Physical interpretation of thermal waves in phototermal experiments

G. Gonzalez de la Cruz and Yu. G. Gurevich
Departamento de Fisica, Centro de Investigación y Estudios Avanzados, Instituto Politécnico Nacional
Apartado postal 14-740, 07000 México, D.F., Mexico

Recibido el 3 de diciembre de 1997; aceptado el 22 de octubre de 1998

The physics of thermal waves propagation created in a fluid or solid medium by a periodic heat generation are examined from the theoretical point of view. The temperature distribution in the sample as a function of both, time and position, is obtained by solving the heat diffusion equation with appropriate boundary conditions, the time dependent heat flux at the surface of the sample and fixed temperature at the opposite surface are used as boundary conditions. The attenuation and reflection of thermal waves in a the temperature fluctuation in a dissipationless sample, is can be explained as a consequence of the periodic time variation of the incident radiation converted into heat at the heat flux at the surface of the sample which can be positive or negative. The thermal waves spectrum is not precisely like electromagnetic wave propagation, features that appear to be shared by all photothermal models of thermal waves propagation. The response of the system to this external pertubation in the limit of high and low modulation frequency ω of the incident light respect to the characteristic time τ of the sample is analayzsed.

Keywords: Thermal waves; attenuation and reflection; characteristic time; longitudinal and transversal waves

En este trabajo se analiza la propagación de ondas térmicas en un sólido generadas por la absorción de la radiación electromagnética en la superficie de la muestra. La distribución de temperatura en la muestra se obtiene resolviendo la ecuación de difusión de calor con apropiadas condiciones a la frontera consistentes con el experimento. La atenuación y reflexión de las ondas térmicas en un medio no disipativo se explica como consecuencia de la variación periódica en el tiempo del flujo de calor en la superficie de la muestra debido a la radiación incidente. Se hace referencia a la diferencia entre las ondas electromagnéticas y el espectro de las ondas térmicas. Finalmente, la respuesta del sistema a esta perturbación externa en el límite de alta y baja frecuencia respecto al tiempo característico de la muestra es analizado.

Descriptores: Ondas térmicas; atenuación y reflexión; tiempo característico; ondas longitudinales y transversales

PACS: 65.90; 78.90

1. Introduction

Diffusion is a process by which matter and/or heat is transported from one part of the system to another as a result of random particle collisions. Transfer of heat was recognized by Fick in 1855 [1], who first put diffusion on a quantitative basis by adopting the mathematical equation of heat conduction derived some years earlier by Fourier (1822). The mathematical theory of diffusion in isotropic substances is therefore based on the hypotesis that the rate of transfer of diffusing substance through unit area of a secction is proportional to the concentration gradient measured normal to the section. and energy conservation law. The two corresponding one dimension equations in heat flow are

$$Q = -k\frac{\partial T}{\partial x} \tag{1}$$

$$\rho c\frac{\partial T}{\partial t} = -\frac{\partial Q}{\partial x} \tag{2}$$

where T is the temperature, k is the heat conductivity, ρ is density, and c is specific heat, so that ρc is the heat capacity per unit volume. The space coordinate is x and t is time. In Eq.(1) Q is the amount of heat flowing in the direction of x increasing per unit time through unit area of a section which is normal to the direction of x. Many authors [2] propose an additional term $\tau_c\partial^2T/\partial t^2$ in Eq. (2) where τ_c is the energy relaxation time, *i.e.*, is the elapsed time at which the extra energy is distributed in the sample. This new equation really represents a wave equation instead of a diffusion one and it is valid when $\tau_c > \omega^{-1}$, where ω being the frequency of the modulation light converted into heat in photothermal experiments. However, it is well known that the temperature T is a thermodynamic parameter which describes the average energy of the system; thus if τ_c satisfies $\tau_c > \omega^{-1}$ T is not a well defined parameter and Eq. (2) loses its physical meaning. It is a good reason why it is not convenient to use the temperature as a boundary condition at the surface of finite samples in heat flux problems; heat conservation used as boundary conditions at the interfaces of the sample is always a well defined quantity. In this work, we shall restrict ourselves to study conduction of heat in isotropic solids in which Eqs. (1) and (2) are valid, *i.e.*, $\tau_c \approx 0$, when compared with the these terms in typical photothermal experimental conditions.

