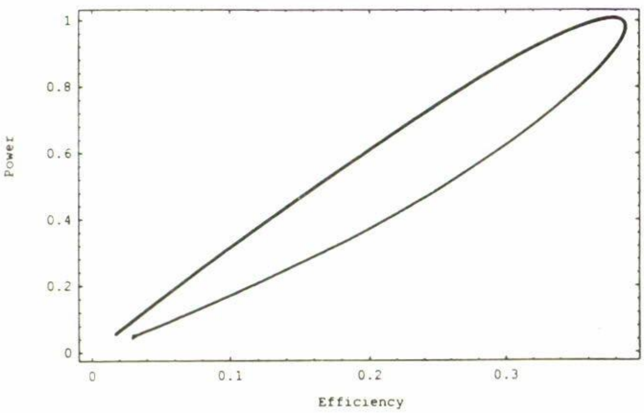


FIGURE 5. Loop-shaped curve for normalized power against efficiency.

in assuming that all irreversibilities in a thermal engine for example, can be ascribed to the couplings between the working fluid and its environment and is permitted that the working substance undergoes reversible transformations. As is well known, the internal irreversibilities are of major importance. One manner to include them in a thermodynamical analysis is by means of the Clausius inequality. Özkaynak *et al.* [19] and Chen [18] have recently proposed equivalent approaches for a nonendoreversible FTT-Carnot cycle. These authors go beyond the endoreversibility hypothesis by means of a parameter defined as [19]

$$R = \frac{\Delta S_{1w}}{|\Delta S_{2w}|}, \qquad (17)$$

which has values in the interval $0 \leq R \leq 1$; ΔS_{1w} is the change in the internal entropy along the hot isothermal branch and ΔS_{2w} is the entropy change corresponding to the cold isothermal compression. Evidently, in the endorreversible limit $R = 1$. By means of parameter R, the Clausius inequality for an internally irreversible cycle: $\Delta S_{1w} + \Delta S_{2w} \leq 0$, becomes an equality

$$\Delta S_{1w} + R\Delta S_{2w} = 0. \qquad (18)$$

Since the Joule-Brayton is formed by two adiabats and two non-adiabats (as it is the Carnot cycle), we can define as in the previous way a non-endoreversible factor given by

$$R = \frac{\Delta S_{1w}}{|\Delta S_{2w}|} = \frac{C_{P_B}}{C_{P_A}} \frac{\ln\left(T_3/T_2^{'}\right)}{\ln\left(T_4^{'}/T_1\right)}, \qquad (19)$$

where $T_4^{'}$ and $T_2^{'}$ are depicted in Fig. 1.

It is easy to show that

$$\alpha \equiv \frac{\ln\left(T_3/T_2^{'}\right)}{\ln\left(T_4^{'}/T_1\right)} < \frac{\ln\left(T_3/T_2\right)}{\ln\left(T_4/T_1\right)} = 1 \qquad (20)$$

