Two-fluid model of the apparent slip phenomenon in Poiseuille flow

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A two-fluid model to analyze the apparent slip phenomenon in Poiseuille flow is presented in this work. The model was formally developed on the basis of the fluid mechanics and provides a theoretical support for the phenomenological correction given by Mooney to account for slip in the Poiseuille flow. In addition, the model predicts a dependence of the slip velocity on the flow geometry, which is not considered in the Mooney correction but has been observed experimentally in some polymeric systems in the presence of apparent slip. Finally, the main differences with previous similar analyses by other authors are discussed.

Keywords: Capillary flow; apparent slip; polymeric solutions

En este trabajo se desarrolló un modelo de dos fluidos para analizar el fenómeno de deslizamiento aparente en el flujo de Poiseuille. El modelo fue desarrollado formalmente con base en la mecánica de fluidos y es un soporte teórico para la corrección fenomenológica propuesta por Mooney para cuantificar el deslizamiento en el flujo de Poiseuille. Además, el modelo predice una dependencia de la velocidad de deslizamiento de la geometría de flujo, la cual no es considerada en la correción de Mooney, pero ha sido observada experimentalmente en algunos sistemas en la presencia del deslizamiento aparente. Finalmente, las diferencias principales con análisis similares previos realizados por otros autores son discutidas.

Descriptores: Flujo en capilar; deslizamiento aparente; soluciones poliméricas

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1. Introduction

It is well known that flow enhancement may take place during the flow of polymer solutions through capillaries [1, 5]. This flow enhancement has been called *apparent slip* to distinguish it from the real slip occurring in some polymer melts. Such an increase in the flow rate has been assumed to be due to the existence of a polymer depleted layer close to the capillary wall, which has a lower viscosity than that of the bulk fluid. Even though in the presence of apparent slip it is not expected a clear demarcation between the fluid close to the wall and that in the bulk, the fluid may have a completely different rheological behavior near to a solid wall than far from it, as shown by the results of Rofe *et al.* [6], where the velocity field was measured and a large velocity gradient was found in a small region close to the capillary wall. On the other hand, direct measurements of the polymer concentration in the neighborhood of a solid wall by Ausserré *et al.* [7], showed that this was continuously decreasing until it reaches an almost well defined constant value close to the solid wall, which clearly demarcates the slip layer. Therefore, on the basis of the previously mentioned results, the assumption of a two-fluid model to analyze the flow in the presence of apparent slip is plausible.

The hypothesis of a two-fluid model has been used by several authors to analyze the Poiseuille flow and the slip layer characteristics for polymer solutions and suspensions [2, 3, 8, 11], as well as for polymer melts [12]. However, some of these works have inconsistencies, in their assumptions or development. On the other hand, some of the above mentioned models do not show a dependence of the slip velocity on the rheological parameters of the core fluid.

The aim of this work is to show the suitability of a two-fluid model to analyze the Poiseuille flow in the presence of apparent slip. The model was formally developed on the basis of the fluid mechanics and provides a theoretical support for the phenomenological correction given by Mooney [1], to account for slip in the Poiseuille flow. In addition, the model predicts a dependence of the slip velocity on the flow geometry, which is not considered in the Mooney correction but has been observed experimentally in some polymeric systems [13, 14]. Also, the model provides an expression for the slip velocity which is a function of the rheological parameters of the fluid. Finally, the differences with previous analyses by other authors are discussed.

2. Model

Assuming that the apparent slip phenomenon is due to the formation of a polymer depleted layer, the capillary flow field can be thought as consisting of two concentric regions. These regions are filled with immiscible fluids, one in the core, which is representative of the polymer solution, region I, and the other one in the boundary slip layer of thickness δ, region II. It is supposed that the flow field is steady, laminar, isothermic, and fully developed. Such conditions are usually fulfilled in any rheometrical experiment. Thus, the velocity field $V(r)$ is divided in,

$$V_{\rm I}(r) \qquad \text{for} \qquad 0 \leq r \leq R-\delta,$$

and

$$V_{\rm II}(r) \qquad \text{for} \qquad R-\delta \leq r \leq R,$$

where r is the radial position and R is the capillary radius.