Carslaw and Jaeger's [2] and other books [3] contain a wealth of solutions of the heat-conduction equation. One of them usually correspond to so called thermal waves. The most important fact about thermal waves is that they are rapidly attenuated. A plane thermal wave has the form $e^{i\omega t-\sigma x}$, where $\sigma = (1\pm i)(\omega/2\alpha)^{1/2}$ (see Sect. 2). It is thus damped by a factor e^{-1} in a distance called the thermal diffusion length $L = (2\alpha/\omega)^{1/2}$. Then any thermal wave generated at the sample depth greater than L will thus have negligible effects on the surface of the sample. Therefore from Eq. (1), the heat associated with this thermal wave also dissapears at the same diffusion length. However, in a dissipationless media where there are not any sources and sinks of heat, a natural question arises; what does happen with the energy conservation? In addition the extra terms, $+i(\omega/2\alpha)^{1/2}$ represent a reflected thermal wave in the sample. If heat is a diffusion process: what is the physical interpretation of these terms? It is worth mentioning that the boundary conditions used by those authors (*e.g.* continuity of the temperature at the surface or at the interface of two different media, a plane periodic heat source at the surface $\Delta Q e^{i\omega t}$, etc.), do not represent the usual experimental conditions in the photothermal experiments (absorption of chopped light produces a periodic heating at the surface of the solid). There is general awareness among scientists and engineers that the phenomena of heat flow and diffusion are basically the same. Nevertheless, many physicists experience difficulty in making the difference between heat flux as a diffusion process and heat transport like a "electromagnetic wave".

A very active area of research in applied physics these days comes under the general heading of photothermal or photoacoustic phenomena. Photothermal techniques in solid materials are becoming a valuable tool in measuring thermal parameters as well in the semiconductor industry for characterizing process in the manufacturing of electronic devices [4]. These techniques are versatile, nondestructive and can be employed under different experimental conditions for determining thermal parameters of solid and liquid materials. Several photoacoustic cells with slight modifications, including the derivative photopyroelectric and photothermal deflection methods, have been used in some special cases with great success [5]. The apparatus for all techniques is basically the same, the modifications being mainly concentrated on the type of detector and on its location relative to the sample inside the cell. Recently, a new technique has been described in which a transient thermoelectric voltage of a semiconductor is measured after a pulse laser radiation, it is generated by heating the semiconductor through absorption of an optical pulse, furthermore of the thermal characterization, it is also possible to obtain inforrmation about the electronic energy spectrum in semiconductors [6]. In all the cases, the photothermal signal depends on the material thermal properties, interaction between the quasiparticle systems as well as on the geometry of the sample. The fact the photothermal signal depends upon how the heat diffuses through the sample, allows us to perform thermal characterization of the sample (*i.e.*, measurments of its thermal conductivity and thermal diffusivity) and carrier transport propierties [9]. The absorption of an incident energetic chopped light beam by the quasiparticle systems and the subsequent relaxation processes gives arise to a periodic heat source in the sample, which may be distributed throughout its volume. This periodic heating of the sample, causes both temperature and/or pressure fluctuations within the sample (thermal waves) propagation inside the sample, which are then detected by thermal or acoustic, or even both, sensing devices.

In the present work, heat diffusion in solids or fluids created by a periodic light beam is examined from the physical point of view for a wide range of modulation frequency. We restrict our analysis by solving the heat-diffusion equation assuming that the sample is optically opaque to the incident light (*i.e.*, all the incident light is absorbed at the surface) and the optical carrier generation or recombination is neglected as well as quasiparticle systems with different temperature *e.g.* electrons and phonons in which $T_e \neq T_p$ [7]. It is clear that when the intensity of the radiation is fixed, the light-into-heat conversion at the surface ($x = 0$) can be used as boundary condition. The heat flux in the time-independent problem must be maintained in addition to the plane heat source with sinusoidal time dependence at the condition instead of the temperature fluctuation at surface, which in general it is an unknown parameter in the experimental conditions [11]. In addition, because the specimen is usually in contact with a heat reservoir at some temperature T_0, the continuity of the temperature distribution is used as boundary condition at the opposite surface ($x = d$). It is important mentioning that the solutions of the heat diffusion equation obtained in this work are not new, they were first obtained by Rosencwaig and Gersho [10] in the interpretation of the photoacoustic effect in solids on the basis of thermal waves. However, the physical meaning of the solutions and the boundary conditions used in this work are not completely clear.