Thus, the parameter R can be expressed as

$$R = \frac{C_{P_B}}{C_{P_A}} \alpha < 1. \qquad (21)$$

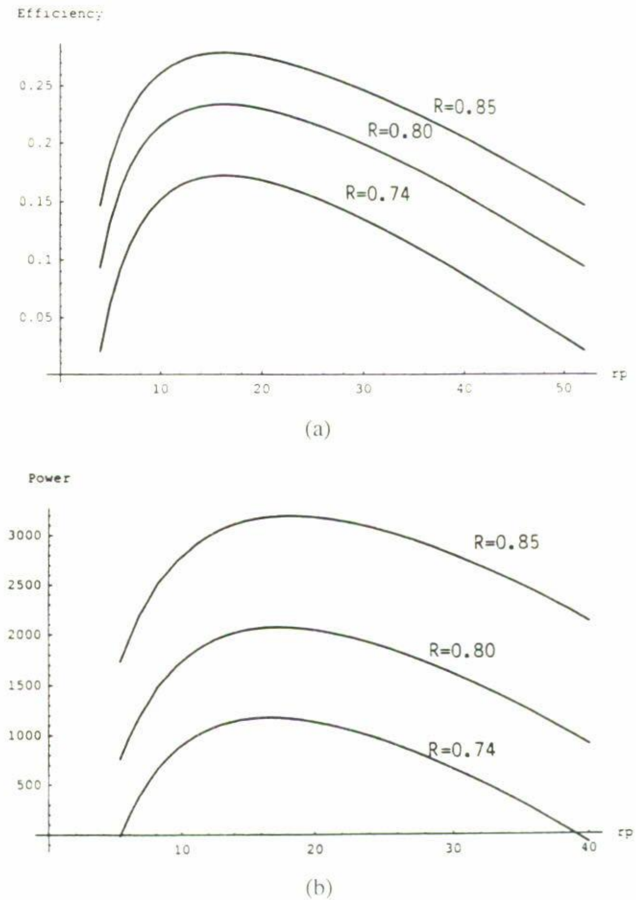


FIGURE 6. (a) Efficiency and (b) power *versus* pressure ratio curves for several values of R.

Through the quotient C_{P_B}/C_{P_A} we can introduce the parameter R in Eq. (16) for the efficiency and we get

$$\eta_{NE} = 1 - \frac{\alpha}{R}\left\{ r_p^{(1-\gamma)/\gamma} + \frac{b[\theta r_p^{(2-\gamma)/\gamma} - 1]^2}{C_{P_A}} \left[K_1 + K_2 r_p^{(1-\gamma)/\gamma}\right] \right\}. \qquad (22)$$

This equation reduces to the reversible efficiency when $R \to 1$, $\alpha \to 1$ and $b = 0$. If we take a typical value for $\theta = 3.7$ ($T_3 = 1100$ K and $T_1 = 300$ K) [15] and the values for b, K_1, K_2, C_{P_A}, C_{P_B} given in the previous section, we can see in Fig. 6a, that the efficiency is very sensitive to parameter R. The same behavior is observed for the power output which expressed in terms of R, becomes

$$P_{NE} = \frac{C_{P_A}\left[\dfrac{R}{\alpha} - r_p^{(1-\gamma)/\gamma}\right]}{K_1 + K_2 r_p^{(1-\gamma)/\gamma}} - b\left[\theta r_p^{(2-\gamma)/\gamma} - 1\right], \qquad (23)$$

In Fig. 6b, we see that also power output is very sensitive to R. We wish to remark for example, that for $R = 0.74$ we obtain $\eta = 0.17$ which is a typical real value for a gas turbine efficiency [23]. In fact the parameter R may be manipulated through its definition [Eq. (19)] for improving both efficiency and power output of gas-turbines.

4. Conclusions

As is well known the CET-models of thermal engines are the reversible limits of the real ones. Thus, the CET-models are of null power output, due to reversible processes (as sucession of equilibrium states) require a very long time to proceed. A first contribution of FTT was to provide no-null power models, that is, thermal cycles undergoing finite-time processes. The FTT-models in general consider dissipative terms and so positive entropy production. For this reason, the FTT-models are more realistic than the CET ones. In standard thermodynamics textbooks only CET-models for thermal engines are presented. For example in Ref. 15, when the Joule- Brayton cycle is discussed only the reversible efficiency $\eta = 1 - r_p^{(1-\gamma)/\gamma}$ is calculated although the author remarks that the true thermal efficiency has a convex behavior with an unique maximum point at certain pressure ratio. In the present paper we propose a Joule-Brayton cycle model that reproduces the real behavior of the efficiency discussed by Haywood [15]. Our model considers dissipative effects through a nonendoreversibility parameter. Our results are consistents with typical values of r_p and η for gas-turbines which are the devices that follow in an approximated manner the Joule-Brayton cycle. Another important feature of our model is that it shows the great sensitivity of power and efficiency to the parameter R. This result permits the suggestion of a procedure for improving power and efficiency of real gas-turbines.

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