The velocity field must satisfy the boundary conditions:

1. The no-slip condition at the wall is satisfied, *i.e.*,

$$V_{\rm II}(r)\,|_R = 0. \tag{1}$$

2. The continuity of the velocity field in the interface between the two regions,

$$V_{\rm I}(r)\,|_{R-\delta} = V_{\rm II}(r)\,|_{R-\delta}\,. \tag{2}$$

The established continuity condition of the velocity at the interface in this work is more natural than the assumed continuous velocity gradient in the interface by Cohen and Metzner [3]. For example, when working with two-immiscible Newtonian fluids of different viscosity their assumption may lead to inconsistent situations.

By mass conservation, the total volumetric flow rate, Q, can be expressed as the sum of the contributions to the flow rate in each region:

$$Q = 2\pi \int_0^{R-\delta} V_{\rm I}(r) r\,dr + \int_{R-\delta}^{R} V_{\rm II}(r) r\,dr. \tag{3}$$

From this equation, and taking into consideration the boundary conditions (1) and (2), the apparent shear rate, $\dot\gamma_A$, can be written as

$$\dot\gamma_A = \frac{4Q}{\pi R^3} = \frac{4}{\tau_w^3}\left(\int_0^{\tau_\delta} f(\tau)\tau^2 d\tau + \int_{\tau_\delta}^{\tau_w} h(\tau)\tau^2 d\tau\right), \tag{4}$$

where $f(\tau)$ and $h(\tau)$ are the constitutive equations in regions I and II, respectively, given by

$$-\frac{dV_{\rm I}}{dr} = f(\tau) \qquad 0 \leq r \leq R-\delta, \tag{5}$$

$$-\frac{dV_{\rm II}}{dr} = h(\tau) \qquad R-\delta \leq r \leq R, \tag{6}$$

τ is the local shear stress, τ_w is the capillary wall shear stress and τ_δ is the shear stress at the interface between the two fluids given by

$$\tau_\delta = \tau_w\left(1-\frac{\delta}{R}\right). \tag{7}$$

Note that up to this point of the analysis there is not any restriction with regard to the type of fluid in each region, then (4) is a general expression for the flow of two immiscible fluids in the capillary geometry.

Now, suppose that $h(\tau)$ corresponds to the shear rate function for a Newtonian fluid, $h(\tau) = \tau/\mu$ where μ is the Newtonian viscosity, which could be the situation in the presence of apparent slip, and that $f(\tau)$ represents the shear rate for a non-Newtonian power law fluid, $f(\tau) = (\tau/m)^{1/n}$ where m and n are the power law parameters. If these assumptions about $f(\tau)$ and $h(\tau)$ are used in Eq. (4) and an expansion in power series of δ/R is carried out, then we have

$$\dot\gamma_A = \frac{4n}{3n+1}\left(\frac{\tau_w}{m}\right)^{\frac{1}{n}} + \frac{4\delta}{R}\left\{\left[\frac{\tau_w}{\mu} - \left(\frac{\tau_w}{m}\right)^{\frac{1}{n}}\right] + \frac{1}{2}\frac{\delta}{R}\left[\frac{2n+1}{n}\left(\frac{\tau_w}{m}\right)^{\frac{1}{n}} - 3\frac{\tau_w}{\mu}\right] + \ldots\right\}. \tag{8}$$

It can be observed that a first order approximation in δ/R in Eq. (8) reproduces the Mooney equation [1] and the result of Funatsu and Sato [12], being the first term of the right hand side the shear rate free from slip, and the apparent slip velocity V_s, is given by the second term in the right hand side of the Eq. (8). At this order of approximation, δ and μ are coupled and they can not be obtained explicitly, unless an assumption on δ or μ is provided. This difficulty is inherent to macroscopic two-fluids models [15]. In general, the apparent slip velocity is given by

$$V_s = \delta\left\{\left[\frac{\tau_w}{\mu} - \left(\frac{\tau_w}{m}\right)^{\frac{1}{n}}\right] + \frac{1}{2}\frac{\delta}{R}\left[\frac{2n+1}{n}\left(\frac{\tau_w}{m}\right)^{\frac{1}{n}} - 3\frac{\tau_w}{\mu}\right] + \ldots\right\}. \tag{9}$$

Observe that this equation for the slip velocity is dependent on the core fluid parameters m and n, the layer thickness δ, and the fluid layer Newtonian viscosity μ, in addition to the known dependence on the shear stress. Also, Eq. (9) predicts a dependence on the capillary radius. Similar equations can be easily obtained for the case of slit flow [16].