2. Heat-flow equations

It is well known that heat transport in solids is carried out by various quasiparticle systems (electrons, phonons, etc.). Frequently the interactions between these quasiparticles are such that each of these systems can have its own temperature and the physical conditions at the boundary of the sample may be formulated separately for each quasiparticle system. It can be shown that under certain conditions on the relaxation frequency of the electron and phonon systems and the size of the sample, the total system can be described by the same temperature T and the total heat flux satisfies $\operatorname{div} Q = 0$ in the static approximation, *i.e.*, the heat flux is independent of time. However, in the photothermal experiments, the incident radiation is modulated on time by the chopper and in this case, it is necessary to consider the dynamic contribution in the heat transport equation Eq. (2). In the case of semiconductors, the absorption of light is accompanied by the generation of electron-hole excitations which have a finite lifetime and which travel a finite distance before they transfer theiry energy to the sample in the form of heat. Consequently, the photothermal signal is governed not only by the absorption coefficient of light but also by the characteristics of the transport processes such as the carrier lifetime, the transport length and surface recombination, so that the photothermal response depends on all these quantities which can be deduced from the experimental data by analyzing the amplitude and phase of the photothermal signal as a function of the modulation frequency [12]. Strong surface recombination will be considered in this work such that, the heat source is quite confined at the surface of the sample, in other words, the incident radiation is totally absorbed at the surcace and converted into heat, therefore, in this approximation the electron-hole excitations are neglected. In the heat diffusion equation, we are considering that the thermal conductivity k is independent of time and coordinates temperature, this approximation is valid when the intensity of the incident radiation is not strong. The solution $T(x,t)$ should be supplemented by boundary conditions at the surface $x = 0$. In the photothermal experiments, the most common mechanism to produce thermal waves is the absorption by the sample of an incident modulated light beam, $I_0 + \Delta I e^{i\omega t}$ where I_0 with is the incident monocromatic ligth of modulation frequency ω. Let β denote the optical absortion coefficient of the solid sample. The heat density produced at any point x due to the ligth absorbed at this point in the solid is given by

$$\beta e^{-\beta x}(I_0 + \Delta I e^{i\omega t}). \tag{3}$$

It is clear that when the intensity of the radiation is fixed, the light-to-heat conversion at the surface may be written as the thermal diffusion equation in the solid taking into account the distributed heat source

$$\rho c \frac{\partial T}{\partial t} + \operatorname{div} \vec{Q} = \beta e^{-\beta x}(I_0 + \Delta I e^{i\omega t}). \tag{4}$$

This full equation is somewhat difficult to solve. However, physical insight may be gained by examining the special case according to the optical opaqueness of the solids as determined by the optical absorption coefficient. Assuming that the sample is optically opaque to the incident light ($\beta\, d \gg 1$, d is the thickness of the sample) so that, all the incident light is absorbed at the surface and converted into heat. Then, it is possible to eliminate the right hand side term in Eq. (4) and write the general boundary condition at the surface of the solid as

$$Q(x,t)|_{x=0} = (Q_0 + \Delta Q e^{i\omega t}) - \eta(T - T_0)|_{x=0}, \tag{5}$$

where η is defined as the thermal surface conductivity [13], T_0 the ambient temperature and T the distribution temperature into the sample. Physically the second term in Eq. (5) represents the heat flux from the surface of the sample to the surrounding media. In the limit when η vanishes (adiabatic approximation), the heat boundary condition is only given by the first term. (We shall use these conditions in this work). Here Q_0 is the average over time of the total heat flux $Q(x,t)$ at the surface of the sample and it is proportional to the intensity of the incident light (it is frequency-independent only if the incident radiation is chopped in equal time-intervals, otherwise it is frequency-dependent). Physically, this static heat flux at the surface gives rise to a dc temperature distribution in the specimen, while the dynamic contribution part in Eq. (3) represents a heat source with sinusoidal time dependence at the surface $x = 0$ and it produces a thermal wave propagation into the sample. The temperature is not used as boundary condition because it is usually an unknown parameter in the photothermal experiments and besides that it is necessary to know the temperature on the back. It is only important in thermoelectric phenomena in semiconductors where the specification of the temperature on the surfaces of the sample must be know. At this point, it is important to compare the boundary conditions used in this work with the other ones used in previous theories; Rosencwaig [10] and many scientists consider the temperature and heat flow continuity at the surface of the sample $x = 0$ in order to describe thermal waves in the photoacoustic experiments. In this work, we only consider the light-to-heat-conversion at the surface since this is a well-known parameter while, the temperature is usually an unknown parameter in all the photothermal experiments. Opsal and Roesencwaig [14], on the other hand, used a plane periodic heat source $Qe^{i\omega t}$ at the surface of a semi-infinite elastic body as boundary condition to study the thermal-wave depth profiling. However, in order to observe the phothothermal effect, the sample must to be illuminated with light which is intensity-modulated by a mechanical chopper to generate thermal waves inside the solid by absorption of radiation. The intensity radiation is usually must to be described as $I = I_0 + \Delta I e^{i\omega t}$, where in general $I(t), I_0 > 0$ and $\Delta I e^{i\omega t}$ can be positive or negative and $\Delta I < I_0$. Thus according to the Opsal and Rosencwaig model the plane periodic heat source is solely determinated by the dynamic part of the incident radiation into heat conversion neglecting the static contribution in the boundary condition at the surface of the sample *i.e.*, $Q_0 = 0$, and the average on time of the heat source at the surface vanishes, *i.e.*, $\langle Qe^{i\omega t}\rangle_t = 0$. This model, however has not physically meaning because the amplitude of the modulation heat is greater than $\langle Q\rangle = Q_0$ *i.e.*, $\Delta Q > 0$.

3. Temperature distribution in the sample

The general solution of the heat diffusion equation for this system is given as

$$T(x,t) = A + Bx + \Theta(x,t). \tag{6}$$

In order to obtain the constants A, B and the dynamic contribution to the temperature distribution $\Theta(x,t)$, it is also necessary to specify the boundary condition at some point in the sample $x = d$. Because the temperature is a thermodynamic parameter, the all system has to be in contact with a heat reservoir at some temperature T_0, *e.g.*, in the standard photoacoustic cell configuration the system consists of a solid sample in a small, gas-filled cell, a transparent window through which a modulated radiation beam is incident and a condenser microphone mounted on the cell walls to detect the acoustic signal produced in the gas chamber, see Fig. 1, all the system is at room temperature. So that, again, it is natural to choose the following general boundary condition

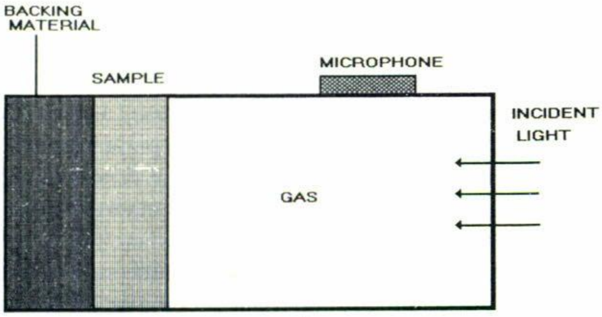

FIGURE 1. Schematic photoacoustic cell configuration.

$Q(x,t)|_{x=d} = \eta(T - T_0)$ which in the limit $\eta \to \infty$ (good thermal contacts) gives the continuity of the temperature distribution in the sample at $x = d$, *i.e.*, $T(x,t)|_{x=d} = T_0$.

It is important mentioning that the one-layer model used in this work, actually represents an effective medium in the photothermal experiments, *e.g.*, in photoacoustic experiments the layer represents the sample to be analayzsed, the gas chamber and the substrate. Our theory allows us to obtain the effective thermal parameters of the sample.