In most cases the δ/R ratio is usually considered to be very small, and therefore is neglected. Nevertheless, there exist physical situations where δ/R can not be neglected, so, more terms in Eq. (9) should be considered. This situation will arise when non-linear Mooney plots are obtained, in this case, the δ and μ can be determined from the coefficients of a non-linear fitting of $\dot\gamma_A$ *vs.* $1/R$. The result may be particularly important when studying the flow through porous media, vein blood flow, flow through membranes and chromatography, where the flow takes place through very narrow channels. This hypothesis is supported by the work of Brunn *et al.* [17], who analyzed the slip in tube and channel flow using a dimensional approach. They introduced a characteristic length of a particle and concluded that the slip velocity V_s does not depend on R, for large enough R, and that there is a dependence upon the sixth power on R for small R. Also, they concluded that if the characteristic dimension is no longer the size of an individual particle, rather the size of the structure, then the slip layer would not appear infinitely thin.

On the other hand, there are some works supporting the fact that a dependence of the apparent slip velocity on the capillary diameter may arise. Badura *et al.* [13], when studying a rubber with a softener system (NBR 3807/DBP 20% Vol%) found that a Mooney plot did not render an straight line, however, a plot of $\dot{\gamma}_A$ vs $1/R^2$ did. This is in agreement with Eq. (9) in the sense that the slip velocity expression can be dependent on the capillary radius. In addition, Mourniac *et al.* [14], when analyzing their data on a SBR compound and assuming that the slip velocity was due to the formation of a thin layer of fluid close to the wall, obtained an expression for the slip velocity which fitted their slip data. Their expression is a function of the fluid parameters in the core, as well as a function of the die geometry.

Finally, the velocity profile $V(r)$ can be obtained by using Eqs. (5) and (6):

$$V(r) = \frac{R}{2}\frac{\tau_w}{\mu}\left[1-\left(\frac{\delta}{R}\right)^2\right], \quad R-\delta \leq r \leq R, \tag{10}$$

and

$$V(r) = V_\delta + \frac{n}{n+1}R\left(\frac{\tau_w}{m}\right)^{\frac{1}{n}}\left[\left(1-\frac{\delta}{R}\right)^{\frac{n+1}{n}} - \left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right], \quad 0 \leq r \leq R-\delta, \tag{11}$$

where V_δ is the interfacial velocity between the two fluids, given by

$$V_\delta = \delta\frac{\tau_w}{\mu}\left(1-\frac{1}{2}\frac{\delta}{R}\right). \tag{12}$$

Note that this interfacial velocity is not the same as the slip velocity given by Eq. (9), which is the total slip contribution to the velocity field. In this point, it is convenient to comment on some other two-fluid models appearing in the literature, since there are differences in the definition of slip velocity with respect to this work. Yilmazer and Kalyon [11] studied highly filled suspensions and obtained an expression to evaluate the slip velocity as a function of the slip layer thickness and its viscosity. Such expression is identical to Eq. (12), however, they considered that such velocity is the Mooney apparent slip velocity, instead of the interfacial velocity as shown above. The slip velocity should be given by Eq. (9), which explicitly includes the flow parameters m and n of the core fluid.

Another two-fluid model given by Kozicki *et al.* [18], postulated a correction term accounting for the anomalous behavior in the vicinity of the capillary wall, *i.e.*, to estimate the slip layer thickness and slip velocity. This term is expressed by the function $g(\delta,\tau,r)$, which is zero at $r<R$. Assuming that in the domain $R-\delta \leq r \leq R$, the fluid properties represented by $f(\tau)$ can be approximated by $f(\tau_w)$, the authors integrated the corrected constitutive function and obtained that the slip velocity was given by

$$V_s = \int_{R-\delta}^{R} g(\delta,\tau,r)dr. \tag{13}$$

This V_s can not be related, directly, to the apparent slip velocity of Mooney. However, the final expression of Kozicki *et al.* [18], to first order approximation in δ/R ratio, agrees with the first order approximation for the slip velocity in this work, see Eq. (9). They obtained this expression assuming that the velocity gradient in the slip layer is constant and equal to τ_w/μ. Note that in the Kozicki *et al.* model, the possibility of having a slip velocity depending on capillary radius is not taken into consideration, in contrast with the result in the present work.