Using the boundary conditions on Eq. (6) the constants A, B may be written by

$$A = T_0 + \frac{Q_0}{k}d, \quad B = -\frac{Q_0}{k}, \tag{7}$$

and the solution $\Theta(x,t)$ satisfies the heat diffusion equation similar to Eq. (2) with the following boundary conditions:

$$-k\frac{\Theta(x,t)}{dx}\bigg|_{x=0} = \Delta Q e^{i\omega t},$$
$$\Theta(x,t)|_{x=d} = 0. \tag{8}$$

The general solution for $\Theta(x,t)$ is

$$\Theta(x,t) = (f_1 e^{-\sigma x} + f_2 e^{\sigma x})e^{i\omega t}. \tag{9}$$

The parameter σ is determinated by forcing Eq. (9) to satisfies Eq. (2) and it is equal to $\sigma = (1+i)\sqrt{\omega/2\alpha}$ and $\alpha = k/\rho c$ is the thermal diffusivity. Using the boundary conditions at $x = 0$ and $x = d$, the constants f_1 and f_2 are given by

$$f_1 = \frac{e^{\sigma d}}{e^{\sigma d} + e^{-\sigma d}}\frac{\Delta Q}{k\sigma}, \quad f_2 = \frac{e^{-\sigma d}}{e^{\sigma d} + e^{-\sigma d}}\frac{\Delta Q}{k\sigma}. \tag{10}$$

As can be seen from Eq. (9) the decreasing exponential term of time-dependent part of the the temperature fluctuation into the sample attenuates rapidly to zero with increasing distance from the surface such that at a distance $L \approx \sqrt{2\alpha/\omega} = \sqrt{2k/\rho c\omega}$, this contribution to the temperature fluctuation is effectively damped out. Physically it represents a propagation of the thermal waves from the surface into the sample, while the growing exponential term is associated with the propagation of these waves from the sample towards the surface of the specimen ("reflected thermal wave"). It is also possible to observe that the heat wave can penetrate deep into the sample in the limit of low frequency and high thermal conductivity, however, the thermal wave attenuates quickly if the heat capacity is high *i.e.* it is necessary to introduce a great amount of heat energy into the sample to elevate one degree its temperature. At this point, an important question arises: why does the temperature attenuates at a distance L from the surface of the sample in a dissipationless media? The answer is the following: The modulation of light-into-heat conversion at the surface given by $\Delta Q e^{i\omega t}$ can be positive or negative. It is positive during a half period of the modulation incident radiation, *i.e.*, for π/ω, the temperature at the surface is higher than inside of the sample and during this time there is propagation of heat from the surface to the bulk of the elastic body. This effect is associated with the decreasing exponential term in Eq. (9). On the other hand, the growing exponential term in Eq. (9) represents the heat flux from inside to the surface of the sample when $\Delta Q e^{i\omega t}$ is negative, *i.e.*, now, during the period of time π/ω the bulk of the sample is hotter than its surface. According to this brief discussion, it is important to note that both, the sinusoidal behavior of the source heat at the surface and the thermal parameters of the sample, give the fundamental characteristics of the propagation of thermal waves.

The temperature distribution resulting from the modulated light-to-heat conversion in the sample is

$$T(x,t) = T_0 + \frac{Q_0}{k}(d-x) + \frac{\Delta Q}{k\sigma}\frac{\sinh\sigma(d-x)}{\cosh\sigma d}e^{i\omega t}. \tag{11}$$

Once we know the temperature distribution in the sample, we can assume some model in order to calculate the photothermal signal (*e.g.*, photoacoustic signal, photothermal deflection, etc.) [5].

As can be seen from Eq. (11), the dynamic contribution to the temperature distribution has a sinusoidal dependence through the imaginary part of the exponential terms, *i.e.*, $\exp\left[\pm i\sqrt{2\alpha/\omega x}\right]$. Nevertheless, the propagation of thermal waves in the sample are not similar to the propagation of electomagnetic waves, the heat flux is a diffusive process and it is described by the heat diffusion equation Eq. (2), while the propagation of electromagnetic radiation satisfies the wave equation.

4. Comparison with electromagnetic waves

We believe, at this point, that its important to emphazise the main differences between the propagation of electromagnetic waves (can also be sound waves) and thermal waves. In the absence of rigorous theoretical guidance from first principles, several workers find it neccesary to introduce arbitrary algebraic factors into their calculations in order to get a desirable fit to the data. As a consequence the controversial analogy between thermal waves an electromagnetic waves arises. The main diferences are the following: The propagation of electromagnetic waves in one direction satisfy the wave equation

$$\frac{\partial^2\varphi}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} \tag{12}$$

which is a hyperbolic equation (second-order derivative on coordinates and time), while the temperature distribution satifies a parabolic equation [diffusion equation with first-order dervative on time and second one on coordinates Eq. (2)], c is a constant, φ in the wave equation represents a component of the a vector field while in the heat flux equation the temperature distribution is a scalar field. The electric or magnetic field associated with the electromagnetic wave can be longitudinal or transversal, the temperature cannot be. In general, the time-dependent solution of the wave equation may be written as

$$\varphi(x,t)=\varphi_1\mathrm{e}^{i(kx+\omega t)}+\varphi_2\mathrm{e}^{-\lambda x}\mathrm{e}^{-i(kx-\omega t)}, \tag{13}$$

where $\varphi_{1,2}$ are constants and λ and k are real numbers. This solution of course simply reflects the fact that a finite medium with energy loses will not transmit and reflect the wave without attenuation. The fields die within the sample over a distance λ^{-1}, in metals this distance is called the classical skin depth, for example. On the other hand, the time-dependentce solution of the heat diffusion equation, Eq. (11), is given by

$$T(x,t)=T_1\mathrm{e}^{\gamma x}\mathrm{e}^{i(\gamma x+\omega t)}-T_2\mathrm{e}^{-\gamma x}\mathrm{e}^{-i(\gamma x-\omega t)}, \tag{14}$$

where $T_{1,2}$ are complex numbers and $\gamma=\sqrt{\omega/2\alpha}=L^{-1}$. Although, Eqs. (13) and (14) are mathematically similar, the physical meaning between them are different; the electromagnetic wave is damped. This means that the energy associated with it is absorbed by a quasiparticle system in the medium; while in a diffusion process, the thermal wave generated by the chopped beam light converted into heat at the surface of the solid is also damped. However it is, of course, due to the periodic time variation of the heat at the surface around Q_0, see Eq. (5), any energy dissipation is involved in the diffusive processes. Another important difference is the change on sign in the solution of the diffusion equation, Eq. (13) as compared with Eq. (14); in the wave problem the total flux density of energy in the sample is the difference of the flux density energy of both, the transmited and reflected waves and it is proportional to $|\varphi|^2$ (Poynting vector), while in the heat diffusion process is the sum, it corresponds to the rate at which heat is generated or absorbed at various places in the sample and because $Q(x,t)=-k\partial t/\partial x$ the energy heat flux can be positive or negative at different points in space at different times. Finally, its important to remark that the following inequality $k>\lambda$ is always fulfilled in the solution of the wave problem; in the diffusion of heat they have the same value, see Eq. (14). All these differences mentioned in the text show that the analogy between thermal "waves" and electromagnetic waves are not correct.

5. Temperature frequency dependence

It is worth mentioning that our model is valid for a wide range of modulated frequenciesy of the incident light on the sample. In the limit when $\omega\tau\ll 1$ where $\tau=d^2/\alpha$ is the characteristic time at which the system responds to an external perturbation (this condition is also equivalent to assume $|\sigma|d\ll 1$), Eq. (11) reduces to

$$T(x,t)-T_0+\frac{Q_0}{k}(d-x)+\frac{\Delta Q}{k}(d-x)\mathrm{e}^{i\omega t}. \tag{15}$$

In this limit the temperature distribution is quasi-static and independent of the diffusivity of the sample, *i.e.*, from any photothermal experiments only information about the thermal conductivity can be obtained. Eq. (9) may be obtained from the static heat diffusion equation $\partial^2T(x,t)/\partial x^2=0$ and the boundary conditions used earlier, note that at low modulation frequency, the thermal diffusion length in the sample $L=(2\alpha/\omega)^{1/2}$, is large as compared with d.