On the other hand, Cohen and Metzner [3] developed an expression for the Mooney apparent slip velocity, which was approximated to zero order in δ/R, yielding

$$V_s = \frac{1}{2}\delta\left[p(\tau_w,c_w) - q(\tau_\delta,c_\infty)\right], \tag{14}$$

where $p(\tau_w,c_w)$ and $q(\tau_\delta,c_\infty)$ are the shear rate functions of a Newtonian fluid in the layer and of a non-Newtonian fluid in the core respectively . These functions depend on the shear stress and polymer concentration. The above expression corresponds to our Eq. (9) considering the same order of approximation, except for a factor of $\frac{1}{2}$. This difference can be explained observing that the Cohen and Metzner model is not physically equal to the present one, being the main difference the boundary conditions, as discussed in the previous section.

Finally, it is very important to point up here the meaning of the slip velocity as introduced by Mooney. The slip velocity was introduced by Mooney as a phenomenological correction to account for the flow enhancement observed in some polymer systems. In the presence of apparent slip in polymer solutions there is not true slip at all, and therefore, the slip velocity is just a correction term.

3. Application

To illustrate the suitability of the model to analyze the Poiseuille flow in the presence of apparent slip, it was applied to the experimental data obtained in the capillary flow of 0.2% Xanthan aqueous solutions, which are presented in Fig. 1 and clearly show evidence of apparent slip.

The plots of $\dot{\gamma}_A$ *vs.* $1/R$ are shown in Fig. 2. It is observed that they appear linear, therefore, Eq. (8) can be used at a first approximation in $1/R$, which reduces to the Mooney result for a power law fluid. Note that the second term in Eq. (8), belonging to the slip velocity, is dependent on the slip layer

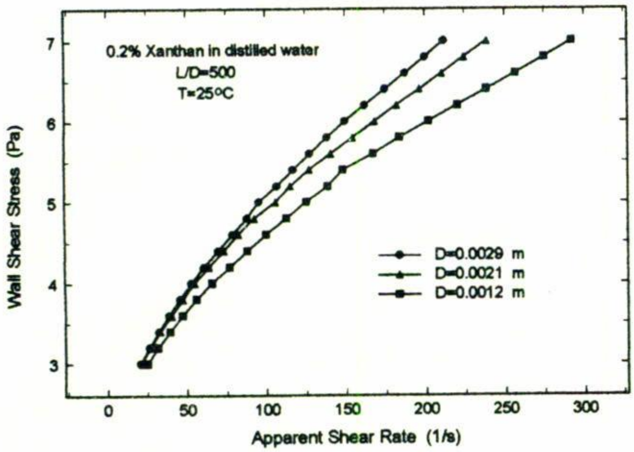

FIGURE 1. Flow curves of 0.2% Xanthan aqueous solution, obtained with a capillary rheometer.

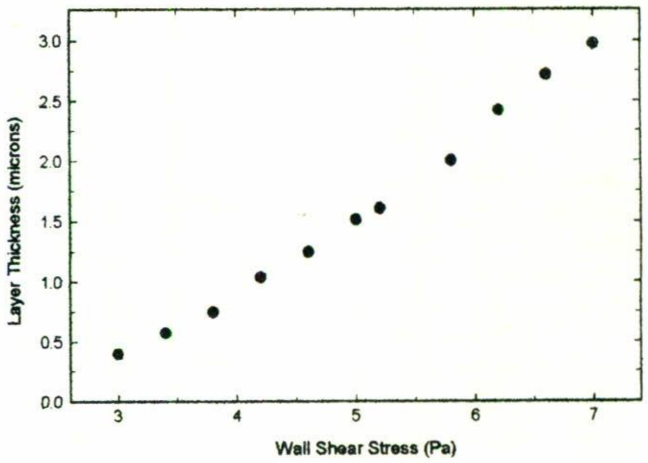

FIGURE 3. Variation of the slip layer thickness with the shear stress for the 0.2% Xanthan aqueous solution calculated from Eq. (15).