On the other hand, in the limit when $\omega\tau\gg 1$ (high modulation frequency) or $|\sigma|d\gg 1$, Eq. (11) reads

$$T(x,t)=T_0+\frac{Q_0}{k}(d-x)+\frac{\Delta Q}{h\sigma}\mathrm{e}^{i\omega t-\sigma x}. \tag{16}$$

In this limit the dynamic part of the temperature fluctuation in the sample is smaller than the static temperature distribution part, *i.e.*, $\Delta Q/\sigma k\ll Q_0d/k$ and it attenuates rapidly to zero with increasing distance from the surface such that, at a distance $L\ll d$, the temperature fluctuation is effectively damped out. Another different but equivalent manner to analize this temperature frequency dependence is the following: from Eq. (11), the amplitude of the heat modulation is $\Delta Q=-kd\Theta/dx\propto\Theta/L$, in the high limit frequency modulation, $L\to 0$ and because ΔQ is finite then $T\to 0$. The thermal diffusion of a two-layer system for high modulation frequency has been analysed in Ref. 11. At this frequency, the system cannot respond to this external perturbation and essentially the temperature distribution into the sample is static.

6. Conclusions

In conclusion, a theoretical analysis of heat diffusion in one temperature approximation has been studied. Using the appropiate boundary conditions, respect to the usual photothermal experiments, we obtain the temperature distribution in the sample. The thermal waves generated into the specimen by the incident modulated radiation are analaysed in terms of the characteristic time of the system $\tau=d^2/\alpha$ according to the modulation frequency. In the limit of high modulation frequency the temperature fluctuation in the sample is small as compared with the static contribution while in the limit of the low frequency, the temperature distribution in the sample fluctuates in a quasi-static variation on time, and is independent of the thermal diffusivity, it only depends on the thermal conductivity.

Finally it is important mentioning that the one-layer model used in this work, actually represents an effective medium in the photothermal experiments, *e.g.*, in photoacoustic experiments the layer represents the sample to be analaysed, the gas chamber and the substrate. Our theory allows us to obtain the effective thermal parameters of the sample. Finally, direct comparison of the diffusion and the wave equation have been made and the main differences between heat flux in solids and the propagation of electromagnetic waves have been described here. Each physical situation must to be described and interpreted independently according to if is a wave problem or diffusive process. In summary, there is not analogy between propagation of electromagnetic wave, for example, or thermal waves.

Acknowledgments

This work is partially supported by Consejo Nacional de Ciencia y Tecnología (CONACyT). One of us (GGC) would like to thank the Centro de Investigación en Ciencia y Tecnología Avanzada del Instituto Politécnico Nacional (CICATA-IPN), for financial support.

1. J. Cranck, *The mathematics of diffusion*, (Clarendon-Press, 1975).
2. H.S. Carslaw and J.C. Jarger, *Conduction of heat in solids*, (Clarendon-Press, Oxford 1959).
3. D. Joseph and Luigi Preziozi, *Rev. Mod. Phys.* **61** (1989) 41.
4. A. Mandelis, *Photoacustic and thermal waves phenomena in semiconductors*, (North-Holland, 1987).
5. H. Vargas and L.C.M. Miranda, *Phys. Rep.* **161** (1988) 43.
6. A. Kulbachinskii *et al. Phys. Rev. B* **52** (1995) 10915.
7. G. Gonzalez de la Cruz and Yu. G. Gurevich, *J. Appl. Phys.* **80** (1996) 1726 .
8. Wang *et al., J. Appl. Phys.* **73** (1993) 7275.
9. E.M. Conwell, *Solid State Physics*, edited by H. Ehrenreich, F. Seitz, and D. Turnbull, (Academic Press, New York, 1967).
10. A. Rosencwaig and A. Gersho, *J. Appl. Phys.* **47** (1976) 64.
11. G. Gonzalez de la Cruz and Yu.G. Gurevich, *Phys. Rev. B* **51** (1995) 2188.
12. V.A. Sablikov and V.B Sandomirskii, *Phys. Status Solidi B* **120** (1983) 471.
13. Yu. G. Gurevich and O.L. Mashkevich, *Phys, Rep.* **181** (1989) 328.
14. J. Opsal and A. Rosencwaig *J. Appl. Phys.* **53** (1982) 4240.