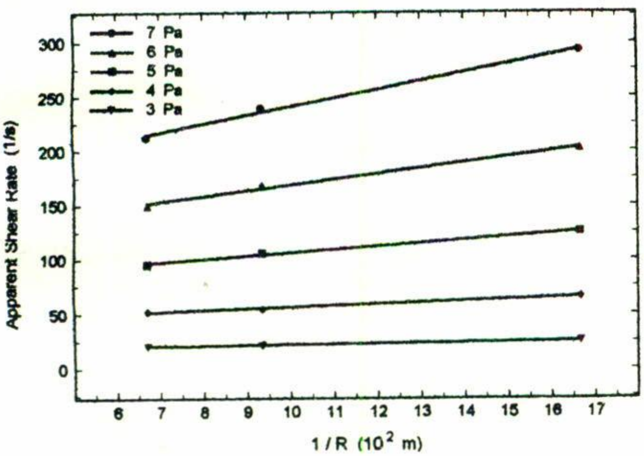

FIGURE 2. Apparent shear rate versus the inverse of capillary radius for the 0.2% Xanthan aqueous solution. The linearity of the plots suggests a first order approximation in δ/R for the model proposed.

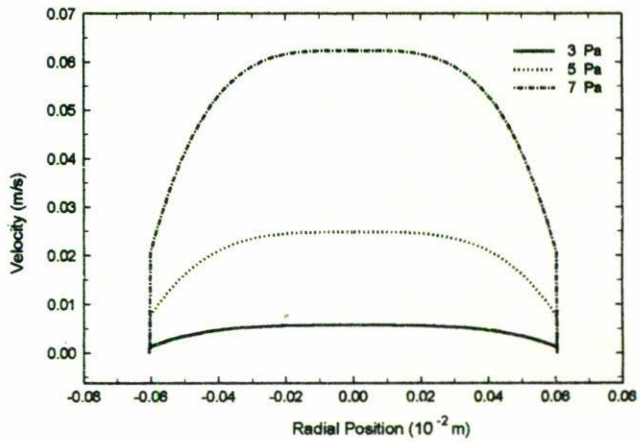

FIGURE 4. Calculated velocity profiles for the 0.2% Xanthan aqueous solution for shear stress 3,5, and 7 Pa, $R = 0.0006$ m.

viscosity μ, its thickness δ and the rheological parameters of the core fluid n and m. Thus, the parameters m and n of the polymer solution can be easily determined, but, due to the constant value of the slip velocity at a given shear stress, μ and δ of the slip layer can not be independently determined. However, the viscosity of the slip layer is bounded by the lowest possible value belonging to the viscosity of the solvent (μ_s), *i.e.*, $\mu \geq \mu_s$. For this polymer solution in the limiting case, $\mu = \mu_s = 10^{-3}$ Pa·s, corresponding to the water viscosity, then the layer thickness δ can be calculated as

$$\delta \geq \frac{V_s}{\dfrac{\tau_w}{\mu} - \left(\dfrac{\tau_w}{m}\right)^{\frac{1}{n}}} \tag{15}$$

The variation of the slip layer thickness with the shear stress, calculated from Eq. (15) is shown in the Fig. 3. It can be observed that the layer thickness is less than 4 microns under the assumptions given above, and increases with the shear stress as reported by Ausserré *et al.* [7].

The velocity profiles obtained by using this model [Eqs. (10) and (11)], are illustrated in Fig. 4 for the shear stresses values of 3, 5, and 7 Pa, assuming the corresponding lowest possible value of the slip layer thickness, *i.e.*, using the viscosity of the solvent. Note that the velocity profiles are qualitatively in good agreement with those of Rofe *et al.* [6], and show a high velocity gradient near to the wall. A quantitative comparison with the work of Rofe *et al.* can not be performed because they assumed that the slip velocity is given by the extrapolation to the wall of their velocity profile, being their closest datum point located at 30 microns to the wall.

4. Conclusions

The model presented in this work, supports the phenomenological correction given by Mooney to account for slip in the Poiseuille flow. In addition, the model allows to know the properties of the bulk fluid, in the present case, the parameters of the power law fluid, but the model does not give in a independently way the parameters that describe the slip layer fluid, δ and μ, when linear Mooney plots are obtained. However, making reasonable assumptions about the value of one of these parameters, either δ or μ, a procedure of flow data reduction for polymer solutions in a capillary or slit geometry can be obtained.

In the case when non-linear Mooney plots are obtained, δ and μ can be determined by using a non-linear fitting. Also, an expression for the slip velocity is obtained, which predicts a dependence in the δ/R ratio, the core fluid, as well as, the layer fluid parameters.

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